Unlocking mathematics’ power: interpreting content and context within word problems

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ABSTRACT

Mathematics is a fundamental subject with wide-reaching importance in education, providing students with the tools to apply mathematical principles in diverse contexts. This study examines the abilities of 60 pre-service mathematics teachers (PSTs) in identifying content and context within mathematical word problems. Utilizing a case study approach, the study employed the mathematics word problems test and the content and context questionnaire. The findings reveal that PSTs generally struggle with error detection and content comprehension in mathematical word problems, as demonstrated by their inability to recognize inaccuracies in two of three test questions. The failure of PSTs to identify errors in mathematical word problems often stems from their tendency to rely solely on the solutions they obtain, without first understanding the entire question presented. In essence, they may focus on finding a solution rather than critically evaluating the problem, which can lead to the oversight of errors or inaccuracies within the problem statement itself. This study emphasizes the need for PSTs to grasp mathematical concepts and contextualize them in everyday life scenarios. Challenges were observed in linking computational results to real-world contexts. Thus, the study calls for future research in pre-service teacher education to explore strategies for enhancing critical thinking, error detection, and the integration of practical context in mathematical problem-solving. Furthermore, the study suggests that assessing the ability of PSTs to formulate problem-solving questions evaluates their capacity to answer questions and their ability to construct questions that can enhance students’ cognitive abilities.

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1. INTRODUCTION

Mathematics plays a fundamental role in diverse facets of human existence, encompassing activities such as commerce, pharmaceutical dosing, software development, and even the exploration of outer space. With the rapid advancement of technology and information, the demand for mathematical proficiency is on an exponential rise [1]. Despite the compartmentalization of mathematical topics in school curricula, it is crucial to recognize that mathematics is an intricate web of interconnected concepts, each reinforcing the others [2], [3]. The skill to forge connections between various mathematical ideas and topics, along with their application in non-mathematical domains, is referred to as mathematical connection ability [4]. In the field of mathematical education, mathematical understanding encompasses conceptual understanding (knowledge of
the concept) and procedural understanding (ability to carry out actions to solve problems) [5]. Engaging in learning experiences that foster these mathematical connections not only enhances students’ comprehension of mathematics but also cultivates an appreciation for its practical utility in everyday life [6], [7]. This is particularly significant since previous research has shown an increase in the number of students struggling with everyday mathematics problems.

In the study conducted by Ying et al. [8], it was found that 92.5% of students faced challenges when solving problems with real-world applications, and they struggled to apply mathematical concepts with unfamiliar contexts correctly. Supporting this, Suseelan et al. [9] further emphasized that most students tend to focus solely on the numerical aspects of a question, often failing to grasp the entire context of the problem. This limited perspective frequently led to errors in problem-solving, particularly in cases involving the application of mathematical concepts in real-life situations. Therefore, educators wield a significant influence in nurturing students’ prowess in making mathematical connections by elucidating the interrelationships among mathematical topics, demonstrating how mathematical concepts transcend traditional disciplinary boundaries, and presenting mathematical challenges that inherently involve these intricate connections [4].

Mathematical challenges involving interconnected concepts often manifest in the form of mathematics word problems. These word problems are mathematical quandaries presented within the framework of everyday scenarios [10], comprising a fusion of mathematical content and real-life context. Numerous research studies have illuminated the predicament posed by many mathematics word problems, primarily stemming from either insufficient information [11] or flawed connections between the provided data points [4]. Saleh et al. [2] found a problem posed by the pre-service teacher, namely, “Mrs. Mira owns land in Bogor with a length of 12 cm and a width of 5 cm. What is the total area of the land?” Viewed from the aspect of mathematical content, this problem needs to contain precise information about the geometric shape of the land. However, the use of the length and width of the terms, as well as the prevalence of landforms in problems that are often found in textbooks, indicate that the land is rectangular. Such problems encompass extraneous contexts misaligned with everyday life and are classified as pseudo-problems.

Mathematics word problems, whether marred by information deficits or encumbered by pseudo-contexts, ultimately fall short in aiding students’ comprehension of mathematical concepts within real-world contexts. Furthermore, according to Amalia and Nuriadin [12], these problems fail to instill an appreciation for the practical application of mathematics in the tangible world, potentially fostering the misconception that mathematics and real-life situations exist in isolation. High-quality mathematics word problems should incorporate a diverse range of topics, encompass crucial mathematical principles, and possess significance for students [13]. Consequently, teachers, including those in the pre-service phase, must excel in discerning the mathematical content and the contextual relevance within mathematics word problems [14]. The primary objective of this research is to scrutinize the pre-service teachers (PSTs) proficiency in identifying the content and context embedded in mathematics word problems. This study seeks to provide a comprehensive evaluation of the capabilities of PSTs, laying the groundwork for potential enhancements in their competence.

2. RESEARCH METHOD

The primary objective of this research is to undertake a comprehensive examination of the PSTs capacity to recognize both the content and context within mathematical word problems. The study’s participants consisted of pre-service mathematics educators affiliated with Universiti Teknologi Malaysia (UTM). In addition to being one of the institutions providing professional education programs in mathematics, UTM was also selected as the site of research since the research being conducted is a collaboration between UTM and the University of Muhammadiyah Makassar. It is essential to clarify that these participants were not selected for comparative purposes but were chosen to serve as illustrative examples of proficiency in identifying content and context within mathematical word problems. The research adopts a case study approach, specifically of the instrumental case type as classified [15]. This case study methodology is employed to elucidate and provide a clear understanding of a particular issue or problem. The selection of research participants followed a critical sampling approach as outlined [15]. Careful consideration was given to the selection of PSTs, who were then subject to in-depth analysis to delineate their abilities in identifying content and context within mathematical word problems.

The research unfolded in three distinct stages: preparation, data collection, and data analysis. The preparatory phase marked the commencement of the research and involved the development of preliminary research instruments. These instruments encompassed the mathematics word problems test and content and context questionnaires. Collaboration among mathematics and mathematics education researchers at UTM was pivotal in refining the instrument drafts. The researcher's observations of the difficulties PSTs encountered in formulating and solving mathematical questions presented in word form led to the development of the mathematics word problem test in this research. Based on the observations, the

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researchers concluded that this word problem test will concentrate on several issues, including fractions that are inconsistent with addition operations, problems unrelated to daily life, and word problems with insufficient details. The researchers also referred to several additional research [2], [11] to make sure that the questions aligned with the issues previously explored in the research. Furthermore, the word problem test was initially tested on various groups of PSTs in a pilot study to determine its reliability.

Meanwhile, the content and context questionnaire consisted of two categories of items: one aimed at evaluating the understanding of mathematical concepts, and the other focused on assessing the comprehension of how mathematics relates to everyday life. Further validation of the instruments was undertaken by content experts, and subsequent revisions were made based on their recommendations until the instruments were deemed both valid and suitable for implementation. Following data collection, the analysis phase unfolded concurrently in three stages: data condensation, data display, and the drawing and verification of conclusions guided by the framework presented [16]. To ensure data validity and reliability, the triangulation method was applied, and member checking was employed to verify the findings.

In this research, two groups comprising 60 PSTs enrolled in the Bachelor of Science in Education (Mathematics) Honors program at UTM were chosen by using a purposive sampling approach as the study participants. Since the primary aim of this investigation was to evaluate the proficiencies of these PSTs in discerning both the context and content within mathematics word problems, the selection criteria for research participants included undergraduate students majoring in mathematics education, as well as first and second-year students. According to Douglas [17], a non-probability method called purposeful sampling is used to specifically choose the most suitable sources of information to fulfill the study's objectives. Therefore, in this study the selection of participants allowed the researcher to examine how the semester of study influenced the PSTs capacity to evaluate the word problem test. In pursuit of the research objectives, the identified respondents were tasked with completing two assessment tools: the mathematics word problems test and the content and context questionnaire. Furthermore, demographic information of the sample, including details such as the semester of study, was systematically collected and subjected to descriptive analysis. Table 1 presents the frequency and percentage distributions for each semester of study to provide researchers with a comprehensive overview of the sample composition. The data presented in Table 1 reveals that the sample of 60 PSTs included in this study originated from two distinct semesters: the second and the fourth. To be precise, 29 PSTs, constituting 48.33% of the total sample, were enrolled in their second semester, while the remaining 31 PSTs (51.67%) were in their fourth semester.

![Table 1. Distribution of PSTs by semester of study](image)

<table>
<thead>
<tr>
<th>Semester</th>
<th>Frequency (f)</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>29</td>
<td>48.33</td>
</tr>
<tr>
<td>4</td>
<td>31</td>
<td>51.67</td>
</tr>
<tr>
<td>Total</td>
<td>60</td>
<td>100</td>
</tr>
</tbody>
</table>

3. RESULTS
3.1. Understanding real-world contents through mathematical word problems

A mathematics word test featuring three distinct problems was administered to pre-service mathematics teachers. Each question highlighted a unique mathematical concept and integrated real-world contexts that encountered challenges, including issues such as incomplete information and difficulties in connecting the provided data with the question's requirements. The primary objective of this test was to evaluate the PSTs competence in recognizing and comprehending both the context and content of mathematical word problems.

The analytical process involved categorizing PSTs into distinct groups based on their proficiency in identifying the mathematical content embedded within the problems. These categories encompassed the following: i) ‘Im’-representing those PSTs who could promptly identify incorrect mathematical information in a problem and, refrained from answering due to the detected error; ii) ‘Ti’-comprising PSTs who exhibited the ability to recognize mathematical content through the solutions they devised; iii) ‘Naw’-encompassing those PSTs who failed to identify inaccurate mathematical information within the questions; and iv) ‘N/A’-designating PSTs who were unable to provide an answer to the presented question. Figure 1 presents illustrates the outcomes of the analysis, providing insights into the PSTs competency in recognizing mathematical content within the context of all three questions.

Figure 1 provides a visual representation of a trend observed among PSTs in their ability to detect mathematical content-related errors. The first question exhibited the highest percentage of the ‘Naw’ category, standing at 96.67%. This category encompassed 29 PSTs from each semester, signifying that all second-semester PSTs were unable to accurately discern the mathematical content within the first question.
The primary aim of the first question was to assess comprehension in calculating the perimeter of a plot of land. However, the provided side lengths did not conform to the triangular inequality theorem, meaning they could not form a valid triangle, rendering the calculation of a perimeter infeasible. Nonetheless, as indicated by the data from Figure 1, a total of 58 PSTs (96.67%) remained oblivious to the erroneous mathematical information in the problem and proceeded to solve it utilizing the perimeter formula. Notably, as shown in Table 2, some of the justifications provided by the respondents implied that the mathematical problem could be resolved if the perimeter formula was applicable concerning the calculation of a land parcel's perimeter that did not adhere to the triangular inequality theorem. Consequently, the formation of a valid triangle, and consequently the calculation of its perimeter, was rendered impossible.

![Figure 1. Distribution of PSTs abilities in identifying mathematical content](image)

**Table 2. Responses from PSTs in identifying the mathematical content**

<table>
<thead>
<tr>
<th>No of PSTs</th>
<th>Im</th>
<th>Ti</th>
<th>Naw</th>
<th>N/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**3.2. Understanding real-world contexts through mathematical word problems**

In this research, we delve into the notion of real-world context embedded within mathematical word problems, which relates to elements of everyday life seamlessly integrated into the presented problems. Specifically, the mathematical word problems employed in this study are intricately linked to tangible real-life scenarios, encompassing situations such as designing a garden, establishing a catfish farming pond, and personal life contexts. These three distinctive contexts were chosen to evaluate the PSTs' ability to discern extraneous details within the everyday life contexts presented within mathematical word problems. Figure 2 furnishes the percentage distribution of students categorized into three distinct groups: 'Aw' (aware), 'Naw' (not aware), and 'N/A' (not applicable). The criteria used for categorizing PSTs under 'Naw' and 'N/A' align with those previously employed in the preceding section, where we assessed teachers' capability to identify mathematical content.

Figure 2 and Table 3 provide insights into the percentage of PSTs who demonstrated the ability to discern intricacies within real-life contexts presented in mathematical word problems. Notably, the results reveal a lower percentage for the first question (30%) when compared to the second (43.33%) and third (60%) questions. The first question revolves around the units assigned to each side of the land that are deemed impractical and excessively small to create a garden. The study found that the majority of second-semester PSTs failed to recognize this peculiarity and considered the question to be pertinent to real-life scenarios. In contrast, fourth-semester PSTs exhibited a heightened level of 'Aw' concerning the incongruity related to the units used in the first question. Specifically, 17 fourth-semester PSTs fell into the 'Aw'
category, while only 13 were classified as 'Naw'. Significantly, the semester of study emerged as a crucial factor, with fourth-semester PSTs displaying greater accuracy in identifying the impracticality within the first question compared to their second-semester counterparts.

![Figure 2](image.png)

**Figure 2. Percentage distribution of PSTs' abilities in identifying real-world contexts**

**Table 3. Responses from PSTs in identifying the real-world context**

<table>
<thead>
<tr>
<th>Mathematics word problem</th>
<th>NO.1</th>
<th>NO.2</th>
<th>NO.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO.1</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO.2</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO.3</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. **DISCUSSION**

Effective mathematics instruction hinges significantly on a deep comprehension of mathematical content [18]. As highlighted by Tran et al. [19], this foundational understanding is particularly critical for PSTs to enable them to deliver instruction that is clear and comprehensive. Beyond mastering core content, cultivating critical thinking in students often involves the utilization of word problems in real-life contexts. Consequently, the training of PSTs with a dual emphasis on mathematical content and real-world context ensures that they impart not only the mechanics of mathematics but also its practical relevance, logical reasoning, and overarching significance [20]. The study assessed PSTs' understanding of mathematical word problems, specifically focusing on the application of geometric concepts and problem-solving approaches.

In the first and second questions, which involved the triangular inequality theorem, 96.67% of PSTs demonstrated a lack of awareness of the theorem's violation in the first question, while only 50% recognized an error in the second question. The study highlighted the influence of derived answers on PSTs' ability to detect errors, emphasizing a potential gap in their comprehension of underlying mathematical principles. This aligns with prior research [21], [22], indicating challenges in translating word problems and applying appropriate mathematical concepts.

Furthermore, given that the triangular inequality theorem falls within the domain of geometry concepts, Dewi and Asnawati [23] emphasized the significance of proficient geometric skills, which encompass not only the ability to articulate geometric concepts with confidence but also the competence to utilize and derive the relevant theorems effectively in mathematical problem-solving. The study suggests the importance of a holistic understanding, emphasizing the multifaceted nature of problem-solving skills. Comprehension of the whole word problem is rooted in the foundational principles being emphasized, rather than being solely reliant on the derived answers. This perspective aligns with study by Pongsakdi et al. [24] who contend that while solving word problems, abilities, including text comprehension, computational proficiency, and the aptitude to establish connections between the provided narrative and pertinent mathematical content, are indispensable. This multifaceted approach underscores the value of holistic understanding, transcending mere computation, as underscored [25].
Furthermore, another critical competence that evolves when dealing with word problems is the ability to choose the appropriate problem-solving strategy. One highly effective approach center on the utilization of visual representation. The study's findings shed light on four PSTs, who adeptly identified errors in the third question by employing visualization techniques. As demonstrated, these graphical tools can aid PSTs in simplifying complex word problems, making them more digestible and enhancing their ability to spot any inconsistencies within the question. However, while this reliance on visuals suggests a proficiency in spatial skills among PSTs, Özsoy [26] indicated that a significant portion of PSTs encounter challenges in effectively translating word problems into visual representations. Furthermore, Barham [27] has added that to convert narrative details swiftly and accurately into visual formats, strong text comprehension remains vital, even when utilizing a visualization approach. This underscores the fundamental concept that the adoption of any problem-solving method hinges on a comprehensive understanding of the problem.

In addition to evaluating the understanding of mathematical content, the study also delved into PSTs capacity to detect inconsistencies within real-life contexts. Results showed a relatively low percentage of PSTs capable of identifying incongruities in the first question (30%), compared to higher percentages in the second (43.33%) and third questions (60%). The first question involved creating a garden on triangular land with impractical small units, and the majority of second semester PSTs failed to recognize the incongruity. Those who displayed awareness noted that the unit, centimeter, used for the garden’s construction was unsuitably minuscule for the task. According to Brown [28], questions of this nature can be categorized under the tapestry category requiring PSTs to navigate between tangible situations and abstract mathematical concepts. While the computed perimeter for a land parcel may make sense from a mathematical standpoint in an abstract sense, the implausibility becomes evident when considering the real-world units involved. Therefore, such problems call for meticulous scrutiny and put the PSTs ability to recognize and evaluate in both the mathematical and real-world domains. This underscores the importance of comprehensive comprehension, where mathematical solutions must align with their context, as emphasized [29].

In addition to the mathematical content findings, the study observed PSTs awareness of the peculiarities in the second question, where a pond with a side length of 0 was impossible. Despite recognizing this issue, a higher percentage of PSTs fell into the ‘N/A’ category for the second question compared to the first and third questions. The PSTs acknowledged that the second question did not appear irrelevant when considered in the context of daily life but failed to explain, indicating an inability to identify the incongruity within the presented context. These results highlight the importance of PSTs reasoning abilities in solving word problems, emphasizing the need for logical justifications, a skill that may often remain at an intermediate level according to previous research [30], [31]. Furthermore, in the context of the third question, the study revealed that a higher percentage of PSTs, totaling 60%. These PSTs, categorized under the ‘Aw’ classification, identified the illogical nature of the word problem due to the inclusion of irrelevant data, specifically the sum of fractions exceeding the initial quantity of one plate of beans. Their ability to establish a connection between this incongruity and the real-life context suggests that PSTs may be more proficient at connecting with real-life contexts closely aligned with their everyday experiences, such as eating, while potentially facing limitations in generalizing mathematical concepts across a broader spectrum of real-world scenarios.

5. CONCLUSION

In conclusion, the justifications provided by PSTs are heavily reliant on their problem-solving approaches and are intimately linked to their comprehension of the involved mathematical content. PSTs who can accurately identify the mathematical content also demonstrate greater proficiency in recognizing the intricacies and incongruities within the presented mathematical problems when viewed through the lens of everyday life. Furthermore, this study underscores the critical necessity for PSTs to possess a dual capability: a thorough understanding of mathematical concepts and the capacity to contextualize these concepts within real-life scenarios. Therefore, enhancing these competencies among PSTs holds significant potential for improving their effectiveness as educators.

The importance of future mathematics educators comprehending mathematical concepts within real-life contexts is twofold. Firstly, it enables them to design complex mathematical problems that can significantly enhance students’ understanding and problem-solving abilities. Secondly, the findings from this study offer practical insights into improving teacher training programs and curriculum design, particularly in mathematical education. This has implications for both educational practice and policy, emphasizing the need to bridge the gap between mathematical theory and real-world applications in teaching and learning programs for PSTs. Thus, university-level curricula need to place a stronger emphasis on integrating concepts that bridge the gap between mathematical theory and real-world applications in teaching and learning programs for PSTs. Ultimately, equipping PSTs with the skills to understand and teach mathematics within real-life contexts can result in more engaging and effective mathematics instruction, leading to improved student learning.
learning as they recognize the practicality of mathematical concepts in real-world situations. Indeed, the current study has its limitations that should be addressed in future research. Future studies could provide the word problem test that focuses on a deeper assessment of the cognitive challenges faced by PSTs. This would provide a more comprehensive understanding of their problem-solving skills and their proficiency in formulating mathematical challenges.

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