Research on mathematical beliefs: systematic literature review

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ABSTRACT

This study uses the systematic literature review (SLR) method to identify knowledge trends in beliefs about the nature of mathematics, learning mathematics and identify new development opportunities for further research. The Scopus database was used, and 41 documents were found as the primary source for further analysis. The results show that mathematical beliefs have become a current research trend. The number of publications increased significantly starting in 1989, accompanied by many citations. Based on these primary documents, it is known that the development of mathematical beliefs research topics. The foundation of this topic begins with studying the topic of knowledge, activities, and learning mathematics. Then it is developed into other, more varied studies, including teacher’s beliefs transition; mathematical beliefs comparison; mathematics teacher’s belief, teaching, and learning; mathematical beliefs and problem-solving; inquiry-based learning (IBL); affective factors; technology utilization; common misconceptions; and development of mathematics teacher’s beliefs instrument. This research investigates the core of scientific work to provide information to researchers and institutions as material for consideration of research to be carried out.

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1. INTRODUCTION

In the educational process, teachers are crucial [1]. Teachers’ beliefs and knowledge of the pedagogical content determine how effectively they communicate lesson content [2]. This includes knowledge of students’ learning styles, teaching and learning strategies, and content mastery [3], [4]. Teachers’ beliefs and commitment to pedagogical practice in the classroom are linked to using appropriate teaching methods and simplifying instruction [2], [5].

Mathematical beliefs about the discipline and how it is learned and taught [6]. The teacher’s experience as a student and the teacher's influence at school shape initial beliefs. Beliefs in mathematics as a discipline, beliefs about teaching, and beliefs about learning mathematics can all be classified [4]. Ernst, as quoted [6], [7] stated three philosophies about mathematics. The first is the Instrumental viewpoint, which holds that mathematics is a set of facts, rules, and abilities. The second is the Platonistic view of mathematics as static and a synthesis of other fields of knowledge. Mathematics was invented, not created. Third, mathematics is dynamic, namely the continuous development process resulting from the human creation. Mathematics is a constant process of knowing, and there is no end.

In addition, four categories of mathematical beliefs are held by students [8]. First, the notion that mathematics is complicated or constrained by rules is one belief. Second, self-confidence is the belief that one
can learn mathematics and succeed or fail. Thirdly, beliefs about teaching include notions of how to assist students in learning mathematics. Fourth are socially relevant beliefs. The belief that mathematics education is competitive and that parents or other outsiders influence it falls under this category. Systematic research is being conducted on teachers’ teaching beliefs and actual practice [2]. According to one group of researchers, teachers’ beliefs, as well as their teaching and learning values, influence their teaching practices [1], [2], [4], [7]. Mathematics is heavily influenced by mathematics learning and teachers’ beliefs about mathematics and how students are taught.

As explained, mathematics education researchers’ contribution to understanding teachers’ beliefs and knowledge is known. In addition, it is also known that this topic has only begun to be massively researched in the last decade. Unfortunately, until now, there is no factual data regarding the many publications and mapping of topics that have been investigated. Academics need to get a map of the development of studies on this topic from year to year. Therefore, it is necessary to examine the latest effects of studies on beliefs about the nature of mathematics, teaching, and learning mathematics.

### 2. RESEARCH METHOD

The systematic literature review (SLR) method is used in this study, which aims to identify knowledge trends in the subject of beliefs about the nature of mathematics, mathematics teaching, and mathematics learning and identify new development opportunities for further research [9]–[12]. In SLR, the systematic mapping study (SMS) method is the initial stage. The SMS method is more descriptive than supported by the SLR method, which is exploratory and exploitative and provides adequate transparency and replication as a research method [11]. However, this article only presents SLR so the research topic can be explored in depth.

There are five research stages: keyword selection, data search, article selection, data validation, and data analysis. Before entering the research phase, the Scopus database was chosen because it is broad in scope, credible, and provides access to a collection of essential information for this research, such as titles, abstracts, and keywords [13], [14]. The keywords “nature of mathematics,” OR “learning of mathematics,” OR “teaching of mathematics,” AND “beliefs” were used in the Scopus database search (i.e., in titles, abstracts, and keywords). This search resulted in 222 documents which were then reduced with various considerations, namely: i) only English documents were selected to facilitate content analysis; and ii) only documents in the form of articles of journals or proceedings were selected because they went through a rigorous review process and had been empirically validated. Based on the reduction, 178 documents were obtained in 168 journal articles and 10 proceeding articles in English.

Furthermore, the examination and selection of titles and abstracts are carried out following the topics studied. It was found that seven documents did not match the topic, so they were removed from the research database, leaving 171 documents in the form of 163 journal articles or eight articles of proceedings. Table 1 shows the document selection stage. The data search results are then selected and validated to read and analyze the data. The primary sources selected based on the SMS method were further analyzed by the SLR method using NVivo to identify new original development opportunities for further research. Coding on the primary source is done to form categories, then presented in tables or diagrams and interpreted.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Description</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1</td>
<td>Select database: Scopus.</td>
<td>-</td>
</tr>
<tr>
<td>Phase 2</td>
<td>Search the database with four keywords: “nature of mathematics,” OR “learning of mathematics,” OR “teaching of mathematics,” AND “beliefs.”</td>
<td>222 documents</td>
</tr>
<tr>
<td>Phase 3</td>
<td>Select only English article publications.</td>
<td>205 documents</td>
</tr>
<tr>
<td>Phase 4</td>
<td>Select only publications in the form of journal articles or proceedings.</td>
<td>178 documents</td>
</tr>
<tr>
<td>Phase 5</td>
<td>Check the title and abstract according to the topic being studied.</td>
<td>171 documents</td>
</tr>
</tbody>
</table>

### 3. RESULTS AND DISCUSSION

#### 3.1. Primary document

The primary document is chosen based on the number of global and local citations. Citations to all documents in the Scopus database are considered global citations, whereas citations to specific documents are considered local citations in this research database (171 documents). Therefore, every document that gets local citations must also get global citations, not vice versa. In Table 2, 41 documents have received local citations and global citations. So, documents that get global citations only and documents that do not get citations are set aside. The documents in this research are in the form of journal articles or proceedings. Stipek et al. [15] received the most local citations (10 times) and most global citations (375 times). Therefore, this article

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becomes the primary reference on this topic. Another article by Beswick was followed by 7 local and 99 global citations [4]. The third position, Felbrich et al. [16] received four local and 33 global citations. Besides these three articles, 38 other articles can be considered primary documents. These essential documents are further explored in greater depth through the SLR [11], [17].

### Table 2. Local and global citation documents

<table>
<thead>
<tr>
<th>Document</th>
<th>Year</th>
<th>Local citation (LC)</th>
<th>Global citation (GC)</th>
<th>LC/GC ratio (%)</th>
</tr>
</thead>
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<td>10</td>
<td>375</td>
<td>2.67</td>
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<td>7</td>
<td>99</td>
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<td>33</td>
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</tr>
<tr>
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<td>3</td>
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<tr>
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<td>2016</td>
<td>3</td>
<td>18</td>
<td>16.67</td>
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<tr>
<td>Viholnamen, 2014</td>
<td>2014</td>
<td>3</td>
<td>18</td>
<td>16.67</td>
</tr>
<tr>
<td>Zakaria, 2010</td>
<td>2010</td>
<td>3</td>
<td>10</td>
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</tr>
<tr>
<td>Cooney, 1999</td>
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<td>2</td>
<td>59</td>
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</tr>
<tr>
<td>Barkatsas, 2005</td>
<td>2005</td>
<td>2</td>
<td>50</td>
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<td>Perry, 1999</td>
<td>1999</td>
<td>2</td>
<td>34</td>
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<td>2013</td>
<td>2</td>
<td>31</td>
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<td>Ren, 2018</td>
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<td>17</td>
<td>11.76</td>
</tr>
<tr>
<td>Forgasz, 2002</td>
<td>2002</td>
<td>2</td>
<td>8</td>
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<tr>
<td>Chouinard, 2007</td>
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<td>118</td>
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<td>67</td>
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<td>Liu, 2009</td>
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<td>Benz, 2012</td>
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<td>Smith, 2016</td>
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<td>Paolucci, 2015</td>
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<td>1</td>
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<td>Zakaria, 2012</td>
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<td>Even, 2005</td>
<td>2005</td>
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<td>10</td>
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<tr>
<td>Ruthvien, 1994</td>
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<td>1</td>
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<td>Liebendörfer, 2017</td>
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</tr>
<tr>
<td>Tarmizi, 2010</td>
<td>2010</td>
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<td>5</td>
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<tr>
<td>Safrudiansyah, 2021</td>
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<td>Szydlik, 2013</td>
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<td>Srimaram, 2004</td>
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<td>Muhtarom, 2018</td>
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<tr>
<td>Tounassis, 1997</td>
<td>1997</td>
<td>1</td>
<td>3</td>
<td>33.33</td>
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<tr>
<td>Spangenberg, 2017</td>
<td>2017</td>
<td>1</td>
<td>2</td>
<td>50.00</td>
</tr>
</tbody>
</table>

#### 3.2. Primary document description

This SLR uses the help of the NVivo software. Based on the analysis results of 41 primary documents, it can be mapped into five themes that are often discussed, namely “mathematics”, “teachers”, “beliefs”, “students”, and “learning” as shown in Figure 1. The most dominant theme mentioned was “mathematics” 1,070 times, followed by “teachers” 866 times, “beliefs” 423 times, “students” 300 times, and “learning” 217 times as shown in Figure 2.

Furthermore, to coding articles into themes, articles are also coded into sentiments so that the trend of the direction of the information can be seen. Figure 3 presents the results of the sentiment coding of the article, and it can be seen that positive information appears 2,651 times and negative information appears 1,329 times. So, it can be concluded that the information in the primary document highlights more positive things related to the research topic than negative things. On the positive sentiment, the researchers emphasized mathematics, teaching, knowledge, and students. On the other hand, although with a smaller proportion, researchers also have negative sentiments regarding these things except for knowledge, negative sentiments also arise regarding teachers, beliefs, and perspectives. To see each article in detail, it can be seen in Figure 4.
A positive researcher's sentiment is that teaching is founded on firmly held views about the nature of the subject, student learning, and the teacher's position. Because beliefs shape individual conceptions of and engagement with mathematics, they play an essential role in creating teachers' distinctive instructional behavior patterns [18]. In contrast, the negative sentiment is that pre-service high school mathematics teachers have difficulty putting their beliefs into practice [19]. Three possible explanations, namely: i) the profundity of the teacher's beliefs and their integration with other knowledge and beliefs, particularly pedagogical knowledge; ii) how much teachers reflect on their teaching techniques and the degree to which they are aware of their beliefs; and iii) the impact of the social setting on the actions and behaviors of teachers [20].
3.3. The development of mathematical beliefs research topics

Currently, studies on mathematical beliefs are increasingly in demand by academics. Research into mathematical beliefs emerged in 1901 that linked them to one's emotions [21]. However, this topic was not trending then, and there were no other publications until 1988. So, this topic was not in demand for more than eight decades. The number of documents on mathematical beliefs began to increase significantly in 1989, with a peak in 2021 of 16 documents. Research on mathematical beliefs began to be re-initiated by Ernest [20], who examined the philosophy of mathematics and education. Views of the nature of mathematics are essential to teaching it and can significantly impact students' mathematics curriculum. However, it is necessary to distinguish between views inferred from actual classroom practice and beliefs expressed as a nature of mathematics [20]. Starting from the results of this study, in the following years, this topic began to be of interest to researchers [22]–[25].

Whitman and Lai [22] discussed the beliefs held by teachers from various sociocultural backgrounds about how to effectively teach mathematics: Hawaii, US, and Tokyo, Japan. Although beliefs regarding what constitutes practical teaching share similarities, the differences are more significant. These differences appear to be a reflection of teachers' sociocultural backgrounds. Classroom management, face-saving measures, and responding to individual variances and needs differ. Ernest [25] then presents a model of belief systems related to teaching mathematics and the issue of the contrast between espoused and enforced beliefs. It is similarly argued [23] that the inseparability of mathematical practice from theory means putting specific knowledge into practical effect. Meanwhile, according to widespread opinion, from the perspective of learning technology [24], computers should be used to: i) allow students to create computer programs that imitate well-known mathematical approaches; ii) allow students to reinforce previously taught topics; and iii) explore the micro-world of mathematics, which includes well-known mathematical ideas and concepts.

This study includes exploratory findings from published research on the research subjects of beliefs about the nature of mathematics, mathematics instruction, and mathematics learning. In this study, 41 articles were identified as the primary documents for the study of this topic. Three of them are articles can be seen as the primary reference [4, 15, 16]. Stipek et al. [15] demonstrated significant coherence between teachers' beliefs and their associations with learning practices. The student's self-confidence as a math learner correlates with the teacher's self-confidence as a math teacher. His studies include: i) the nature of mathematics (tools for thinking versus procedures for solving problems); ii) learning mathematics (concentrating on precise responses rather than grasping mathematical principles); iii) who should be in charge of students' mathematical activity; iv) the nature of mathematical aptitude (fixed versus soft); v) the importance of extrinsic rewards for getting students involved in mathematical activities; and vi) teacher confidence and enjoyment of teaching. Further, mathematics teachers' opinions about the nature of mathematics influence their teaching [4].

In most cases, school mathematics is distinct from mathematicians' mathematical pursuits. In a matrix framework that accommodates possible divergent perspectives on school mathematics and the subject, beliefs about mathematics can be helpful. In line with this, Felbrich et al. [16] stated that beliefs are essential to professional ability and are very important for perceiving the situation because they influence our actions. Without ignoring the other 38 documents, which also contributed significantly, these three documents became the primary source for information on this subject. The following are the contributions of other documents to the topic of mathematical beliefs' development.
3.3.1. Knowledge, activities, and learning mathematics

Ruthven and Coe [26] researched mathematical knowledge, activity, and learning. The emergent constructs can provide heuristic benefits for understanding student beliefs. The findings indicate no direct and systematic link between beliefs about the nature of mathematical knowledge and activity and mathematics education. In contrast, how mathematics is taught in schools is heavily influenced by philosophical and epistemological beliefs about the nature of mathematics [27]. The primary conceptual tool used is the model of the two dichotomies of the philosophy of mathematics: absolutism versus fallibilism and its relationship to mathematics pedagogy. Sriraman [28] stated that there is an age-old debate about whether mathematics was discovered or created in the philosophy of mathematics. This debate has four popular points of view: platonism, formalism, intuitionism, and logism. Interviews with five professional research mathematicians were conducted and revealed four mathematicians leaning towards Platonism, which contradicts the popular belief. According to the findings of this study, mathematicians' research methods are influenced by their beliefs about the nature of mathematics, which are, in turn, influenced by their theological beliefs. Liu [29] observed how students’ epistemological beliefs about mathematics developed during a calculus course. The findings indicate that while most students taking this course experience relatively significant shifts in their mathematical epistemological beliefs, the patterns and extents of these shifts vary between groups and individuals. Viholainen et al. [30] examined the epistemological beliefs of mathematics teaching assistants on the nature of mathematics and the aims of teaching and learning mathematics. Teaching assistants frequently view mathematics as a static system. Nevertheless, the following characteristics are present in their conceptions of the objectives of mathematics instruction: application-related, process-related, schema-related, and formalism-related issues. Because education can impact future teachers' beliefs, it is essential to acknowledge the results.

3.3.2. Teacher’s beliefs transition

Cady et al. document the progress of mathematics teachers as they move from being in-service to being experienced [31]. Teachers’ epistemological perspectives and beliefs towards mathematics teaching and learning have evolved. Nevertheless, many participants do not carefully examine students’ thought processes when making instructional decisions. As previously taught, changes in teachers’ beliefs and practices have exceeded expectations and resulted in effective teachers. A similar study was conducted [32] comparing the beliefs of mathematics majors and faculty about the behavioral nature of mathematics at the beginning and end of an inquiry-based mathematics course with those of general education students. The initial survey’s findings align with those of previous studies of the same group, such as elementary school teacher candidates; the subject is viewed by participants as a collection of loosely connected facts and procedures. However, the statistics demonstrate that general education students’ mathematical ideas changed dramatically by the end of the course; general education students scored similarly to mathematics majors post-survey. This implies that the theme beliefs of these students are flexible. Paolucci [33] considered the ability of advanced mathematical studies to impact pre-service teachers' beliefs about mathematics. According to the findings, many teacher candidates’ beliefs continue to reflect narrow interpretations of critical terminology and a lack of respect for the theoretical and conceptual networks that underpin secondary mathematical rules and procedures, even after completing a degree that includes advanced mathematical studies.

Additionally, many of their beliefs about mathematics fail to understand its ability to foster creative and analytical thinking. In circumstances where this is the case, this study analyzes the role of advanced mathematics coursework for pre-service teachers in generating well-developed beliefs. Liebendörfer and Schukajlow [34] examined the development of first-year student interest in junior high school teacher programs and the relationship between belief systems and interests. The findings demonstrated that: i) students’ interest in mathematics remained stable throughout the first academic year; ii) beliefs in the application were positively correlated with interest in the first semester but not in the second; and iii) beliefs in the application at the start of the semester predicted student interest at the end of the semester in the second semester but not in the first. Furthermore, process beliefs, schemas, or formalism did not predict the first or second periods of interest. These studies are about the impact of belief systems on interest and possible consequences based on distinctions between school mathematics and university mathematics.

3.3.3. Mathematical beliefs comparison

Perry et al. [35] reported an investigation of teachers’ beliefs regarding the nature of mathematics and the learning and teaching of mathematics. This study seeks to compare the beliefs held by the head mathematics teacher with those held by the class mathematics teacher at the same school. A different subject [18] was investigated, reporting studies on the history of the teacher’s professional life and her beliefs about the subject content necessary for teaching and learning mathematics. As a result of firmly defined national viewpoints on education in general and mathematics education in particular, the two groups demonstrated significant differences. Teachers in the UK tend to see mathematics as a number that can be applied and a means by which
students prepare for the world outside of school. Teachers prioritize mathematics in Hungary as a problemsolving and logical reasoning skill. Spangenberg and Myburgh [36] compared male and female pre-service instructors' perceptions of mathematics' nature. While both genders hold the same Platonic and instrumentalist beliefs, women adhere to lesser experimenter beliefs than men.

Meanwhile, the relationship between mathematics and everyday life, and perspectives on the development of mathematical knowledge should be the focus of study on potential teachers' opinions about the nature of mathematics [37]. According to the findings, most instrumentalist pupils believe mathematics is an exact science constituted of laws that do not change, are proven to be accurate, and are not interconnected but valuable in life. That knowledge of mathematics is fixed and unchanging. This belief is founded on the notion that mathematics is a precise science. Others believe that mathematics is constantly changing, creative, and dynamic.

### 3.3.4. Mathematics teacher’s belief, teaching, and learning

Cooney [38] discussed how teachers learn and teach mathematics and how these methods relate to their professional development. The teacher's mathematical experience must match the type of teaching expected of a reflective and adaptive teacher. Next, Barkatsas and Malone [19] investigate mathematics instructors' views on the nature of mathematics and its teaching and learning. It looks into the numerous links between these ideas and learning techniques. According to the data, teachers' ideas about learning and teaching mathematics were less traditional than their actual teaching practices. Focusing more on assessment techniques, it examines two issues related to teachers' expectations to use contemporary assessment techniques [39]. Teachers' comprehension of assessment data is the first issue. The teacher's interpretation of the student's math understanding, knowledge, and learning refer to a vast knowledge base of beliefs, attitudes, and understandings.

Consequently, there is ambiguity and difficulty in the students' mathematical understanding sense-making process. The second issue involves assisting educators in implementing modern assessment methods. It would appear that modern assessment tools receive attention, but this is linked to traditional assessment goals. Aspects of relearning assessment are highlighted [40]. Still, the link between mathematics instructors' beliefs and how they organize classroom activities, interact with students and evaluate student learning is investigated in more complex situations. The findings revealed that beliefs largely influence teachers' day-to-day pedagogical decisions. Their assumptions regarding pedagogy and student learning are primarily based on their assumptions regarding the nature of mathematics.

On the aspect of the learning approach, Zakaria and Musiran [41] analyzes teacher apprentices' assumptions about the nature of mathematics, teaching, and learning. According to the study, the intern teacher's beliefs lead to a constructivist approach. According to intern teachers, math problems can be answered in various ways. Furthermore, intern teachers feel that for pupils to study mathematics, they must understand its concepts, principles, and procedures. Mathematical instruction should include opportunities for students to apply their knowledge to real-world scenarios. Finally, the data revealed significant gender disparities in attitudes regarding the nature of mathematics and studying mathematics. Zakaria and Maat [42] further investigate secondary school mathematics teachers' beliefs and teaching practices; researchers did not consistently report the relationship between the two. In terms of mathematical beliefs, there was no difference between teachers with less experience and those with more experience. Their teaching methods and beliefs in mathematics are correlated in a moderately significant way. Teachers will be more likely to use positive and efficient teaching methods if they have reasonable beliefs about mathematics.

Meanwhile, Purnomo et al. [43] investigated the relationship between pre-service elementary school teachers' views and mathematics classroom learning practices. The research findings indicate that instructional approaches may not necessarily mirror the views espoused. However, assumptions about the nature of mathematics have the most significant influence on learning processes.

### 3.3.5. Mathematical beliefs and problem-solving

Classroom discursive practice greatly influences students' mathematics learning [44]. While shared macro cultural values and beliefs may frame classroom social interactions similarly, meta-discursive rules in classroom microculture dictate opportunities for student learning in mathematics based on similarities and differences across the three classrooms. Siswono et al. [6] investigated secondary school teachers' ideas regarding three mathematical concepts: the nature of mathematics, mathematics teaching and learning, and mathematical problem-solving expertise. All teachers do not consistently respond to their students' opinions on math-related ideas and have shortcomings, particularly in problem-solving topic knowledge. Furthermore, the data demonstrate that instructors' opinions strongly relate to their problem-solving knowledge. Instrumental teachers' beliefs are primarily compatible with their lack of problem-solving knowledge.

However, the attitudes of platonic and problem-solving instructors are congruent with their reasonably firm grasp of pedagogical content or problem-solving. Safrudianmurr and Rott [45] investigate how students' skills influence instructors' ideas about teaching, learning, and problem-solving mathematics, as well as the
relationship between these beliefs and teachers' opinions about mathematics. The findings reveal that teachers' opinions on teaching and learning can change in response to the abilities of their students. They also identified a correlation between teachers' opinions about teaching and learning mathematics with low-ability students rather than high-ability pupils.

3.3.6. Inquiry-based learning

Engeln et al. [46] stated that schools must adopt a pedagogical change to overcome science and mathematics teaching shortcomings and enhance literacy standards in these disciplines. According to teachers, inquiry-based learning (IBL) is the preferable technique for characterizing the current state of IBL in everyday instruction and assessing the challenges instructors expect when implementing IBL. They provide a subset of the study's findings for the first time, which provide an overview of teachers' attitudes and reports on implementing current IBL techniques in a European environment. From the perspective of a practicing teacher, the findings facilitate cross-cultural comparisons of IBL's potential and challenges. Furthermore, this research reveals significant differences in how science and mathematics are taught. The findings of a baseline study can be used to assess the effects of interventions on improving educational quality. Dorier and García [47] investigated the conditions and constraints that may aid or hinder the widespread adoption of inquiry-based science and math education. In this research method, the teacher is viewed as an institutional actor embedded in the educational system, represents various fields, and shares some societal pedagogical concerns. The approach is structured around the four layers of institutional organization that influence the content and didactic features of mathematics and science teaching: discipline, education, society, and pedagogy. Instead, they investigate how conditions and restrictions operate, explain their primary findings, and draw numerous views utilizing their four stages of didactical determination.

3.3.7. Affective factors

Liljedahl [48] researched the impact of the AHA experience. The positive emotions accompanying such enlightenment are examined explicitly for their role in altering the beliefs and attitudes of "hold" students. Pre-service teachers who believe they are disabled or afraid of math and learning it are required as a prerequisite for admission to a teacher preparation program to take an undergraduate mathematics course. According to the findings, AHA had a transforming influence on students' affective domain, fostering positive attitudes and beliefs about mathematics and their ability to do it. The basis of mathematical achievement behavioral models includes social agent support, competence beliefs, utility value, and achievement targets [49]. The data suggest that effort in mathematics is mainly explained by a belief in competency and achieving goals.

Regarding the function of social agents, the findings indicate that teacher support acts on competence beliefs, while parental support perceptions primarily explain mathematics assessment-related variables. The goal of completeness significantly influences students' efforts to learn mathematics. In mathematics, age and gender had no significant impact on the nature or strength of the relationship between competence beliefs, utility value, goal attainment, and effort. Tarmizi [50] investigated the teaching and learning of mathematics, particularly its beliefs, attitudes, and values. These affective factors include, among others, beliefs about learning contexts, how social context affects mathematics performance, motivational beliefs, mathematical competence, and beliefs about mathematics. Three significant predictors were found to impact students' perceptions of their mathematical competence significantly: gender, grades in mathematics, and students' beliefs regarding teacher characteristics for teaching mathematics. Phelps [51] investigates the resources that pre-service teachers use to develop their learning goals and self-efficacy beliefs, which shape their motivational profiles. According to the findings, participants' past performance, vicarious experiences, verbal persuasion, career goals, and unity between participants' views of mathematics and the nature of mathematics in the classroom were among the sources used to construct their efficacy beliefs and goals. Benz [52] evaluated the feelings, beliefs, and attitudes of kindergarten teachers about "math," "mathematics teaching and learning," and "mathematics in the early years." The findings present a broader and distinct perspective on early kindergarten mathematics. Ren and Smith [53] investigate the relationship between various teacher characteristics and contextual factors and early elementary school teachers' attitudes toward and beliefs about learning mathematics. The findings revealed a link between teacher-centered beliefs, teachers' motivation to learn mathematics, and anxiety. Teachers' mathematical beliefs and attitudes were also influenced by their levels of teacher certification, the number of college mathematics courses taken, and their perception of peer and administrative support. The findings indicate that using math knowledge in the classroom can improve teachers' attitudes and beliefs about math.

3.3.8. Technology utilization

The effects of using computers to teach and learn mathematics are the subject of Forgaz's investigation. Ownership, professional development, perceptions of technology skills, beliefs regarding the efficacy of using computers in mathematics, and data regarding how teachers use computers to teach secondary
mathematics are all related to the findings of this study [54]. Laviczka [55] argued that teachers' beliefs and conceptions about using technology in teaching are critical factors in understanding why technology integration is slow. The focus has shifted away from learning and technology and toward teachers' beliefs and conceptions. The extensive use of computer algebra systems (CAS) in research and teaching by mathematicians, as well as the documentation of teaching practice and the conduct of research at this level, will benefit the university and contribute to improved comprehension of technology integration at all levels.

Digital technologies such as multi-representation tools could improve mathematics learning [56]. Due to their potential, there are numerous calls to incorporate such tools into mathematics instruction. Teaching with technology is problematic because it relies on teacher competencies like knowledge and beliefs. Professional development is essential to support teachers' meaningful use of technology and professionalize them. The outcomes revealed: teachers' beliefs about using technology to teach, their self-efficacy in using technology to teach, their epistemological beliefs, and the frequency with which they self-reported using technology using quantitative questionnaires before and after the tests. Teachers' attitudes regarding technology were where professional development programs had the most significant impact. During the professional development program, the experimental group appeared to use technology more frequently over time. Self-efficacy and epistemological beliefs were not found to be affected by the professional development program. Teachers must overcome two obstacles to use technology effectively in the classroom, which is becoming increasingly common: access to resources, including knowledge expansion and rational belief formation [57]. Understanding the connections between these obstacles is essential for teachers to overcome. The relationship between prospective middle-school teachers' technological, pedagogical, and content knowledge (TPACK) framework and their subject beliefs. The outcomes show the connection between their convictions about arithmetic, learning and showing math, and their utilization of innovation and their individual substance information, instructive substance information, and innovation educational substance information.

Bennison and Goos [58] stated that digital technology can improve student mathematics learning and that secondary school mathematics curriculum documents now encourage or require computer and graphic calculator use. However, prior research has demonstrated that teacher knowledge, self-confidence, experience, and beliefs, as well as access to resources and participation in professional development, influence the uptake and implementation of technology in the classroom. Bennison and Goos [58] comprehensively survey secondary mathematics teachers' technology-related experiences and professional development requirements. It was discovered that teachers who had participated in professional development had a greater level of self-assurance when it came to utilizing technology and were more convinced of its advantages in assisting students in learning mathematics. Expert math teachers in large metropolitan schools are more likely than other teachers to attend technology-related professional development. Still, many teachers find that they do not have enough time or access to resources. Teachers favored professional development because it helped them effectively incorporate technology into lessons to help students learn specific math concepts.

3.3.9. Common misconceptions

Lee and Ginsburg [59] identified and address nine typical misconceptions about learning and teaching mathematics to children that prospective and actual early childhood teachers share. These misconceptions include: i) math education is inappropriate for young children; ii) some intelligent children have math genes; iii) simple numbers and shapes are sufficient; iv) literacy and language skills are more important than math; v) teachers must create a stimulating physical environment, step back, and let the kids play; vi) mathematical concepts should not be taught separately; vii) mathematical concepts should not be taught separately; vii) math evaluations for young children are worthless; viii) children can only learn mathematics through interaction with natural things; and ix) utilizing computers to teach and learn mathematics is a horrible concept. These misunderstandings frequently make it challenging to comprehend and interpret the most recent recommendations for the education of young children in mathematics. They also develop into covert (and sometimes overt) obstacles that prevent teachers from implementing the new methods in the classroom.

3.3.10. Development of mathematics teacher's beliefs instruments

Purnomo [60] developed and validated teachers' belief scales, including notions about the subject's nature, how it is taught, and how math learning is assessed. The scale design had 54 items, of which 15 were related to beliefs about assessment in learning mathematics, 23 to ideas about teaching mathematics, and 16 to opinions about the essence of mathematics. The exploratory factor analysis (EFA) examines it at the initial stage to assess the scale factor's structure. As a consequence of the investigation, each scale has two elements. Confirmatory factor analysis (CFA) was employed in the second stage to validate the factors identified by the EFA. The CFA findings demonstrate that the provided model has a high index of good fit. Each factor also has a suitable internal consistency coefficient between 0.715 and 0.787. As a result, this scale may help evaluate teachers' beliefs about mathematics.
4. CONCLUSION

This study presents the findings of an exploratory research topic of beliefs about the nature of mathematics, mathematics teaching, and mathematics learning, as reflected in published research. The results show that the topic of beliefs research about the nature of mathematics, mathematics teaching, and mathematics learning has attracted the global community’s attention. The number of publications increased significantly starting in 1989, accompanied by many citations. The main document is in the form of a journal article or proceedings. Based on these primary documents, it is known that the development of mathematical beliefs research topics. The foundation of this topic begins with studying the topic of knowledge, activities, and learning mathematics. Then it is developed into other, more varied studies, including teacher’s beliefs transition; mathematical beliefs comparison; mathematics teacher’s belief, teaching, and learning; mathematical beliefs and problem-solving; IBL; affective factors; technology utilization; common misconceptions; and development of mathematics teacher’s beliefs instrument. This research investigates the core of scientific work to provide information to researchers and institutions as material for consideration of research to be carried out. Future research should develop a mathematics teacher’s belief instrument based on information technology. The confidence data collected is essential for the government to make policies related to continuing professional development (CPD) for mathematics teachers and teacher training institutes in developing strategies to prepare their graduates.

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