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Mathematical Writing Errors in Expository Writings of College Mathematics Students

Ivee K. Guce
Mathematics Department, De La Salle Lipa, Philippines

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ABSTRACT

Despite the efforts to confirm the effectiveness of writing in learning mathematics, analysis on common errors in mathematical writings has not received sufficient attention. This study aimed to provide an account of the students' procedural explanations in terms of their commonly committed errors in mathematical writing. Nine errors in mathematical writing were pre-defined -- namely, misuse of mathematical terms, misuse of mathematical symbols, incorrect notation, incorrect grammar, incorrect capitalization, no or incorrect punctuation, vague term, incorrect term, and lack of term or phrase. This study used qualitative method of research to keep a record of errors in mathematical writing. Conducted in the College of Education Arts and Sciences of De La Salle Lipa, the study involved twelve BS Mathematics students enrolled in Advanced Calculus 1 class as respondents. Results revealed that the most committed errors done in mathematical writing are incorrect grammar and misuse of mathematical symbols. Certainly, intervention programs on mathematics writing will bring favorable outcomes. Language courses in the students' curriculum which tackle proper grammar usage may be integrated with writing about mathematics as part of the student activities. Such will provide the students with writing experiences fitted to their discipline.

Corresponding Author:

Ivee K. Guce,
Mathematics Department,
De La Salle Lipa,
Pres. J.P. Laurel Highway, Lipa City, Batangas, Philippines.
Email: ivee.guce@dlsl.edu.ph

1. INTRODUCTION

Writing, as a form of language, plays an essential role in mathematics learning. Apart from understanding concepts and principles, students should be able to write mathematical solutions clearly and logically. However, students in a mathematics class tend not to be mindful of how they would explain, in writing, their solution to a given problem as they think that the teacher focuses only on the correctness of their answer. They become too concerned with the computations, paying little or no attention to the clarity of their overall solution. It therefore becomes essential for them to realize the importance of writing skills in mathematics class as it is in their English or other subjects.

Integration of writing in mathematics has received considerable attention in the research literature. The role of communication continues to be addressed in current reports and studies. In fact, one of the process standards identified by the National Council of Teachers of Mathematics [1] for students, aside from becoming mathematical problem solvers, is communicating knowledge. Before, math classes relied on skill-building and conceptual understanding activities. But in the recent years, teachers are realizing that inclusion of writing in a mathematics class is "more than just a way to document information" (p. 3); it is a way of deepening the students' learning and a tool for helping them gain new insights [2]. In spite of the few studies

which cited some apprehensions of teachers in using writing in mathematics (e.g. [3],[4]), a number of researchers articulated its benefits (e.g. [4]-[9]).

Writing-to-learn methodologies require students to analyze, compare, and synthesize information. It develops a skill that is grounded in the cognitive domain [10]. Daily writing activities increase mathematics achievement and metacognition [5]. Rather than focusing on sole mathematical computation, students pay attention also to how they express ideas and deepen their understanding when they are engaged in mathematical writing tasks [11]. The process of writing slows the students down—allowing them to process information [6]. It gives them the opportunity to communicate the way they understand mathematics and use mathematics vocabulary in writing [12]. This leads to the development of their writing skills helping them make deep and meaningful connections between mathematical concepts [4],[7].

In a case study conducted by Defazio, et al. [10], he stressed that students in all levels should not only be good in written communication but also must understand the importance of good writing skills. Observations by Seo [8] suggest that writings in a mathematics class comprise mainly of symbols, using minimal number of words. He pointed out that calculations are important but they should not be the main activity in any mathematics class. Thus, mathematics teachers should promote writing in their classes by explicitly assigning writing activities [8] to constantly strengthen lines of communication between the teacher and students as their activities progress [13]. It has been proposed that using both language and symbolism in communicating mathematical ideas should be a main concern in any mathematics curriculum [5]. Writing exercises integrated in problem solving give the teacher assessment of the knowledge gained by the students [8]; thus, influencing his mathematics teaching [12]. They can be used to give the teacher a hint on how to help students succeed in the class [14].

While it is a primary concern that mathematics students learn the mathematical principles and concepts taught to them, it is very important also that they become skilled at articulating their thoughts and ideas effectively in writing. However, with the birth of text messaging, students nowadays often use informal language in trying to communicate in writing. Grammar, punctuation, capitalization and the like are often ignored [10]. In mathematical expository writings, symbols are at times misused and overused. It is thus a challenge for teachers to hone their students to become effective communicators.

Apart from solving mathematical problems, mathematicians (should) actually spend a great deal of time writing because mastering the ability to write clear mathematical explanations is very essential. According to Lee [15], if a mathematician wants to contribute to the greater body of mathematical knowledge, he must be able to communicate his ideas in a way which is comprehensible to others. Thus, being able to write clearly is an equally important mathematical skill along with the ability to solve equations.

This study concerns the expository writing of third year BS Mathematics students, that is, writing which is intended to explain mathematical ideas and procedures. The aim of this paper is to provide an account of the students' procedural explanations in terms of their commonly committed errors in mathematical writing. Despite the mentioned studies showing the effectiveness of writing in learning mathematics, analysis on common errors in mathematical writings has not received sufficient attention. Little is known of how mathematicians and students understand and use language in technical writing [16] since many of these researches focused on describing the effects of using writing in mathematics learning (e.g. [4]). Documenting the commonly committed errors of the students when doing expository writing in mathematics is presently not much of a concern of many researchers. The result of this study not only provides awareness on the nature of college mathematics students' expository writing but it also gives basis on how their written communication skills in mathematics may still be improved by the teachers who train them.

1.1. Commonly Committed Mathematical Writing Errors

Different mathematical writings can be grouped into formal and informal writing. The former involves writing some terms or bits of explanation on a blackboard during a lecture or explaining something to a friend on a piece of scratch paper. The latter, however, includes the kind of writing expected in a paper. There are differences in what is acceptable [17]. For this study, nine errors in mathematical writing have been regarded by the researcher as the most common that students commit.

1.1.1. Misuse of Mathematical Terms

Correct use of mathematical terms is very important when doing expository writing. Mathematics requires a very precise use of language [15]. Misuse of a mathematical term, if not corrected, will create confusion among the reader of the text. In a study by Moore [18], he noted that "while attempting to write formal proofs, students do not necessarily understand the content of relevant definitions or how to use these definitions in proof-writing." An example of misuse of mathematical terms is to say or write "the limit

converges" or the use of the word "equation" to mean a mathematical expression which is not considered an equation.

1.1.2. Misuse of Mathematical Symbols

In mathematics, symbols are integral components of communication and thus must be used appropriately. Further, part of being able to write well in mathematics is to know when to use symbols or when to use words instead [15]. A common mistake is to misuse the "=" symbol. Many students tend to use equal signs to connect several lines of solution which are not actually equal. Another concern on mathematical symbols is the fact that mathematics is case-sensitive [19]. As a rule, upper-case and lower-case versions of the same letter are not used interchangeably unless it has been specified that they represent the same quantity.

1.1.3. Incorrect Notations

Notations in mathematics are the symbolic expressions which have precise and established meaning. Examples of notations in calculus would be the difference between $\{a_n\}$ and a_n. The former is a notation for a sequence while the latter refers to a term in a sequence. Using the two notations interchangeably would certainly confuse someone reading in a mathematics context.

1.1.4. Incorrect Grammar

Good quality writing observes correct grammar. This also applies to expository writing in mathematics. Lee [15] emphasized that when one writes in a math class he is expected to use correct grammar and spelling as his writing should be clear and professional. Like other texts, mathematics is written with sentences and paragraphs. While formulas and equations are elements of a math paper, one should keep in mind that standard grammatical rules apply in mathematics writing. Aside from subject-verb agreement, another common grammatical error is mistaking a phrase for a sentence like writing "Since *f* is a function."

1.1.5. Incorrect Capitalization

In this study, this error refers to two ideas namely, (i) capitalizing words which should not be capitalized and (ii) *not* capitalizing words which should be capitalized. In writing, it is important to observe the rules in capitalization in that it conveys to the readers the importance of specific words and the change in meanings of words. In writing a mathematical explanation, a student usually tends to capitalize a word which she wants to highlight although the word is not a proper noun at all.

1.1.6. No or Incorrect Punctuation

Punctuation is one of the most important aspects one should be conscious of when writing. In the Importance of Punctuation (https://thewritecorner.wordpress.com), it was pointed out that this feature of writing "gives meaning to the written words, much like pauses and changes in tones of the voice when speaking." Further, an error in punctuation may convey an entirely different meaning to the one that is intended. As stated by Su [17], "all mathematics should be written in complete sentences." Thus equations, even displayed ones, should have punctuation if used in a sentence.

1.1.7. Vague Term or Phrase

A vague term or phrase in this study is one that is not clear or one that suggests different meaning resulting in lack of certainty and distinctness in a mathematical statement. Since mathematics requires precise language, the use of indistinguishable words or terms is highly discouraged. A very common mistake one makes in mathematical writing is the use of the word "it" in giving an explanation to a solution. Appropriate specific terminology or mathematical expression must always be used so as to deliver the correct information to a reader [19].

1.1.8. Incorrect Term or Phrase

A term or phrase which is not appropriately used in a mathematical statement leading to the incorrectness of thought or idea in that statement is referred to as incorrect term or phrase. In this study, this error is taken as a consequence of two possibilities: (i) writer's misunderstanding about certain mathematical principle and (ii) his lack or subpar of choice of word.

1.1.9. Lack of Term or Phrase

While a good mathematical explanation is characterized by a detailed solution, one should not discount the importance of using words or phrases to connect the ideas behind the mathematical expressions. A mathematical problem sure does have a specific answer; however, it makes perfect sense to say that the

answer could be obtained from two or more different solutions. Thus, presenting only the mathematical solution without any explanation is like assuming that the solution will speak for itself which, in essence, is not a purpose of expository writing. Further, not supplying the needed term or phrase leaves the reader to be in-charge of guessing the story behind the writer's solution which is dangerous and very likely to cause misinformation. Also under this type of error is the writer's failure to define a variable that he declares in his solution. Example: To find the sum of a series, one may evaluate $\lim_{n\to\infty} s_n$ where s_n is the n^{th} partial sum of the series.

2. METHOD

This study used qualitative method of research to keep a record of BS Mathematics students' errors in mathematical writing. The study was conducted in the College of Education Arts and Sciences of De La Salle Lipa.

2.1. Participants

The participants in the study were twelve third year BS Mathematics students enrolled in Advanced Calculus 1 in the second semester of school year 2016-2017. As mathematics major students, they are expected to have a reasonable level of mastery in the content of the subject and the ability to deliver the content to their audience, both orally and in writing.

2.2. Writing Task Procedure

Throughout the first half of the semester, explanatory essay type of questions were given to the students as a regular writing activity in the subject. Seo [8] explains how explanatory essays can be used as a writing activity for the mathematics classroom. Aside from simply asking a student to explain certain concepts in his own words, this type of activity required the students to explain a mathematical process in a form of essay. At the time of data collection, no specific program or curriculum is used to enhance the students' mathematical writing skills.

Activities were given at the end of certain topics. To give the students enough time, they were given two days to finish each activity. In solving a problem, they were instructed to show their detailed solution and write a clear explanation about it. They needed to note in words the process or concepts that they used in solving the problem and the rationale for using such method.

2.3. The Instrument

Below are the five activities that the students completed.

Activity 1: Your friend Joseph did not attend today's meeting as he is still processing his enrollment. You lent him your notes. He told you he understood most of what he read except for the latter part of the lecture. He was overwhelmed with the past topics (like indeterminate forms) that emerged during the lecture. He asked if you could

(1) explain to him the convergence of $\frac{2^n}{3^{n+1}}$. Also, to prepare him for the next discussion, he further requested for you to

(2) discuss the next example in the handout which is finding whether the sequence n^3e^{-n} converges or diverges.

Activity 2: What did I miss today in Advcal1? What transpired during the meeting? Looking at your notes, I cannot understand the Squeeze Theorem. Can you explain to me, please?

Activity 3: Your classmate Jen attended the last lecture. However, she could not fully understand the concept of greatest lower bound and least upper bound. She is having difficulty (1) determining if a sequence is bounded and (2) bounding a sequence using its greatest lower bound (GLB) and least upper bound (LUB). How would you explain it to her using the sequence $\left\{\frac{n}{3^{n+1}}\right\}$?

Activity 4: You want to come up with a reviewer for an upcoming quiz. How would you explain to yourself in detail the following?

(1) Determine if the series $\sum_{n=1}^{\infty}\frac{8}{n^2+3n+2}$ converges or diverges. If it converges, what is the sum?

(2) Write 0.272727… as a ratio of two integers.

Activity 5: Your teacher scheduled for a graded recitation next meeting. She gave a list of possible questions to be asked during the activity. You wanted to really prepare for it so you decided to write a *detailed* script on how you would explain the answer to each question.

Questions: Are the following statements true? Why or why not?

(1) Because $\frac{1}{n^4}$ approaches 0 as n approaches ∞, $\sum_{n=1}^{\infty}\frac{1}{n^4}=0$.

(2) Because $\lim_{n\to\infty}\frac{1}{\sqrt[4]{n}}=0$, the series $\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{n}}$ converges.

(3) The series $\sum_{n=1}^{\infty}\frac{n}{1000(n+1)}$ diverges.

2.4. Writing Task Assessment

All the submitted activities of the students were compiled per topic. There were a total of fifty-seven outputs examined. Three students were incidentally absent during the lecture for the second activity and were excluded in the corresponding writing task. All outputs were read thoroughly for at least two times. They were carefully investigated for occurrence of mathematical writing errors. The number of times that each student commits an error was recorded and tallied. For easy reference during the tallying of data, a coding process was developed. The nine pre-defined mathematical writing errors were given codes which appear in Table 1.

Table 1. Pre-defined Mathematical Writing Errors

Code	Mathematical Writing Errors
MMT	Misuse of mathematical terms
MMS	Misuse of mathematical symbols
IN	Incorrect notation
IG	Incorrect grammar
IC	Incorrect capitalization
NIP	No or incorrect punctuation
VT	Vague term
IT	Incorrect term
LTP	Lack of term or phrase

3. RESULTS AND DISCUSSION

3.1. Results

Of the nine mathematical writing errors, incorrect grammar (n=36) showed to be the most committed error in Activity 1 as illustrated in Figure 1 (Mathematical writing errors in Activity 1). Eleven out of the twelve students had at least one grammatical error. Many of these errors relate to subject-verb agreement (e.g. "Since the limit *exist*…") and use of prepositions (e.g. "raise $\frac{2}{3}$ *by* n…"). Next to incorrect grammar is the vague term error which is also relatively high (n=28). The use of the word "it" occurred frequently under this type of error. To cite two instances, Student 5 wrote, "This means that the limit exists and therefore *it* converges" to refer to the given sequence that is being evaluated for convergence, and not the limit mentioned in the sentence. While, based on the writings of Student 9 he wrote, "Since $\frac{2}{3}$ is less than 1, it means that *it* converges to 0" referring actually to $\left(\frac{2}{3}\right)^n$. Misuse of mathematical terms ranked third in the first activity.

Incorrect grammar remained first (n=20) in the second activity while incorrect capitalization came out to be the second (n=14) most-committed error as presented in Figure 2 (Mathematical writing errors in Activity 2). Of the seven students (58%) who had incorrect capitalization error, only one got six error marks in the tally. The other six had one or two error marks only. Examples of which under this activity are capitalizing the words "sequence" and "limit", and capitalizing a word following a comma (e.g. "In $\left\{\frac{1}{n^3}\sin n^2\right\}$, *We* will…").

In the tally for the third activity, Figure 3 (Mathematical writing errors in Activity 3) shows that there is a big gap between the first and second-ranking errors. As in the second activity, some students where tricked into capitalizing words which are actually *not* proper nouns (e.g. least upper bound and greatest lower bound). Incorrect grammar and no or incorrect punctuation both placed second (n=11) in the ranking. This is followed closely by errors in misuse of mathematical symbols (n=10). Some of these are wrong use of

ellipsis (e.g. "The first four terms of the sequence are $\frac{1}{9}, \frac{2}{27}, \frac{3}{81}, \frac{4}{243}$,…") and use of material implication symbol "$\Rightarrow$" to mean an equality.

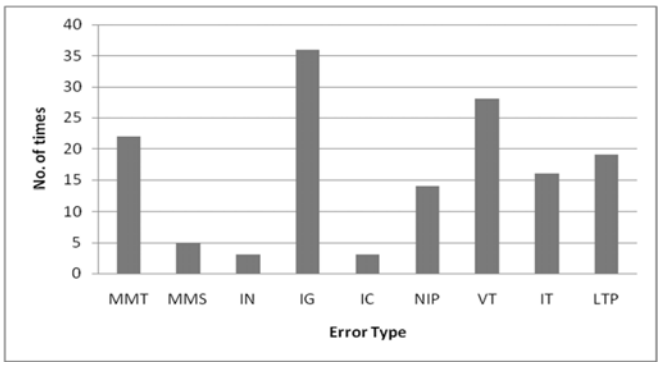

Figure 1. Mathematical writing errors in Activity 1

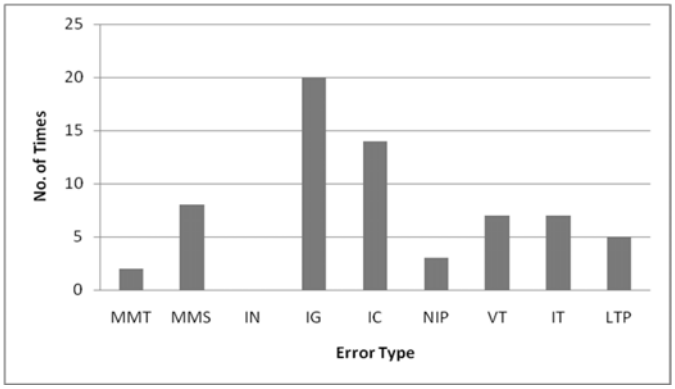

Figure 2. Mathematical writing errors in Activity 2

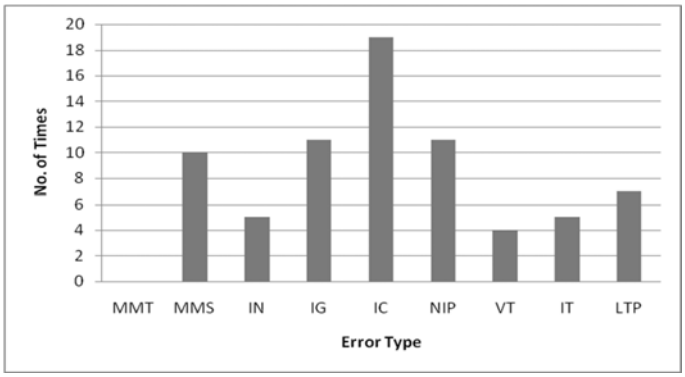

Figure 3. Mathematical writing errors in Activity 3

Of the five activities given to the students, the fourth one came out to have the least number of errors in mathematical writing committed by the students. Only fifty percent (n=6) of the respondents account to the highest frequency (n=18). This can be seen in Figure 4 (Mathematical writing errors in

Activity 4). Because the nature of questions in this activity requires lengthy solutions, student tended to err much on the use of different mathematical symbols (e.g. not using equal "=" signs and parenthesis when necessary and use of material implication symbol "$\Rightarrow$" to indicate a statement or explanation in a solution). The second-ranking type of error, incorrect grammar, placed far below the first with just six counts done by only two (17%) of the twelve respondents.

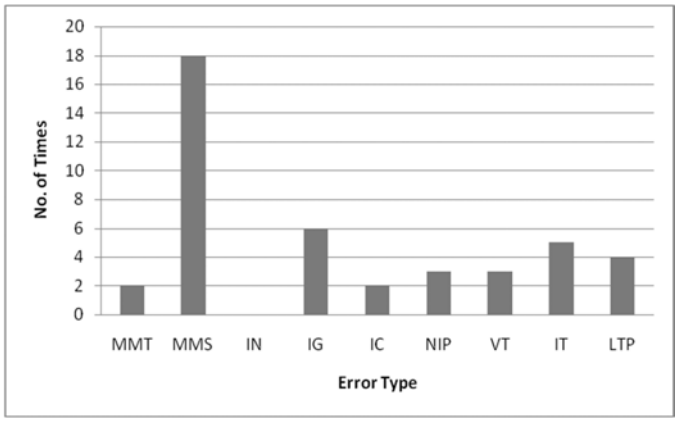

Figure 4. Mathematical writing errors in Activity 4

Like in the first two activities, the last activity had incorrect grammar as the most committed type of error by the students. It is worth noting though that in Activity 5 as shown in Figure 5 (Mathematical writing errors in Activity 5), the frequency (n=10) in grammatical errors, is relatively low compared to Activity 1 (n=36) and Activity 2 (n=20). The second to rank is the lack of term or phrase error with a frequency of 8 from only five students (42%) followed by vague term errors placing third (n=6). Lack of term or phrase errors mostly involved defining the variables used. For instance, Student 1 wrote, "…in the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$, we should find the $\lim_{n\to\infty} s_n$" without defining what s_n is.

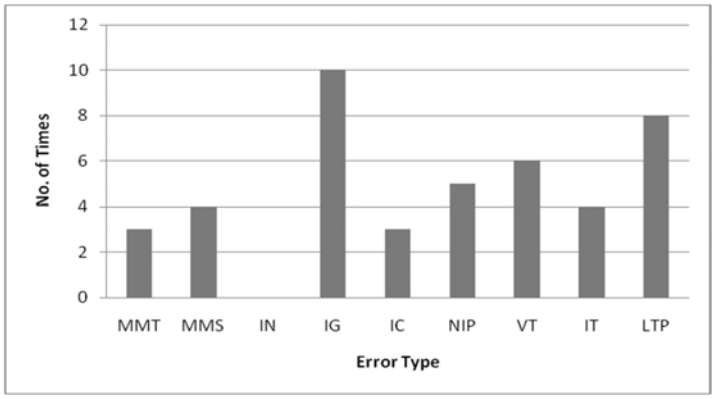

Figure 5. Mathematical writing errors in Activity 5

3.2. Discussion

The NCTM standards implicitly invite researchers to engage in studies that will promote effective mathematical communication to enhance mathematical pedagogy [20]. The nine mathematical writing errors which served as the framework of the study emerged to be viable in that all were present in the students'

procedural explanations. Figure 6 (Average percentages of mathematical writing errors in the five activities) presents the average percentage that each type of error was committed by the students in the five activities.

Incorrect grammar ranked first among the nine errors with an average of 21%. Lew and Mejia-Ramos [16], in their qualitative study, found that undergraduate mathematics students are inclined to think that the rules of English do not apply in mathematical settings although for the most part, mathematicians in the study believed that grammar and parts of speech in mathematical words should be carefully attended to in mathematical writing. The need for complete sentences and attention to grammar is supported by a number of mathematicians [21]-[25]. Similar to this study, dela Peña [26] did an analysis of errors but in those essays written by math, science, and engineering faculty. His study revealed that language problems such as grammatical errors confront people in the field of science and engineering.

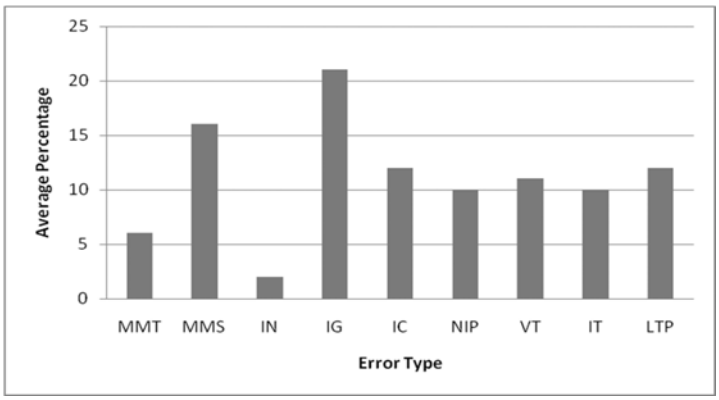

Figure 6. Average percentages of mathematical writing errors in the five activities

Following errors on grammar is misuse of mathematical symbols which averaged to 16% across the five activities. According to Rubenstein and Thompson [27], achievement in mathematics may improve if students understand and properly communicate using math symbols and notation. Chirume [28] who investigated how the use of mathematical symbols influences understanding of math concepts found that most students fail to understand or interpret the meaning of math symbols due to the way they are taught to read, pronounce and use them. Further, this misuse (and also abuse) of symbols may essentially hinder formation, understanding and communication of concepts and might affect achievement in the subject. As stated by Vincent, Bardini, Pierce, and Pearn [29], understanding that symbols do have meaning and making it a habit to check the meaning of symbols used is an aspect of mathematical writing that needs to be cultivated from primary to tertiary levels. Usiskin [30] (p. 4) asserts that "mathematics is both a written language and a spoken language, for—particularly in school mathematics—we have words for virtually all the symbols. Familiarity with this language is a precursor to all understanding." Being familiar with symbols and fully understanding their meaning need to be cultivated at all levels: primary, secondary and tertiary.

Two types of error placed third in the overall ranking--- incorrect capitalization and lack of term or phrase. It was noticeable in the data gathered that many students capitalize words they want to put emphasis on. Also, the students seemed to find it difficult to differentiate common and proper nouns when it comes to mathematical terms. For lack of term or phrase error, it usually stems from the students' thinking that there is no need to define a *commonly used* notation or variable should they decide to include such in their procedural explanation. They have this assumption that it will automatically be picked-up and understood by the reader the way they meant it to be delivered, which should not be the case in mathematical expository writing.

Fourth in rank is the vague term error with an average of 11%. This is followed by no or incorrect punctuation and incorrect term errors, both with average of (10%). In the study of Lew and Mejia-Ramos [16], results showed that there is a lack of punctuation and capitalized letters to indicate the ending and beginning of sentences. Lee [15] stated that to make sure that the paper flows smoothly, equations and formulas, aside from the words, should have correct punctuation.

The errors with the lowest average percentages are misuse of mathematical terms (6%) and incorrect notation (2%). Even though all the solutions in the five activities required high mathematical content, it was evident from the data that the students were very careful in using the correct notations. In fact, for Activities 2, 4, and 5, there was no recorded error in notation.

4. CONCLUSION

This study investigated the frequencies at which the nine pre-defined mathematical writing errors were committed by the students. From the findings of the study, there is evidence that the most committed errors are incorrect grammar and misuse of mathematical symbols. Certainly, intervention programs on mathematics writing will bring favorable outcomes. Language courses in the students' curriculum which tackle proper grammar usage may be integrated with writing about mathematics as part of the student activities. Such will provide the students with writing experiences fitted to their discipline. A mathematics student should not only know the theories and principles behind solving a given problem. Equally important to finding the answer is articulating clearly and coherently how to arrive at that answer.

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BIOGRAPHY OF AUTHOR

Dr. Ivee K. Guce is a mathematics teacher and a researcher. She is an associate professor of the Mathematics Department of the College of Education, Arts and Sciences of the De La Salle Lipa. She finished her BS Mathematics Major in Actuarial Science at the University of Sto. Tomas. She obtained her Master in Mathematics degree from the De La Salle University, and earned the degree Philosophy in Education Major in Mathematics from the University of Batangas. Dr. Guce is currently the Program Chair in Mathematics. Besides teaching, she is actively engaged in research. She has published a number of papers at international journals and presented her works at several conferences both locally and internationally. Most of her works are focused on mathematics education. Her passion for teaching mathematics is her motivation in sharing her knowledge to other educators through the studies she conducts.