Mathematical Model for Mapping Students' Cognitive Capability

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Article Info	ABSTRACT
	The quality mapping of educational unit program is important issue in education in Indonesia today in an effort to improve the quality of education. The objective of this study is to make a mathematical model to find out the map of students' capability in mathematics. It has been made a mathematical model to be used in the mapping of students' capability. Demonstration of the use of models performed in accordance with the data of the results from the math test given to 147 students in grade XII, state senior high school, science program, and academic year 2015-2016. The map of students' capability can be known that 48 test items are derived from 16 sub topics of three cognitive domain, only 19 test items are achieved. The achieved map lies in 8 sub topics for knowledge domain and 6 sub topics for comprehension domain and application domain has 5 sub topics. So that sub topic and cognitive domain which can not be achieved can be done further corrective action to obtain the maximum results. This paper demonstrates how operational research techniques can be applied for problem solving in education.

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1.   INTRODUCTION

Improving the quality of education is the issue of a trend that promoted by the government of Indonesia in order to follow the development of science and technology. Some of the efforts already made are to improve the quality of educators [1], curriculum improvements [2]. It cannot be denied that Indonesian educational outcomes are not well [3] because it is still under the sate ranking participants observed [4],[5]. Including the result of mathematics education is not satisfactory viewed from the rank compared to participating country observed [6], [7]. Even Bawesdan [8] said that the education in Indonesia was an emergency. Other facts show that the average mathematics national examination results the decrease of the last 4 years (2012-2015) [9], [10].

Many factors that affect students' mathematical abilities, internal factors such as students' belief [11], lack of motivation and poor attitudes by students [12], self efficacy, academic resources, self regulation, and learning styles [13], students were less consistent in studying mathematics [14], study habits, time management and attitude towards mathematics [15], the mathematics anxieties related to numerical anxieties on students' mathematics achievement [16]. External factors, such as modifying the linguistic structures in math word problems [17], teaching practices and teacher attribution [18], less effective teacher in choosing learning strategies [19], poor teaching methods [20], teachers and students' self motivation, interactive method of teaching [21].

One of the instruments to know the quality of education nationally is through national exam as well as the determinant of graduation requirement educational unit level [22], [23]. Mathematics is one of the lessons tested nationally and target of students learning outcomes based on Bloom's Cognitive Taxonomy [24] which includes knowledge, comprehensive, and application [25]. In accordance with the development of future, national examination is no longer as the measurement of graduation requirements at the unit level of education in 2016 but as the quality mapping of educational unit program in order to improve the quality of education [26],[27]. Implementation of quality mapping is not only nationally but also any region. Therefore, educator confronted with a new case which should have the competence to perform the mapping capability of students toward educational program unit or instructional material in order to know which material has or has not been mastered by students. It can be taken further to obtain optimal results.

This research aims to make a mathematical model from the mapping problem of the ability of students' mathematical cognitive domain. The research approach used is operational research (OR). It has been widely and successfully used in solving the case in education, such as: student project allocation [28], solution of the university time tabling problem [29],[30], scheduling of room [31], allocate the optimal marks for each chapter of engineering mathematics [32], evaluate the examination question in engineering mathematics course [33]. Modeling carried out by sub topic of teaching material and cognitive domains will be achieved and the data from the results. Based on solving model can be known the map of the ability of students, so it can be take further action to achieve the better quality of the educational unit program.

2. RESEARCH METHOD

Research method is consisted of three stages. The first is to make mathematical model based on sub topic and domain achieved in mathematics national examination at high school. The second is data collection and the last one is solving the model by using LINDO 6.1 package.

2.1. Mathematical Model

Modeling is important in OR. If a problem can be translated into the language of mathematics, it is called a mathematical model from the issue. The mathematical model a system is the collection of mathematical relationship which is for the purpose of developing a design or plan; characterize the set of feasible solutions of the system [34]. Based on recapitulation of the national examination at high school, it can be categorized into p sub topic and q cognitive domain, so the mathematical modeling of the problem is described as follows

Notation use are define as follows

Set:

X_i : Main variable test items to i, $i = 1, p$

C_j : Cognitive to j, $j = 1, q$

X_{ij} : Variable test items to i in domain C_j

a_{ij} : The number of students who answer correctly on each test items to i for all domain C_j

n_{ij} : The maximum number if all the students answer correctly on each test items to i for all domain C_j

b_i : The number of test items to i for all domains C_j

b_j : The number of all test items to i for each domain C_j

S_i : The number of students who answer correctly tets items to i for all domains C_j

S_j : The number of students who answer correctly tets items to i for each domain C_j

T_j : The maximum number if all students answer correctly test items to i for all domain C_j

T_j : The maximum number if all students answer correctly test items to i for each domain C_j

The mapping problem can be presented model as follows. The objective function (1) is to maximize item test in accordance with students' capability. Constraint equation (2)-(10) constraint are to search the map of students' capability. Whereas the role of each constraint, equation (2) states that the number of test is as many as I items. Constraint equation of (3), (5), and (7) are to ensure item test, the number of students who answer the items correctly and the maximum number if students answer all the item correctly for all cognitive domains. Constraint equation (4), (6) and (8) are to ensure the number of items, the number of students answering the item test and maximum number if students answer correctly all the item test for each cognitive domain. Constrain equation (9) and (10) are the difference between the maximum numbers if all students answer correctly and the number of students who answer correctly for each and all cognitive domains.

Therefore the mathematically model can be expressed as follows.

$$Maks \sum_{i=1}^{p} \sum_{j=1}^{q} X_{ij} \tag{1}$$

Subject to

$$\sum_{i=1}^{p} \sum_{j=1}^{q} X_{ij} \leq I \tag{2}$$

$$\sum_{j=1}^{q} X_{ij} \leq b_i, i = 1,...,p \tag{3}$$

$$\sum_{i=1}^{p} X_{ij} \leq b_j, j = 1,...,q \tag{4}$$

$$\sum_{j=1}^{q} a_{ij} X_{ij} \leq S_i, i = 1,...,p \tag{5}$$

$$\sum_{i=1}^{p} a_{ij} X_{ij} \leq S_j, j = 1,...,q \tag{6}$$

$$\sum_{j=1}^{q} n_{ij} X_{ij} \leq T_i, i = 1,...,p \tag{7}$$

$$\sum_{i=1}^{p} n_{ij} X_{ii} \leq T_j, j = 1,...,q \tag{8}$$

$$\sum_{j=1}^{q} n_{ij} X_{ij} - \sum_{j=1}^{q} a_{ij} X_{ij} \leq T_i, i = 1,...,p \tag{9}$$

$$\sum_{i=1}^{p} n_{ij} X_{ii} - \sum_{i=1}^{p} a_{ij} X_{ij} \leq T_j, j = 1,...,q \tag{10}$$

Where $X_{ij =}$ integer.

2.2. Data collection

Data collection instruments use objective test taken from the national exam test domain, assuming that the test that meets the requirements of the analysis of items nationally. Criteria for selection of test item performed based on recapitulation of distribution of teaching materials so it is taken 16 sub topics of 3 cognitive domains namely: knowledge (C1), comprehension (C2), and application (C3). Data collection is carried out by putting 48 test items for 147 students, grade XII, science program, state senior high school, and academic year 2014/2015. The obtained test result is the number of students who answer correctly for each test item and cognitive domain, as the following Table 1.

Table 1. Test Result

Sub Topic	Main Variable	Cognitive Domain			Total
		Knowledge	Comprehension	Application	
Exponentiation and Logarithm	X_1	108	98	75	281
Equation and inequalities	X_2	104	81	87	272
Linear programming	X_3	104	60	107	271
Equation and quadratic function	X_4	96	78	100	274
Trigonometry	X_5	115	67	39	221
Mathematics logic	X_6	92	115	70	277
Statistics and probability	X_7	96	95	55	246
Polynom	X_8	94	81	69	244
Inverse and composition function	X_9	112	106	80	298
Sequence and series	X_{10}	118	83	88	289
Matrix	X_{11}	107	121	76	304
Vector	X_{12}	102	77	93	272
Transformation	X_{13}	107	71	102	280
Three dimensions	X_{14}	54	81	77	212
Limit	X_{15}	82	86	65	233
Differential and Integral	X_{16}	99	75	65	239
$\sum$		1590	1375	1248	4213

3. RESULTS AND ANALYSIS

Based on the test data and subsequently replaced to all the parameters on the constraints equations (2) -(10). The result of the model solution by applying LINDO 6.1 package is summarized in the following Table 2.

Table 2. The Map of Students' Capability Cognitive Domain

Sub Topic	Main Variable	Cognitive Domain		
		Knowledge	Comprehension	Application
Exponentiation and Logarithm	X_1	1	0	1
Equation and inequalities	X_2	0	0	0
Linear programming	X_3	1	1	0
Equation and quadratic function	X_4	1	0	0
Trigonometry	X_5	0	1	0
Mathematics logic	X_6	0	1	0
Statistics and probability	X_7	0	1	0
Polynom	X_8	0	0	1
Inverse and composition function	X_9	0	0	1
Sequence and series	X_{10}	1	0	0
Matrix	X_{11}	1	0	1
Vector	X_{12}	1	0	0
Transformation	X_{13}	0	1	0
Three dimensions	X_{14}	1	1	0
Limit	X_{15}	0	0	1
Differential and Integral	X_{16}	1	0	0

From 48 test items includes 16 sub topics and 3 cognitive domain on Table 2 are only 19 variables valued i. It means that test items in sub topic and cognitive domain is accordance with students' ability. Thus, it can be stated that the map of students' mathematics capability for domain knowledge lies on 8 sub topics and 6 sub topics in domain comprehension and 5 sub topics in domain application. While variable valued zero is 39, it means that the students' capability in sub topic and cognitive domain has still problem. Thus, it can be known that the map of students' mathematics capability is in accordance with demands of mapping program quality [26],[27], so the problem can be performed further action to obtain the maximal result.

4. CONCLUSION

Mapping of educational unit program is important issue in education in Indonesia today in order to improve the quality of education. Specifically, the real implementation of mapping includes the mapping of students' capability for each subject and topic in educational unit program. Therefore, the educators are confronted to new task in order to be able to map students' capability for all subjects and topics being taught. In this research, it has been made mathematical model for mapping of students' cognitive domain capability.

Demonstration of model testing is based on students' test result shows that 48 items based on 16 sub topics and 3 cognitive domains can be known the map of students' capability, namely: 19 items are achieved, 9 sub topics in knowledge domain, 6 sub topics in comprehension domain , 5 sub topics in application domain. So the students' capability in sub topics and cognitive domain which is not achieved, it needs further improvement to obtain maximal result

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