

## Design of Learning Model of Logic and Algorithms Based on APOS Theory

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### ABSTRACT

This research questions were 'how do the characteristics of learning model of logic & algorithm according to APOS theory' and 'whether or not these learning model can improve students learning outcomes'. This research was conducted by exploration, and quantitative approach. Exploration used in constructing theory about the characteristic of the learning model of logic and algorithms based on APOS theory. The results showed that characteristics of learning model used APOS theory had achievement map of competency be classified into four levels. They are action; process; object; and scheme. Action is ability to understand concept by using external stimulus. Process is ability to think about the same action based on specific input and output. Object is ability to think about transformation on action and process, so that they can associate between action and process, process and process itself based on a specific output. Scheme is ability to construct relationship of action, process, object, and other concept to solve the problem in making business document. It is proven that mean value is greater than the other. Especially in lesson plan of action and process.

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## 1. INTRODUCTION

Logic and algorithms are courses that put the basics of the ability to compose an application program [1]. All of knowledge on logic and algorithms are classified into conceptual and metacognitive knowledge [2]. Therefore; the level of logic and algorithms understanding must be relational. Relational understanding is knowledge, which is obtained by connecting between one concept to another [3]. To achieve relational understanding is needed the ability of mathematics logic.

The above requirements are opposite with students condition in STMIK Surabaya recently. Based on database, it was discovered that more than 70% new students have low mathematics logic ability. Consequently, the ability level of logic and algorithms did not meet the requirements needed. Those difficulties are found in some higher education in Indonesia too [4]-[6]. They improve learning by making software. Nevertheless, their research did not pay attention to the learning aspect.

On the other hand, according to Vygotsky [7] in the view of Constructivism, knowledge must be constructed by individuals. Construction in this research is mental construction. Dubinsky found that mental construction consists of action, process, object, and scheme, which are abbreviated in APOS [8]. The theory said that individual constructs a mathematical concept through stages such as; action, process, object, and scheme. It is proven that APOS can be used directly to analyze the success or failure of students in understanding the concept of Division [9]. So, in this research APOS will be used as theories framework for development of logic and algorithms learning model.

The problem researched is how we apply APOS theory for development of logic and algorithms learning model. The problem details are (1) how did the characteristics of logic and algorithms learning model based on APOS theory?, (2) did those model improve students' understanding?.

## 2. RESEARCH METHOD

The research was done through exploration and quantitative approach. The objective of exploration was to construct the characteristics of logic and algorithms learning based on APOS theory. Then, learning model was developed by using Plomp theory [10].

Learning model was tested to new students, who are learning logic and algorithms at odd semester 2013/2014. So, data collection instruments using lesson sheets. The sampels of research were P1 and P2 class. P1 class was a class which was designed based on APOS theory and P2 was designed without using APOS theory. There were 36 students in P1 and 25 students in P2.

Data analysis used F statistics test with  $\alpha$  equal to 5%. The hypothesis is mean values of P1 is not equal to P2. So, the statement hypothesis is [11]:

$$\begin{aligned} H_0: \mu_1 &= \mu_2 \\ H_1: \mu_1 &\neq \mu_2 \end{aligned}$$

$\mu_1$  is mean values of P1 class, and  $\mu_2$  is mean values of P2 class.  $H_0$  is not rejected when F count less than F values of table, that is between 4 and 4,08. With F count is as following equation (1) [11].

$$F_{count} = \frac{\frac{SST}{(k-1)}}{\frac{SSE}{(N-k)}} \dots \dots \dots \text{equation (1)}$$

Before the F test is done, firstly the second variance of sample should be tested to see do both samples come from the same population. With the hypothesis tested as follow [11].

$$\begin{aligned} H_0: \sigma_1^2 &= \sigma_2^2 \\ H_1: \sigma_1^2 &\neq \sigma_2^2 \end{aligned}$$

$\sigma_1^2$  is variance of P1 class and  $\sigma_2^2$  is variance of P2 class.

With  $\alpha$  equal to 5%,  $H_0$  is not rejected when F count less than F table. Because of there are no degree of freedom numerator and denominator which are values 37, so from P1 class was taken 31 students as a sample. Consequently, F table used is  $F_{(24,30)} = 1,94$ . With F count is as equation (2) below [11].

$$F_{count} = \frac{s_1^2}{s_2^2} \dots \dots \dots \text{equation (2)}$$

## 3. RESULTS AND ANALYSIS

The knowledge, which is learned in logic and algorithms contains: data processing, modular technique included; variable, parameter, data, constanta, arithmetics and logic operator, and mathematics logic relationships; sequential, branching, looping process and three of their combination; algorithm development used in flowchart and pseudocode approach; array; some of searching and sorting algorithm [1],[12]-[14]. All of those knowledges are classified into conceptual and metacognitive knowledge [2].

Conceptual knowledge is a knowledge, which is constructed from relationship between base concept and wider knowledge, so it formed a specific function [15]. That knowledge contains factual and conceptual knowledge which is organized into more complex conceptual knowledge [16]. It means that in learning the logic and algorithms must connect between one of concept to another in order to achieve Learning Outcome (LO). The logic and algorithms LO is 'students can develop algorithms to make business document which is shown in flowchart and pseudocode, for example sold transaction form (it did not include database), they can make flowchart and pseudocode both in group and individual'. Nevertheless, this research only focuses in developing algorithms by using flowchart.

To understand the conceptual knowledges is needed relational comprehension not memorizing [3]. Relational comprehensions are individual knowledge which is attained by finding relationship between one of concept to another [3].

Based on those above propositions, so which is meant of logic and algorithm understanding in this research is the ability to relate the data, constanta, parameter, variable, array, arithmetics and logic operator, mathematics logic relationship, data processing, flowchart symbols, otomation process (such as; sequential, braching, looping), searching, and sorting to develop business document algorithm (it is shown in flowchart or pseudocode).

According to APOS theory, a comprehension of the concept (in mathematics) began with subjects ability in manipulating object both mental and physic in cognitive structure to establish action. Then, actions are interiorized into process. Then, the processes are encapsulated into object. The objects are de-encapsulated into process. Finally, action, process, and object to be organized into scheme [17].

APOS is Piagets' cognitive theory which is expanded to understand reflective abstraction on children mathematics learning [18],[19]. This theory was noticed by all of mathematics entitas which can be showed in terms action, process, object, and scheme. APOS can be used directly to analize data by researcher. The students' sucession or failure on mathematics homework can be compared with specific mental construction.

### A. Action Conception

Action is object transformation which is understood by individual with external stimulus assistance. According to Asiala [17], one of those stimulus types is details of stage solution. As an illustration, the students who understand in action level can interpret a situation as a function if they are given unique formulation to calculate values of function. If it does not comply with a request, they cannot do it. Their ability is only to evaluate the function of specific problem and manipulate function formulation.

According to Dubinsky, McDonald, & Brown, an action is object transformation or ordered command to do specisific operation [19]. For instance, to understand coset at action level is an individual only can make left coset from concrete group  $Z_{20}$ . Its manner construct subgroup  $H = \{0, 4, 8, 12, 16\}$  which is multiple of 4. Then, they write left coset from 5 as set  $5+H = \{1, 5, 9, 13, 17\}$ , it covers elemens  $Z_{20}$  with modulo 1 when divide by 4. The students at action level will be in difficulty when they manipulated left coset from permutation group.

DeVries said that to develop each concept is started from individuals' mind with an action. At this level, the students can only do action one step at the time [18]. For example, students are demanded to find alternative solution from linier equation system with n variables. If they begin with substitution each tuple separately, so they are predicted cannot imagine in advanced whether or not a tuple which is given can be substituted and give result of prospective solution (candidate solution). According to DeVries, their incapability shows that the students at action level can fullfil through action, but they did not think overall and predict the results [18].

Based on that literature study, action level in this research is subjects' ability to know and transform concept physically with external stimulus assistance that is software. Table 1 shows actions' indicator each concept which is learned by students.

Table 1. Indicator of Action Conception

Concept in Logic and Algorithm	Indicators
Variable	<ol style="list-style-type: none"> <li>1. The students can find input and output variable include its type (contain: simple variable, 1-D and 2-D array) based on external stimulus in form specific software.</li> <li>2. The students can transform all of variables into flowchart form use preparation symbols. They did not do without external stimulus.</li> </ol>
Constanta	<ol style="list-style-type: none"> <li>1. The students can find constantas include their values based on external stimulus in form specific software.</li> <li>2. The students can transform all or variable into flowchart form use preparation symbols. They did not do without external stimulus.</li> </ol>
Data	The students can make example data from input and output variable (minimal two examples) based on software is run by themselves.
Operator	<ol style="list-style-type: none"> <li>1. The students can name all of operators used in transforming input to output based on software is run by themselves.</li> <li>2. The students can calculate all of values of operation or and relation which are used in transforming input to output.</li> <li>3. The students can transform all of operators into flowchart form with process or decision symbols.</li> </ol>
Data Processing	<ol style="list-style-type: none"> <li>1. The students can differentiate between input and output variable including their type from softwares' display.</li> <li>2. The students can transform all of input and output variable into flowchart symbols (using input or output symbols).</li> <li>3. The students can name all of transformation process which are used in transforming input to output design. They are introduced to three otomation process: sequential, branching, and looping.</li> <li>4. The students can transform all of transformation process into flowchart symbols.</li> </ol>
Flowchart	The students can relate one flowchart symbol to another so that those symbols can form sequential, branching, and looping process based on software is run by themselves.

## B. Process Conception

When the students succeed to predict their results and they can find a shorter way than previous and can draw their action in a verbal manner without doing it. It means that concept developed in their mind at process level [18]. For instance, the students can find tuple approach as solution candidate on linier equation systems with  $n$  variables. They can image substitution action without doing it.

According to Dubinsky, when an action repeated, and individu reflects to his own action, so it is said that he has made internal mental construction, which is called process [8]. Process is internal mental construction on the same action without external stimulus. Individu in this mental process can reflect action itself without doing it. As an illustration, the students think about formulation sub group  $H$  to form left coset from permutation group  $S_4$  [8]. To think about forming those set is an indicator that the understanding of coset concept on the individu at process level.

Individuals' understanding at level process occurs when an action repeated, and then individu reflects on those actions, furthermore he interiorizes it into process. In other words, process is internal construction by doing the same action but without external stimulus [17]. An individu at process level can reflect, draw, or reverse transformation order without doing it. In this condition, a transformation is understood by individu in his mind, and under his control, more than his respon to external stimulus. For instance, to understand function of  $\sin(x)$ , individual needs conception of function process since there is no explicit instruction to find output from input, relating to implement the function. An individu must image relationship between real number and values of sinus. In abstract aljabar, a comprehension of coset including thought about formation of set by operating a certain element in which each elements in certain sub-group. Initially, they do not need to operate, but only to think about certain operation. So, with process conception, coset can be formed on situation where formulation is not given.

Based on the above literature study, the understanding of concept at process level in this research is subjects' ability in thinking about the same action based on input and output design which is given without external stimulus. The following Table 2 is indicator process from each concept in logic and algorithms.

Tabel 2. Indicator of Process Conception

Concept in Logic and Algorithms	Indicator
Variable	<ol style="list-style-type: none"> <li>1. The students can relate input and output variable include its type (contain: simple variable, 1-D and 2-D array) based on input and output design.</li> <li>2. The students can transform all of relationship between input and output variable into mathematics model and exactly flowchart symbols without external stimulus.</li> </ol>
Constanta	The students can think about all of constantas based on input and output design, and they can transform all of constantas to appropriate flowchart symbols without external stimulus.
Data	The students can make example of data from input and output variable (minimal two examples) based on input and output design.
Operator	<ol style="list-style-type: none"> <li>1. The students can think about all of operators are used in transforming input into output based on input and output design.</li> <li>2. The students can calculate all of values of operation or and relation which are used in transforming input into output based on input and output design.</li> <li>3. The students can transform all of operators into flowchart form with process or decision symbols without external stimulus.</li> </ol>
Data Processing	<ol style="list-style-type: none"> <li>1. The students can think about relationship between input and output variable based on input and output design.</li> <li>2. The students can think about mathematics model (such as; equation, not equation, equal to, and not equal) which transform input into output based on input and output design.</li> <li>3. The students can think about transformation all of variabels and mathematics model into flowchart symbols.</li> <li>4. The students can think about transformation process which is used in changing input into output. They are introduced to three otomation processes. They are sequential, branching, and looping process.</li> <li>5. The students can transform all of transformation processes into flowchart symbols.</li> </ol>
Flowchart	The students can think about relationship between one of flowchart symbol to another which can form sequential, branching, and looping process based on input and output design. There are minimal two structures.

## C. Object Conception

When individu views process as elemen from a collection of processes where rules or operation in set can be applied to the processes, so it is said that concept has developed in his mind into object [18]. For example, when students are faced on a system of linear equations and they are asked 'is the solution visible?', he can draw a solution from system. In this case, the process has been encapsulated into an object.

According to Dubinsky, McDonald, & Brown, object is constructed from a process when individu awares the process as a totality and he awares that transformations can be done on that process [19]. For example, an individu understands coset as object when he thinks about a number of cosets from specific subgroup, and can imagine the comparison between two cosets to know the equality or cardinality itself, or can make binary operation on set of all of subgroup coset.

According to Asiala et all, when an individu reflects an operation on certain process, he awares process as totality, and he awares transformation (action or process) can be done on that process, and he can construct some transformation, so he thinks about process as object [17]. In this case, it is said that the process has been encapsulated into an object. While individu do an action or process on an object, he often need deencapsulation object into process again relating to the use of those features to manipulate an object.

Based on the above literature study about object, object conception in this research is individuals' ability encapsulate action or process, so he able to think about transformation on action and process and he can relate action to process, process to process based on certain output. Action or process encapsulation is individuals' ability understand action or process as totality. Individu become aware that an operation can be done to action or process. He can construct appropriate transformation on action or process. While he transforms action or process, he often needs deencapsulation object to process again relating to the use of those features to manipulate object. Individu in this case viewed process as an object, which is static. The following Table 3 is object indicator of each concept in logic and algorithms.

Table 3. Indicator of Object Conception

Concept in Logic and Algorithms	Indicator
Variable	<ol style="list-style-type: none"> <li>1. The students can think about other variable, not include input and output variable, which is used in transforming input into output based on output desired by user or in question.</li> <li>2. The students can think about relationship between one transformation variable to another into flowchart symbols.</li> </ol>
Constanta	The students can think about all of constanta based on output desired by user or in question, they can think about transformation all of constanta into flowchart symbols.
Data	The students can make example of data from each variable which is used in transforming based on output was desired by user or in question.
Operator	<ol style="list-style-type: none"> <li>1. The students can think about all of operator needed on data processing based on output desired by user or in question.</li> <li>2. The students can think about the operation or relation result.</li> <li>3. The students can think about transformation all of operators into flowchart form by using process or decision symbols without external stimulus.</li> </ol>
Data Processing	<ol style="list-style-type: none"> <li>1. The students can think about input and output design include variable, contain: simple variable, 1-D and 2-D array, based on output desired by user or in question.</li> <li>2. The students can think about mathematics model that involved 1-D and 2-D array variables as form the input transformation into output based on output desired by user or in question.</li> <li>3. The students can think about transform all of variabel and mathematics model into flowchart symbols based on output desired by user or in question.</li> <li>4. The students can think about transformation process which is used in changing input into output based on output desired by user or in question. They are introduced to three otomation process: sequential, branching, and looping.</li> <li>5. The students can transform all of transformation process into flowchart symbols.</li> </ol>
Flowchart	<p>The students can think about modular technique to predict otomation process (i.e. sequential, branching, looping process, and their combination) based on output desired by user or in question which is presented in flowchart symbols.</p> <p>There are minimal two structures, and they can compare between one structure to another.</p>

#### D. Scheme Conception

A collection of process and object are organized into cognitive structure which is called scheme [17]. The complete statement is 'A collection of processes and objects can be organized in a structured manner to form a schema'. According to Dubinsky, McDonald, & Brown, the scheme of certain mathematics concept is a collection of actions, processes, objects, and other schemes, which are linked with universal principle to form work frame in individuals' mind [19]. They are centered on problem situation which involve those concepts. The idea of scheme according to Dubinsky, McDonald, & Brown [19] is close to concept image from Tall and Vinner [20]. According to Tall and Vinner, concept image is total cognitive structure relates to concept, include all of mental image and relationship between their features and process [20]. Concept image is constructed in years through their experiences and changes. Individuals' image concepts are different from formal theory and contain some factors which cause cognitive conflict.

Based on the above literature study of scheme, so scheme in this research is individuals' ability to organize a collection of actions, processes, objects, and other schemes is linked some universal principles to shape work frame in individuals' mind. They are centered on problem situation which involve all of concept in logic and algorithms. There for, individual can construct relationship among action, process, object, and

other concept to form business document at this level. The following Table 4 is scheme indicator from each concept in logic and algorithms.

Concept in Logic and Algorithms	Indicator
Variable	<ol style="list-style-type: none"> <li>1. The students can design simple relation between 1-D and 2-D array variables to make business documents.</li> <li>2. The students can think about relationship between one transformation variable to another to make business document into flowchart symbols.</li> </ol>
Constanta	The students can find all of constanta which is used in making business document, and they can think about transformation all of constanta into flowchart symbols.
Data	The students can make example of data to evaluate the truth of flowchart logically.
Operator	<ol style="list-style-type: none"> <li>1. The students can think about all of operators needed to make business documents.</li> <li>2. The students can transform all of operators into appropriate flowchart symbols.</li> </ol>
Data Processing	<ol style="list-style-type: none"> <li>1. The students can design input and output model to make business documents which need simple variable, 1-D or 2-D array variable.</li> <li>2. The students can design mathematics model which concern with simple variable, 1-D or 2-D array variable such as equation or not, and equal to or not to make business documents.</li> <li>3. The students can transform mathematics model into flowchart symbols.</li> </ol>
Flowchart	The students can construct flowchart to solve the problem associated with the making of business documents.

### E. The Characteristics of Competency Achievement Map

The above discussion of study literature shows that the characteristics of learning logic and algorithms according to APOS theory situated on competency achievement map. There for, competency achievement map is classified into four levels. They are action, process, object, and scheme conception. Action is ability to understand a concept with external stimulus like software. Process is ability to think about the same action based on certain input and output design. Object is ability to think about transformation on action and process, so they can relate action to process, process to process based on output desired by user. Scheme is ability to construct relationship among action, process, object, and other concepts to solve the problem associated with the making of business documents.

This paper only informs the learning design on action and process conception. We assumed that those conceptions are very important, because they are the lowest level in APOS theory. So, their truth must be proven.

#### a) Lesson Plan of Action

Based on those characteristics, the learning on an action conception requires software. Those softwares are used in learning basic concepts in logic and algorithms. They are variable, constanta, data, a variety of otomation process, input and output design. The students are requested to run it and observe each changing that occurs. The following figure is an example of display from software used in learning.

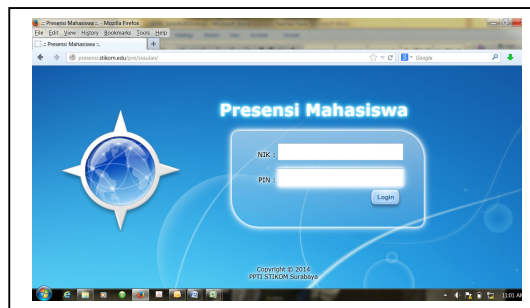


Figure 1. An example of display from software used in learning

Furthermore, they are asked about several things that are: input variables and their types; example of their data; name of otomation process used in changing input into output variables and input into output design. This learning model repeated until they can do it correctly. After that, they are introduced several flowchart symbols that are input, output, process, modul, branching, looping, and terminator. The activity repeated until they can do it correctly. All of activities repeated are different examples.

Software functions to change abstract into concrete concept. Thus, those activities are important in learning model on action conception. So, this model must be done in laboratory.

### b) Lesson Plan of Process

Learning model on process conception requires imagination of input and output design and some information which explain about rules which are used in transforming input into output design. The following figure is an example of learning model on process conception.

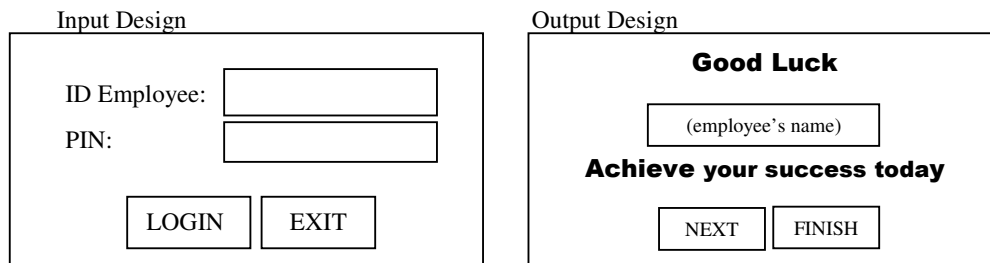


Figure 2. An example of input and output design was used to learning

The information given to students in learning model are as follow. The employees who succeed to login are: name 'susanti'; pin '001'; and she must push button 'login'. If these conditions are not fulfilled, so input display does not change. If button 'exit' is pushed by employee, then the program is finished. On the contrary, output design is displayed.

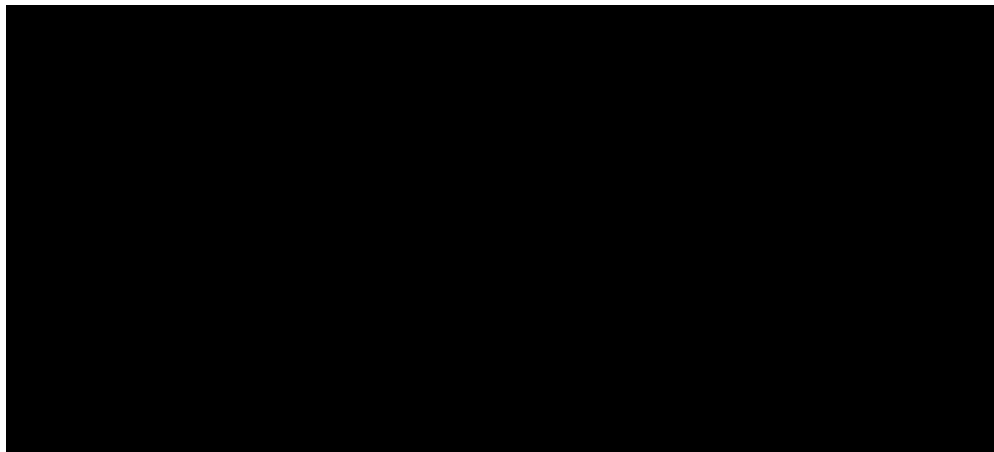
On those conditions, employees' name is displayed too. Furthermore, if button 'next' is pushed, so the next users can enter their identity again. But, if button 'finish' is pushed the program is finished.

The questions that are asked to students covers: input and output variables and their types; their example of data; name of transformation which is used in changing input into output design; and their appropriate flowchart symbols. After that, they are introduced about how to assemble flowchart symbols correctly.

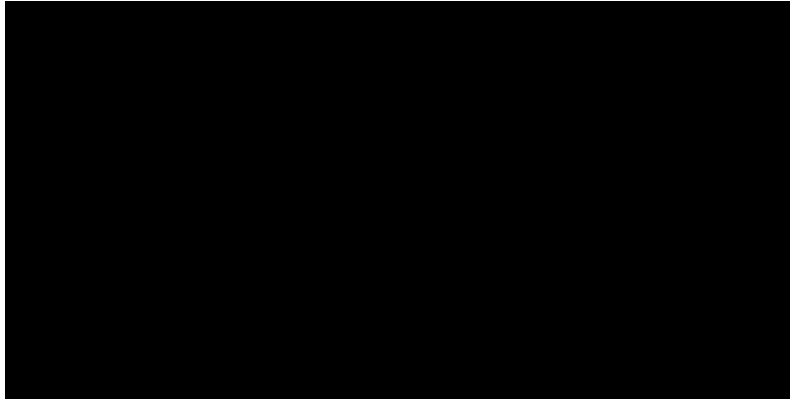
Those activities repeated until they can do it correctly. All of activities repeated are different examples. Input and output design that is followed by some information function to change abstract into semi-concrete concept. Thus, those activities are important in learning model on process conception. So, this model can be applied in class.

### F. Learning Outcomes

The following data are list of values of logic and algorithms on middle semester (MS). The achievement of sub competency defined only until process conception.

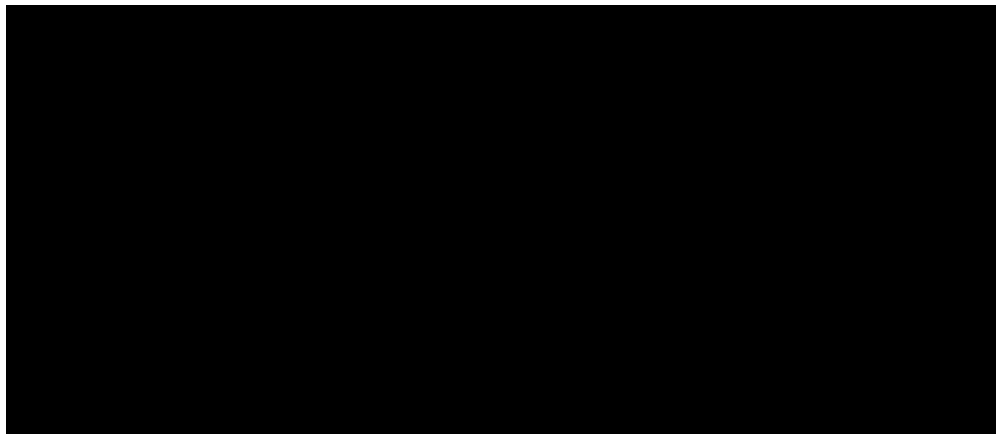


To test of varians were taken randomly 31 students from P1 class, because  $F_{table}(35,24)$  is not found. The  $F_{table}(30,24)$  which is available is 1.94. Those results are shown in the following table.



The result of varians test shows that  $H_0$  is not rejected on  $\alpha$  equal to 5%.  $F_{\text{count}}$  is 0.79 and  $F_{\text{table}}$  is 1.94. These result shows that both samples is from the same population. So, the next test can be done.

Because of number of students in P1 and P2 class are not equal, so not all of students in P1 class are taken. Therefore, to test mean of two mutually independent samples taken randomly 25 students from the class of P1. The result of sampling in P1 class is shown as follow.



The result of calculate of analysis of varians (ANOVA) is shown on the following table.

**Table 8. Anova Table**

Source of Variance	(1) Number of Square	(2) Degree of Freedom	(3) Middle Square (1)/(2)
Between treatment	SST = 5,060.18	k-1= 2-1 =1	MSTR = (SST/(k-1)) = 5,060.18
Error in treatment	SSE = 21,517.84	N-k= 50 - 2 = 48	MSE = SSE / (N-k) = 488.75
SS total	26,578.02		
$F_{\text{count}} = \text{MSTR}/\text{MSE} = 7.08$ ; $F_{\text{table}} = 4.32$ .			

Based on F table for  $\alpha$  equal to 5%, degree of freedom numerator equal to 1 and degree of freedom enumerator equal to 48 then  $F_{(1,48)}$  = (between 4 and 4.08). The rules of taking its decision is not rejected  $H_0$  when  $F_{\text{count}}$  less than or equal to (between 4 and 4.08). Because of  $F_{\text{count}} = 7.08$ , so  $H_0$  is rejected. Thus, values of P1 class greater than P2 are proven. It can be concluded that learning by using APOS theory increases the average value of logic and algorithms.

#### 4. CONCLUSION

Based on that literature reviewed, there are four classes of sub-competency achievement in the characteristics of logic and algorithms learning based on APOS theory. They are action, process, object, and scheme. Action is ability to understand a concept with external stimulus assistance like software. Process conception is ability to think about the same action based on specific input and output design. Object



conception is ability to think about transformation on action and process, so they can relate to action to process, process to process based on specific output. Scheme conception is ability to construct relationship between action, process, object, and other concept to problem solving to make business documents.

Based on the result test on the learning model, it is acknowledged that the model has been proven to students' learning outcomes achievement, especially on action and process level.  $H_0$  is rejected at  $\alpha$  equal to 5%, and  $F_{count} = 7.08$ . So, mean value of the class which apply APOS theory on their learning model proven greater than the other.

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