International Journal of Evaluation and Research in Education (IJERE)
Vol.2, No.4, December 2013, pp. 163~174
ISSN: 2252-8822

Incorporating Diagnostic Aspects to Mathematical Affects Inventory Development

Sun Hee Kim[1], Soojin Kim[2]
[1] Departement of Mathematical Education, Silla University
[2] Division of Educational Evaluation, Korea Institute for Curriculum & Evaluation

Article Info

Article history:

Received Jun 15, 2013
Revised Aug 29, 2013
Accepted Sep 10, 2013

Keyword:

Mathematics
Affects
Cognitive Diagnosis Theory
MAI

ABSTRACT

Many measures have been developed for the affective domain in mathematics, such as the Fennema-Sherman Mathematics Attitudes Scales (1976), Program for International Student Assessment (PISA), and Trends in International Mathematics and Science Study (TIMSS). However, it is difficult to find an inventory of affective attributes in mathematics that includes all affective factors based on recent theories. And affective attributes cannot be strictly differentiated, so one item can measure several factors. Thus, cognitive diagnosis theory was applied to this study, in which each item of assessment test can be allocated several attributes. The purpose of this study is to develop a Mathematical Affects Inventory (MAI) that measures student's specific affective attributes and to verify MAI using Cognitive Diagnosis Theory. Research results will report some statistical information with affective profile of Korean students i.e. the mathematical affects of individual students.

Corresponding Author:

Soojin Kim,
Division of Educational Evaluation,
Korea Institute for Curriculum & Evaluation ,
Jeongdong Bldg, 15-5, Jeong-dong, Jung-gu, Seoul, Korea.
Email: soojin76@gmail.com

1. INTRODUCTION

Usually, students' achievement of mathematical learning can be assessed in the cognitive domain. As the study of relationship between cognitive and affective domains was undergone, many of instruments or questionnaires that assess students' affective domain were developed and used. The instrument of Aiken (1974) or Fennema-Sherman (1976) were assessing the affective domain very broadly, but has developed 40 years ago [1];[2]. Recent developed instruments are assessing very narrow area such as self-control ability. The purpose of this study is to develop the new Mathematical Affects Inventory (MAI) that also contains the recent theoretical concepts.

To develop the questionnaire, the reliability and the validity of that instrument should be confirmed before using. The instrument of affective domain has been tested the validity by using the factor analysis. Therefore, each item has to be corresponded to each factor.

McLeod (1992) concluded that affective domains of mathematical learning are belief, attitude, and emotion [3]. These three factors are located on a spectrum from long lasting and stable to temporary and unstable. If we saw these three factors in this way, belief, attitude, and emotion are not strictly divided concepts. This study starts from this point. The items of MAI are not defined in one factor but assess several affective attributes. This can be confirmed from previous items. For example, one item from PISA 2003 student questionnaire, 'I make good marks at mathematics' is defined to assess self concept. However, this item relates with self confidence [4]. It is not conceptually clear and difficulty to divide self-concept and self-confidence. The attributes composing affective domain can be conceptualized but those factors cannot be strictly divided from other concepts and also are not necessary to be divided. Therefore, it is necessary to find out whether the results of the MAI can be used to make a decision regarding the affective domain in a right way.

Thus, in this study, cognitive diagnosis theory was used to validate MAI. Using cognitive diagnosis theory, each of the attributes can be related to several items and vice versa and these can be the foundation of analyzing students' mastery of each attributes qualitatively. It was proved that students' several mathematical attributes can be related to one item by already conducted study [5]. Each item can measure not only one item such as reasoning, problem solving and so on, but it can measure several attributes at once. The researchers were tried to apply cognitive diagnosis theory to MAI item and validate MAI instrument.

We focus on the specific identification of the attributes of the affective domain in the mathematical learning. Also how much students are aware of such attributes is examined. By challenging the previous belief and theory saying that each item of the questionnaire for the affective domain asks only one attribute, we will focus on the fact that one item is related to several attributes through the cognitive diagnosis theory. As a result, it is possible to state that the attributes of the affective domain in the mathematical learning are closely related to one another instead of being separate.

2. THE ATTRIBUTES OF THE AFFECTIVE DOMAIN

Until now, previous studies included mathematical affective domain topics about self-confidence in mathematics, interest in mathematics and usefulness of mathematics. Aiken (1974) developed a testing tool for the mathematical behavior with 33 items by classifying the behavior into two sub-factors such as the pleasure for the mathematical learning and the mathematical values [1]. In order to measure one's attitude about mathematics, Fennema-Sherman (1976) developed categories with nine factors including attitude to success in math, math as a male domain, confidence in learning math, math anxiety, and motivation [2]. Such a testing tool extensively influences every study related to the behavior about mathematics. In PISA 2003, five factors such as mathematical interest, instrumental motivation, self-efficacy, mathematical anxiety, self-concept were included in the background student questionnaire [4]. In TIMSS 2007, students' self-confidence in learning mathematics, students' valuing mathematics, students' positive affect toward mathematics were included in the background questionnaire [6].

Regarding the affective domain, a test instrument has been developed to investigate the mathematical belief of students. Schoenfeld (1989), who focused on the influence of the mathematical belief possessed by students, developed a tool which could be used to test a large group of students. He executed an overall inspection for the causes of success and failure in mathematics learning [7]. Kloosterman & Stage (1992) developed a test tool with five-point scale in order to measure the belief possessed by middle-school and high-school students as well as those studying at university [8]. In Pintrich & De Groot(1991), eight factors were included with three motivational components that may be linked to self-regulated learning [9]. Those are expectancy components which concerns students' beliefs about their expected success in performing a task, value component with students appreciation of and beliefs about the importance of the task, an affective component with students' emotional reaction. In Schoenfeld (1989), three factors were included in the instrument such as success and failure attributes, motivation, recognition about mathematics and school activities [7]. Lee & Kim (2005) developed a test tool for the measurement of one's self-directed learning capability with ten factors and 57 items based on the Vygotsky's theory for the conceptualization of one's self-directed learning capability, by considering motives, strategies and meta-cognition and analyzing such factors in ten different types in the process of preparing, executing and self-inspection learning [10].

In this study, six factors observed in 'affective domain of mathematical learning' structure were selected from Kim & Kim (2011) [11]. The affective representations which can be shown by students include learning directivity, anxiety, cognition of values, self-control, confidence and interest. Some of the representations could be regarded as the attitude and belief suggested by McLeod (1992) [3]. Anxiety, confidence and interest can be included in the attitude, while cognition of values can be included in the belief. Also, learning directivity and self-control can be regarded as the factors of the meta-affect [12]. The contents of inventory will be set according to the way the affective achievement of each student is established in the mathematical learning. By considering various studies (McLeod, 1992; DeBellis & Goldin, 2006; Hannuula, 2004), it is possible to conclude that affective attributes can be observed in various ways according to the direction taken [3];[13];[14].

It is not easy to check the results of the affective achievements shown by students. Even if it is possible for teachers to closely observe each student and find out their ideas through in-depth interviews, it would be more appropriate to provide students with inventory which require them to show their own responses in a written form like the cognitive test, in order to control a large size of students within the large framework of school education. Therefore, it is necessary to think about the attributes of affective achievements which can be checked through inventory. In this study, six attributes have been selected in order to reflect the concepts contained in the previous tests for the affective domain, while enabling students to recognize and represent their own state.

The first one is learning directivity, which is the attitude to actively challenge for any difficult and unfamiliar problem or task in the situation related to mathematical learning. It could be regarded as the will to learn mathematics. It includes the attitude to study mathematics hard and never give up on it. Learning directivity is related to the level of difficulty for each task. According to Lee & Kim (2010), the level of difficulty is the standard which can be used by students to select their own task, continuously try to solve such a task and be patient enough to overcome any difficult situation [15]. In particular, according to Schunk (1984), the preference for the level of difficulty is expressed in the process of selecting a challenging task which students believe that they could control and handle [16]. However, unlike the preference for the level of difficulty, it is more deeply related to the will of each student in regard to the mathematical learning, i.e. the attitude to solve any unfamiliar task or problem in the environment of mathematical learning.

The second attribute is self-control, which is the ability of each student to be aware of his or her learning method and control himself or herself performing any mathematical learning activity. In this study, regarding learning directivity and self-control, items suggested by Pintrich & De Groot (1990), Zimmerman & Martinez-Pons (1986) and Lee & Kim (2010) have been referred to [9];[17];[15].

The third attribute is anxiety, which is the psychological concern related to the poor performance of mathematics or the difficulty and inconvenience felt by students in the environment of mathematical learning. This attribute has been widely studied in the affective domain of mathematics. The anxiety shown in the mathematical learning could be related to the anxiety or tension caused by tests or public presentation carried out with other students in the class [18]. Therefore, it can be said that the mathematical anxiety is a conceptualized self-schema in regard to the experience of having achievements in the mathematical learning through the self-evaluation process.

The fourth attribute is interest. Interest means the kind of attention, preference and curiosity felt for mathematics and the activities related to the mathematical learning. It is a topic which has been widely dealt by various behavioral studies about mathematics. In various behavioral studies for the mathematical learning, including those carried out by Sandman (1974) and Fennema-Sherman (1976), interest is an important factor and one of the essential criteria [19];[2]. A high level of interest and preference for mathematics could be regarded as the media that make it possible for students to focus on their current learning and continuously carry out their learning in the future.

The fifth is the cognition of values about mathematics. By considering such a factor, it is possible to make a decision about or evaluate the mathematical function, usefulness and importance in the social, occupational and academic contexts or the one related to the life of each student. As the values and necessity about mathematics are highly evaluated, it is possible that the cognition of necessity about the mathematical learning becomes well. By understanding that mathematics is a necessary tool for the change and development of human life and culture and can play the role of a language, it would be possible to recognize the importance of mathematics and form a kind of expectation about its usefulness. Therefore, the cognition of values about mathematics could be regarded as an important attribute for the establishment of a positive affect about mathematics.

The last thing is confidence, which means the positive expectation about one’s own mathematical ability. Even if it has the opposite meaning of anxiety, it does not mean an objection. It is the concept of evaluating and confirming one’s own ability regarding what can be currently done for the mathematical learning at the moment and what can be done in the future. Also, it is the attribute that mutually influences the mathematical achievement of each student [20];[21]. Therefore, the establishment of confidence about mathematics could become an attribute that forms a sense of mathematical self-effectiveness or a positive affect about mathematics.

Such six attributes cannot be regarded as being separate from one another. For example, in case of the item that ‘I like solving difficult mathematical problem even if I get wrong answers.’, it would be possible to think about such two attributes as the involvement of learning directivity and the interest about mathematics regarding the process of solving a challenging mathematical problem. In this study, various attributes have been granted to MAI, while verifying the validity of each attribute according to the Cognitive Diagnosis Theory.

3. COGNITIVE DIAGNOSIS THEORY

Cognitive Diagnosis Theory was developed to evaluate examinees with respect to their levels of competence in each attribute such as knowledge or skills. The purpose of this theory is to provide students, teachers, or parents individual feedback regarding each student's mastery of the attributes measured by the assessment. Using this theory, students' mastery of each attribute can be diagnosed and their progress of learning can be estimated [22];[23];[24].

In Cognitive Diagnosis model, within one item, there can be several attributes measured. These attributes are described on Q-matrix, a n×k matrix containing ones and zeros, where k indicates the number of attributes we wish to assess and n indicates the number of items on the test [24]-[26]. If the attribute is needed to aware the item, the element of Q-matrix is 1; if not, it is 0. Among the models of the Cognitive Diagnosis approach, the Reparameterized Unified Model (RUM) also called Fusion Model, developed by Hartz, Roussos, & Stout (2002), is considered to be successful [27]. The Cognitive diagnostic models based on item response theory define the probability of observing the response of examinee j to item i, given the examinee's ability parameters and item parameters. The equation of the Fusion model is as follows:

$$P\left(X_{ij} = 1 \middle| \alpha_j, \theta_j\right) = \pi_i^* \prod_{k=1}^{K} r_{ik}^{*(1-\alpha_{jk})\times q_{ik}} \, P_{C_i}(\theta_j)$$

In this equation, qik is the attribute k which is measured by item i. If $q_{jk} = 1$, that means attribute k is measure by item i; otherwise $q_{jk} = 0$. If $\alpha_{jk} = 1$, it indicates that examinee j has mastered attribute k; otherwise, $\alpha_{jk} = 0$. The symbol Xij=1 indicates the response of examinee j to item i, where x = 1 indicates an agreement response and x = 0 indicates a disagreement response. The parameter π_i^* is the probability of correctly applying all items i with the required attributes. It can be explained as the probability of an examinee, who has mastered all attributes for item i, to correctly apply those attributes when responding item i. This application is interpreted as the Q-based item i difficulty [27]. The parameter r_{ik}^* is the proportional parameter representing the ratio of the likelihood of a agree answer, given mastery versus non-mastery [27]. It can be interpreted as the item i discrimination parameter for attribute k [27];[28], $P_{C_i}(\theta_j)$ is the probability of applying the skills correctly, particularly the skills not specified by the Q-matrix.

Recently, there are many studies being conducted about Cognitive Diagnosis Theory. For example, the study analyzing students' ability using TIMSS data was conducted using cognitive diagnostic model [29]. Dogan & Tatsuoka (2008) reported a comparison of the mastery of the mathematics attributes between students in Turkey and the United States using TIMSS-R results [30]. Kim, Kim & Song (2008) analyzed the mastery of the mathematics attribute for Korean grade 9 students using the Fusion Model [31]. Instead of giving students a total score which represents an overall ability, it was more effective to plan their learning when more specific information regarding the mastery of each attributes was given. Kim (2009) identified what cognitive attributes are required of eighth graders to solve geometrical problems such as 'Recall,' 'Analyze,' 'Justify,' 'Synthesize/Integrate,' and 'Solve Non-routine Problems' by using the cognitive diagnostic theory [32]. The five attributes are proved as the skills for solving the geometric problems. Many students have not fully mastered the attributes of 'Justify' and 'Synthesize/Integrate'. 'Analyze' best predicted the changes in the geometric achievement.

Here affective attributes in mathematics will be analyzed using the Fusion Model. There are a series of processes needed to use the Fusion Model. First, the proper items that can measure students' affective attributes should be developed. Second, each attribute measured by each item needs to be set. The attributes that measure students' mastery are related to several behaviors; therefore, it needs to be detailed and concrete. This step is for subject specialists who develop and analyze the items and teachers who evaluate students. Third, once the attributes are set, the Q-matrix should be created. Q-matrix is the matrix showing the relationship of attributes that items need to measure and the items. Fourth, the item parameters regarding each attribute and student parameter of mastery of each attribute needs to be estimated.

4. DEVELOPMENT OF MAI

MAI was developed and validated as follows. Initially, we have developed 63 survey items about these six attributes. Each item was selected from those which had been used in the previous affective inventories of mathematics. They were related to learning strategies, self-concept, cognition of values, attitudes, motives, anxiety, interest and confidence. However, the concept related to the factor became more sophisticated in the process of selecting the final items. According to the Exploratory Factor Analysis (EFA) and the Confirmatory Factor Analysis (CFA), the concept of each attribute became materialized and the number of items was reduced to a total of 28 items at the end for MAI.

The affective achievement test was executed in December, 2010 for 1,320 students in the grade 8 at eight middle schools located in Korea. The students made their responses to the investigation by fully using the time of a 45-minute class.

4.1. Development of the Q-matrix

To apply the cognitive diagnosis theory, Q-matrix that shows the relation between each item and each attributes are needed. The Q-matrix based on 28 items and six attributes was established by eight specialists including three professors of mathematical education, one high-school math teacher, two specialists in the field of measurement evaluation, and two specialists with a master's degree in the field of mathematical education. At first, it was established with the attributes based on the opinion of each specialist. It was rearranged to be included in more than two attributes in an intersectional way based on the factor analysis. Based on various parameters about the Q-matrix, the matrix was adjusted several times. The final items of MAI are shown in Table 1, where reverse item will be represented as (R).

Table 1. Mathematical Affects Inventory (MAI)

attribute			item
Learning Directivity	1	Q2	It is exciting to challenge for complicated and difficult mathematical problems.
	2	Q5	I like solving difficult mathematical problems even if I get wrong answers.
	3	Q28	I like to solve one difficult mathematical problem rather than solving several easy problems.
	4	Q30	I enjoy challenging for unfamiliar mathematical problems.
	5	Q34	I like to solve the mathematical problems which make me deeply think about them, even if it would take more time to solve them.
Self-control	6	Q8	I can concentrate in the process of learning the mathematical contents which I do not like.
	7	Q15	I know how to become effective when studying mathematics.
	8	Q22	When it is necessary to study mathematics, I never delay my study about the subject.
	9	Q36	I can solve every difficult problem of a math test patiently until the end.
	10	Q54	I study mathematics myself without being ordered by anyone.
	11	Q59	Once I start to study mathematics, I always study it hard until the end.
Anxiety	12	Q4	I often become anxious that I would have a bad grade before having a math test.
	13	Q6	I cannot sleep well when a math test is coming.
	14	Q25	I often become anxious when I make a presentation in the math class and afraid that I would make mistakes.
	15	Q43	I often become anxious that I would make mistakes when I solve questions in front of others in the math class.
Interest	16	Q35	Mathematics is an interesting subject.
	17	Q37	I do not like to study mathematics. (R)
	18	Q40	Mathematics is boring. (R)
	19	Q46	When I concentrate in the math class, I cannot stop thinking that the class ends too quickly.
	20	Q63	I like the math class.
Cognition of Values	21	Q24	The students with a good grade for mathematics will become more successful in their future jobs.
	22	Q39	The students with a good grade for mathematics will enter better universities.
	23	Q44	Mathematics is an absolutely necessary subject for everyday life.
	24	Q48	Mathematics helps people to have logical ideas.
	25	Q58	Mathematics is one of the important subjects learned at school.
	26	Q62	Mathematics will be useful for various jobs in the future.
Confidence	27	Q53	I could study mathematics really well.
	28	Q57	I am not good at mathematics. (R)

In Table 1, there will be 1 attribute per each item. The first item Q2 is about learning directivity, but if the students have interest in math, they might give a positive response to this item. The item Q4 is about anxiety. However, when students have no confidence in mathematics, they will answer yes to this item. Each item belonged to only one attributes should be questioned.

With expert's opinion, Q-matrix was created as Table 2. In Q-matrix, column represents attributes and row represents items. If students mastered the attribute, 1 was presented and if not 0. For example, if a student with learning directivity answers Q2, then 1 was presented, and otherwise, 0 was presented. In each item, there were 1 to 3 attributes.

Table 2. Q matrix of MAI

Item number	Learning directivity	Self-Control	Anxiety	Interest	Cognition of Values	Confidence	Number of Attributes
Q2	1	0	0	1	0	0	2
Q4	0	0	1	0	0	1	2
Q5	1	0	0	1	0	0	2
Q6	0	0	1	0	0	0	1
Q8	1	1	0	0	0	0	2
Q15	0	1	0	0	0	0	1
Q22	1	1	0	0	0	0	2
Q24	0	0	0	0	1	0	1
Q25	0	0	1	0	0	0	1
Q28	1	0	0	1	0	0	2
Q30	1	0	0	1	0	0	2
Q34	1	0	0	1	0	0	2
Q35	0	0	0	1	1	0	2
Q36	1	1	0	0	0	0	2
Q37	0	0	0	1	0	0	1
Q39	0	0	0	0	1	0	1
Q40	0	0	0	1	0	0	1
Q43	0	0	1	0	0	0	1
Q44	0	0	0	0	1	0	1
Q46	1	0	0	1	0	0	2
Q48	0	0	0	0	1	0	1
Q53	0	0	0	0	0	1	1
Q54	1	1	0	0	0	0	2
Q57	0	0	0	0	0	1	1
Q58	0	0	0	0	1	0	1
Q59	1	1	0	1	0	0	3
Q62	0	0	0	0	1	0	1
Q63	0	0	0	1	0	0	1
Total	11	6	4	11	7	3	42

4.2. Method of Analysis

The inventory consisted of four-point criteria originally. However, in order to apply the Q-matrix, it was necessary to have the type of ‘Yes or No’ response for each item. In this study, an analysis was executed by converting the 3-point and 4-point scale to ‘Yes’ and the 1-point and 2-point scale to ‘No’,

In regard to the application of the cognitive diagnosis theory, the Markov Chain Monte Carlo method of estimation was used to estimate the parameters for the items and subjects of the fusion model. Regarding the analysis, the Arpeggio3_1 program [27] was used. Based on the results of the analysis, whether the Q-matrix could be established according to the cognitive diagnosis theory in the affective evaluation was considered.

Given a set of scored item response data and an associated Q matrix, examinee skills level mastery classification can be calculated by using Fusion Model. To explain more details, based on the examinee's item responses, examinee level mastery classification can be inferred examinee's mastery level of each attribute. Two mastery levels of each attribute are considered as master and non-master. The term used for the two mastery levels can be calculated as like this. Assume that a student's competencies are modeled by a profile of mastery levels. The vector can be represent by 0/1 vector: $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_k)$. Here k is the total number of attributes in the test and α_k is known as students' level of mastery of attribute k represented as 1 or 0 according to mastery.

The correlations between attributes were also examined. The inter-attribute correlations were calculated using Pearson's correlation coefficient as applied to subscores of each attribute. Subscores were calculated by adding up the responses of each item that belongs to the related attribute. Also, a frequency analysis was conducted to determine what attributes students have mastered according to gender and differentiated mathematics instruction which is special to Korean education.

5. RESULTS

5.1. Validity of the Cognitive Diagnosis Theory

According to the Markov Chain Monte Carlo method of estimation, it can be said that the parametric estimations with the autocorrelation of less than 0.2 could be generally accepted [28]. In this study, the autocorrelation of most of the parametric estimations for the items and students that are subject to the Q-matrix was found to be less than 0.2. As a result, most of the parametric estimations were accepted.

Most of the items showed reasonable parametric estimations. In Table 3, all of the parametric estimations for the item i.e. π_j^* showed values higher than 0.6. As a result, it could be said that such items strongly require the affective attributes specified in the Q-matrix. In case of π_j^*, the parametric estimations of the items were relatively lower than the values of other cognitive tests. Accordingly, it could be said that the attributes related to each item can be identified well.

Table 3. Verification of MAI using Cognitice Diagnosis Theory

Number	Item number	π_j^*	r*1	r*2	r*3	r*4	r*5	r*6	c
1	Q2	0.91	0.24			0.45			10
2	Q4	0.74			0.57			0.62	10
3	Q5	0.75	0.24			0.42			10
4	Q6	0.90			0.78				10
5	Q8	0.75	0.77	0.45					10
6	Q15	0.75		0.19					10
7	Q22	0.67	0.73	0.19					10
8	Q24	0.85					0.42		10
9	Q25	0.83			0.21				10
10	Q28	0.80	0.26			0.71			10
11	Q30	0.88	0.17			0.57			10
12	Q34	0.91	0.22			0.50			10
13	Q35	0.88				0.16	0.63		10
14	Q36	0.90	0.72	0.57					10
15	Q37	0.90				0.22			10
16	Q39	0.84					0.47		10
17	Q40	0.88				0.25			10
18	Q43	0.92			0.09				10
19	Q44	0.85					0.34		10
20	Q46	0.64	0.70			0.59			10
21	Q48	0.89					0.40		10
22	Q53	0.92						0.03	10
23	Q54	0.63	0.71	0.22					10
24	Q57	0.75						0.33	10
25	Q58	0.97					0.59		10
26	Q59	0.95	0.70	0.47		0.59			10
27	Q62	0.94					0.30		10
28	Q63	0.71				0.12			10

Table 4 shows the population estimations of each subject group regarding various attributes. The cognition of values was the highest.

Table 4. Estimates of each attributes

Attributes	Learning directivity	Self-Control	Anxiety	Interest	Cognition of Values	Confidence
p_k	0.44	0.46	0.54	0.48	0.64	0.40

The purpose of this study was to develop MAI that can assess students' mathematics achievement on the affective domain and validate using cognitive diagnosis theory. Using cognitive diagnosis theory, we found that attributes of affective domain are not divided and discontinued but related to each other. More analysis regarding this result was performed on next part.

5.2. Correlation for Each Attribute

Each attribute of affective domains were found to be related to each other. The Pearson's correlation coefficients were calculated to examine the correlation between each attribute.

Table 5 Correlation coefficients between attributes

	Learning directivity	Self-Control	Anxiety	Interest	Cognition of Values
Self-control	.870**				
Anxiety	.248**	.258**			
Interest	.933**	.746**	.269**		
Cognition of Values	.502**	.458**	-.024	.531**	
Confidence	.539**	.551**	.508**	.528**	.188**

** p<.01

On Table 5, it can be said that each one of the six attributes has a statistically significant correlation with one another, except for the relationship between anxiety and the cognition of values. Regarding the values of mathematics, there is no relationship between the cognition of the usefulness of mathematics for the future job or learning and the possession of anxiety.

Also, since anxiety is a negative concept, it needs to have a negative correlation with other attributes. However, in this study, students were divided into two groups including those for master and non-master. As a result, it showed a positive correlation with other attributes.

5.3. Results of the Affective Achievements of Students

The master (1) or non-master (0) related to the affective attributes of students was analyzed based on the classes for each gender and differentiated instruction. Please refer to Table 6. The differentiated instruction is the part of the government policy for educational curriculums in Korea. As a result, a number of schools are applying this policy. The mathematical achievements of most students were classified into upper/middle/lower levels. The students at the same level were put in the same class.

Table 6. Number of Students mastered for Affective attributes

( ): %

attributes	Boy (N=777)	Girl (N=525)	Total (N=1302)	χ2(1)	Diff (N=1110)	Not Diff (N=199)	Total (N=1299)	χ2(1)
Self Directivity	368 (47.4)	252 (48.0)	620 (47.6)	.051	531 (47.8)	90 (45.2)	621 (47.8)	.272
Self-control	372 (47.9)	258 (49.1)	630 (48.4)	.201	552 (49.7)	77 (38.7)	629 (48.4)	3.093*
Anxiety	445 (57.3)	274 (52.2)	719 (55.2)	3.271*	616 (55.5)	101 (50.8)	717 (55.2)	.008
Interest	438 (56.4)	294 (55.6)	732 (56.2)	.017	630 (56.8)	101 (50.8)	731 (56.3)	.050
Cognition of Values	511 (65.8)	353 (67.2)	864 (66.4)	.304	728 (65.6)	133 (66.8)	861 (66.3)	4.338**
Confidence	394 (50.7)	284 (54.1)	678 (52.1)	1.440	589 (53.1)	87 (43.7)	676 (52.0)	1.518

*p<.10 **p<.05

The missing was deleted for gender and differentiated instruction. Therefore the sizes of samples are different.

From the results of achievement of affective domain of grade 8 students in Korea, we can see that the cognition of value about mathematics (66.4%) recorded the highest which was followed by interest (56.2%).

Regarding gender, there were proportional differences within the significant level of 0.10 in terms of anxiety. 57.3% of the male students were identified as those with anxiety, while 52.2% of the female students had anxiety. In the Korean society, the level of expectation on male students is high, while female students have recently shown that they advance superior to their male counterparts in terms of mathematical achievements. As a result, it could be said that male students tend to show a high level of anxiety.

In regard to the execution of the differentiated classes, there was a proportional difference within the significant level of 0.05 for the cognition of values about mathematics. 65.6% of the students who were in the differentiated instruction and 66.8% of the students who were not in that, recognized the values of mathematics. Simply by receiving the differentiated instruction, students were not motivated to study mathematics harder. It could be said that there would be more students who recognize the values of mathematics when not receiving the differentiated instruction. Also, there was a proportional difference for the differentiated instruction in regard to self-control within the significant level of 0.10. 49.7% of the students who were receiving the differentiated instruction and 38.7% of the students who were not receiving such instruction showed mastery for self-control. The students participating in the differentiated instruction tended to make and implement their own learning plans, showing a high level of master for self-control.

Table 7 shows the number of affective attributes mastered by students. The percentage of students who master one attribute only were 18.0% and highest. 14.6% of the students mastered all of the six attributes, while 4.1% of the students did not master any attribute in Korea. For gender, students who mastered all six attributes were very similar, 14.8% and 14.3%, for boys and girls respectively. There were no significant findings for different levels mathematics class.

Table 7. Frequency of mastery

( ): %

# of mastered attributes	boy (N=777)	girl (N=525)	total (N=1302)	Diff (N=1110)	Not Diff (N=199)	total (N=1299)
0	31 (4.0)	23 (4.4)	54 (4.1)	48 (4.3)	6 (3.3)	54 (4.2)
1	146 (18.8)	88 (16.8)	234 (18.0)	196 (17.5)	36 (19.8)	232 (17.9)
2	128 (16.5)	87 (16.6)	215 (16.5)	180 (16.1)	36 (19.8)	216 (16.6)
3	118 (15.2)	89 (17.0)	207 (15.9)	184 (16.5)	22 (12.1)	206 (15.6)
4	113 (14.5)	79 (15.0)	192 (14.7)	165 (14.8)	27 (14.8)	192 (14.8)
5	126 (16.2)	84 (16.0)	210 (16.1)	186 (16.7)	23 (12.6)	209 (16.1)
6	115 (14.8)	75 (14.3)	190 (14.6)	158 (14.1)	32 (17.6)	190 (14.6)

The missing was deleted for gender and differentiated instruction. Therefore the sizes of samples are different.

The results can be explained by giving students a profile scores. Out of six attributes, mastered attributes were reported, 1 if student mastered the attribute, 0 otherwise. Table 8 shows examples for the individual profile of each student. Regarding genders, male students represent 1, while female students represent 2. Regarding differentiated instruction, implementation represents 1, while non-implementation represents 2. In case of ID 18, this student is female who is receiving differentiated mathematics instruction with her profile of 111001. This student has mastered learning directivity, self control, anxiety, and confidence, but she has not mastered interest and cognition of value, therefore lack of these two attribute. From this profile results, teachers can give very useful information to each of the students about their affective domain achievement. Also, teachers can have meaningful communication with each of the students regarding education and their learning process.

Table 8. Example of each student' affective profile

Student ID	gender	Diff	Item response	profile
18	2	1	0100101101001001010110110000	111001
19	2	1	0001001111100011010010000101	111100
20	2	1	1110110001111111111111101111	111111
21	2	1	0101011100010111110110010011	011111
22	2	1	0100101101000110010000110101	111101
23	2	1	1001001111000101101110101110	110010
24	2	1	0001110110000110101011110000	011101
25	2	1	1111011101111111101111111111	110111
26	2	1	0010111100010001001010011111	110011
27	2	1	0001010100000101001110001010	000010

Table 9 shows the proportions of Korean students by considering the mastered attribute as 1 and the no-mastered attribute as 0. Regarding six attributes, those mastered by students show various proportions. The highest proportion is 14.58% which shows those who mastered all the six attributes. The second highest proportion is 11.51% which shows those who only mastered the cognition of values. For each student, the mastered affective attributes are very various.

Table 9 Distribution of profiles

Learning directivity	Self-Control	Anxiety	Interest	Cognition of Values	Confidence	N	proportion (%)
0	0	0	0	0	0	54	4.14
0	0	0	0	0	1	11	0.84
0	0	0	0	1	0	150	11.51
0	0	0	0	1	1	19	1.46
0	0	0	1	0	0	11	0.84
0	0	0	1	0	1	3	0.23
0	0	0	1	1	0	30	2.30
0	0	0	1	1	1	12	0.92
0	0	1	0	0	0	48	3.68
0	0	1	0	0	1	13	1.00
0	0	1	0	1	0	58	4.45
0	0	1	0	1	1	12	0.92
0	0	1	1	0	0	15	1.15
0	0	1	1	0	1	24	1.84
0	0	1	1	1	0	21	1.61
0	0	1	1	1	1	11	0.84
0	1	0	0	0	0	4	0.31
0	1	0	0	0	1	2	0.15
0	1	0	0	1	0	15	1.15
0	1	0	0	1	1	7	0.54
0	1	0	1	0	0	3	0.23
0	1	0	1	0	1	6	0.46
0	1	0	1	1	0	9	0.69
0	1	0	1	1	1	10	0.77
0	1	1	0	0	0	9	0.69
0	1	1	0	0	1	8	0.61
0	1	1	0	1	0	18	1.38
0	1	1	0	1	1	6	0.46
0	1	1	1	0	0	11	0.84
0	1	1	1	0	1	32	2.46
0	1	1	1	1	0	12	0.92
0	1	1	1	1	1	38	2.92
1	0	0	0	0	0	10	0.77
1	0	0	0	0	1	7	0.54
1	0	0	0	1	0	23	1.77
1	0	0	0	1	1	12	0.92
1	0	0	1	0	0	12	0.92
1	0	0	1	0	1	5	0.38
1	0	0	1	1	0	15	1.15
1	0	0	1	1	1	17	1.30
1	0	1	0	0	0	2	0.15
1	0	1	0	0	1	2	0.15
1	0	1	0	1	0	9	0.69
1	0	1	0	1	1	2	0.15
1	0	1	1	0	0	5	0.38
1	0	1	1	0	1	19	1.46
1	0	1	1	1	0	13	1.00
1	0	1	1	1	1	28	2.15
1	1	0	0	0	0	5	0.38
1	1	0	0	0	1	11	0.84
1	1	0	0	1	0	13	1.00
1	1	0	0	1	1	15	1.15
1	1	0	1	0	0	4	0.31
1	1	0	1	0	1	19	1.46
1	1	0	1	1	0	10	0.77
1	1	0	1	1	1	60	4.60
1	1	1	0	0	0	3	0.23
1	1	1	0	0	1	8	0.61
1	1	1	0	1	0	4	0.31
1	1	1	0	1	1	10	0.77
1	1	1	1	0	0	14	1.07
1	1	1	1	0	1	59	4.53
1	1	1	1	1	0	15	1.15
1	1	1	1	1	1	190	14.58
Total						1,303	100.00

6. DISCUSSION

We initiated this study based on the fact that the existed inventories related to the affective evaluation of the mathematical learning had been developed a long before and failed to reflect the recent theories. As a result, the MAI, which included such attributes as directivity or self-control about the mathematical learning, was developed. In the previous study about this tool, validity was verified using EFA & CFA [33].

In this study, it was assumed that the items asking the affective attributes could be involved in various attributes. Therefore, the Q-matrix was used for analysis and the related validity was secured by using the cognitive diagnosis theory. Even if the cognitive diagnosis theory is usually used to identify cognitive attributes, in this study, it can be used on the analysis of affective domain. The affective attributes are not separate from one another, but mutually related to one another. As a result, regarding the decision related to the master or non-master of students for the affective attributes, the latest method such as the Cognitive Diagnosis Theory must be utilized. Since this method has not been popularized due to its exclusiveness, it will be necessary to develop the software which can be used to identify the affective attributes related to the mastery of students by using this MAI.

This study focuses on the inspection which was carried out by targeting the students in the grade 8 in Korea. The attribute which was mastered by the highest number of students was the cognition of values about mathematics (66.4%). Such attributes as the interest and confidence about mathematics were also mastered by more than 50% of the students. In Korea, the level of achievement in the affective domain has been always reported to be lower than those of other countries through such studies as TIMSS and PISA. Such results are based on the comparison with other countries. Since modesty is often regarded as a virtue in the Orient, students might express their affective state lower than what it is. Therefore, as shown in the analysis of the results of this study, it is necessary to compare the affective attributes for the master or no-master of the students in other countries with those for the students in Korea, and make second decisions if necessary.

7. CONCLUSION

Lastly, it is necessary to study the attributes of the affective domain based on the level of academic achievement. In Korea, the differentiated instruction has been applied since the second half of 1990. Regarding the current curriculums, the mathematical classes have been expanded to include those based on different levels. Since the differentiate instruction materializes various curriculums based on the different levels of students, the level of satisfaction could be high for the classes. However, since the enthusiasm for education in Korea is very high, there seem to be complaints made by the parents whose children are in the low-level classes. Nevertheless, it is important to think about the fact that the students who were not receiving the differentiated instruction in Korea showed a higher proportion for the cognition of values about mathematics. Also, it is important to consider the fact that the level of anxiety shown by male students in regard to mathematics is higher than the one shown by their female counterparts. Regarding such a fact, it will be necessary to carry out an anthropological study in the future.

REFERENCES

[1] Aiken, L. R. “Two Scales of attitude toward mathematics”, *Journal for Research in Mathematical Education*, Vol. 5. Pp. 67-71, 1974.

[2] Fennema, E. & Sherman, J. A. “Fennema-Sherman Mathematics Attitudes Scales: Instruments designed to measure attitudes towards the learning mathematics by females and males”, *Catalog of Selected Documents in Psychology*, Vol. 6. Pp. 31, (MS. No. 1225), 1976.

[3] McLeod, D. B. “Research on affect in mathematics education: a reconceptualization. In D. A. Gouwes” (ed.), *Handbook of research on mathematics teaching and learning*, Macmillan, NY. Pp. 575-596, 1992.

[4] Mullis, I. V. S., Martin, M. O., & Foy, P. TIMSS 2007 International Mathematics Report: Findings From IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grade. MA: Boston College, 2008.

[5] Schoenfeld, A, H. “Explorations of Student's Mathematical beliefs and behavior”, *Journal for Research in Mathematical Education*, Vol/Issue: 20(4). Pp. 338-355, 1989.

[6] Kloosterman, P & Stage, F. K. “Measuring beliefs about mathematical problem solving”, *Journal of Korea Society of Educational Studies in Mathematics: School Science and Mathematics*, Vol/Issue: 92(3). Pp. 109-115, 1992.

[7] Pintrich, P. R., & De Groot, E. V. “Motivational and self-regulated learning components of classroom academic performance”, *Journal of Educational Psychology*, Vol. 82. Pp. 33-40, 1990.

[8] Lee, C. H. & Kim, S. H. “Development of the self-directed mathematics learning test based on Vygotsky”, *Journal of Korea Society of Educational Studies in Mathematics: School Mathematics*, Vol/Issue: 7(3). Pp. 253-268, 2005.

[9] DeBellis, V. & Goldin, G. “Affect and meta-affect in mathematical problem solving: a representational perspective”, *Educational Studies in Mathematics*, Vol. 63. Pp. 131-147, 2006.
[10] Schunk, D. H. “Self-efficacy perspective on achievement behavior”, *Educational Psychology*, Vol. 19. Pp. 48-58, 1984.
[11] Zimmerman, B. J., & Martinez-Pons, M. “Development of structured interview for assessing student use of self-regulated learning strategies”, *American Educational Research Journal*, Vol. 23. Pp. 614-628, 1986.
[12] Bandura, A. *Social foundations of thought and action: A social cognitive theory*. Englewood Cliffs, NJ: Prentice-Hall, 1986.
[13] Embretson, S. “Diagnostic testing by measuring learning processes: Psychometric considerations for dynamic testing”, 1990. In N. Frederiksen, R. L. Glasser, A. M. Lesgold, and M. G. Shafto (Eds.), *Diagnostic monitoring of skills and knowledge acquisition*. Hillsdale, NJ: Lawrence Erlbaum Associates.
[14] DiBello, L., Stout, W., & Rousses, L. “Unified cognitive/psychometric diagnostic assessment likelihood-based classification techniques”, 1995. In P. D. Nichols, S. F. Chipman, and R. L. Brennan (Eds.), *Cognitively Diagnostic Assessment*. Hillsdale, NJ: Lawrence Erlbaum Associates.
[15] Tatsuoka, K. K.. “Architecture of knowledge structure and cognitive diagnosis: A statistical pattern recognition and classification approach”, 1995. In P. D. Nichols, S. F. Chipman, and R. L. Brennan (Eds.), *Cognitively Diagnostic Assessment*, 327-359. Hillsdale, NJ: Lawrence Erlbaum Associates.
[16] Tatsuoka, K. K.. “Rule space: An approach for dealing with misconceptions based on item response theory”, *Journal of Educational Measurement*, Vol/Issue: 20(4). Pp. 345-354, 1983.
[17] Hartz, S., Roussos, L., and Stout, W. *Skills Diagnosis: Theory and Practice. User Manual for Arpeggio software*. Princeton, NJ: ETS, 2002.
[18] Hartz, S. “A Bayesian framework for the Unified Model for assessing cognitive abilities: blending theory with practice”, *Doctoral thesis.* The University of Illinois at Urbana-Champaign, 2002.
[19] Tatsuoka, K. K., Corter, J. E. & Tatsuoka, C. “Patterns of Diagnosed Mathematical Content and Process Skills in TIMSS-R Across a Sample of 20 Countries”, *Educational Research Journal*, Vol/Issue: 41(4). Pp. 901-926, 2004.
[20] Dogan, E. & Tatsuoka, K.. “An international comparison using a diagnostic testing model: Turkish students' profile of mathematical skills on TIMSS-R”, *Educational Studies in Mathematics*. online first. DIO 10.1007/s10649-007-9099-8, 2008.
[21] Kim, S. H., Kim, S., Song, M. Y. “Using Cognitive Diagnosis Theory to analyze the test results of mathematics”, *School Mathematics*, Vol/Issue: 10(2). Pp. 259-277, 2008.
[22] Kim, S. H. “Assessing Cognitive Attributes in the 8th grade Geometry”, *Journal of Korea Society of Educational Studies in Mathematics: The Journal of Educational Research in Mathematics*, Vol/Issue: 19(4). Pp. 531-543, 2009.
[23] Lee, C. H. , S. H. Kim, B. Kim, K. Kim, S. Kim, S. Yun and Y. Kim. “Development and verification of an affective achievement test instrument in mathematical learning”, *Korea Society of Mathematical Education: The mathematical Education*, Vol/Issue: 50(2). Pp. 247-261, 2011.