International Journal of Evaluation and Research in Education (IJERE)
Vol. 15, No. 3, June 2026, pp. 2096~2110
ISSN: 2252-8822, DOI: 10.11591/ijere.v15i3.39348

Beyond procedural mastery: a semiotic analysis of students' understanding of the first derivative definition

Beni Asyhar[1], Ummu Sholihah[1], Elva Yohana[2], Anwaril Hamidy[3]
[1]Mathematics Education Study Program, Universitas Islam Negeri Sayyid Ali Rahmatullah Tulungagung, Tulungagung, Indonesia
[2]English Language Education Study Program, Universitas Islam Negeri Sayyid Ali Rahmatullah Tulungagung, Tulungagung, Indonesia
[3]Department of Mathematics Education, University of Bremen, Bremen, Germany

Article Info

Article history:

Received Mar 5, 2026
Revised May 20, 2026
Accepted May 26, 2026

Keywords:

Conceptual understanding
First derivative
Mathematical interpretation
Peirce's triadic theory
Semiosis

ABSTRACT

In calculus learning, students often rely on procedural methods when studying the first derivative, which limits their ability to grasp its deeper conceptual meaning. This procedural focus represents a significant learning challenge that needs to be addressed. This study aims to analyze students' semiotic processes in interpreting or expressing the definition of the first derivative using Peirce's triadic framework, which emphasizes the interrelation among the representamen, the object, and the interpretant. A qualitative case study approach was employed with 21 mathematics education students, using written tests and in-depth interviews with three students who demonstrated the most complete semiotic competence. Thematic analysis identified three categories of understanding: comprehensive, partial, and pseudo. Results indicate that students with comprehensive understanding can coordinate symbolic, visual, and verbal representations, forming stable interpretants that conceive the derivative as the slope of a tangent line or an instantaneous rate of change. In contrast, students with partial or pseudo understanding exhibit fragmented semiotic processes and a dominance of procedural approaches. These findings provide important educational value by offering diagnostic insight into students' conceptual difficulties, informing instructional design that integrates multiple representations, and supporting the development of more valid assessment strategies that capture students' conceptual understanding beyond procedural performance.

Corresponding Author:

Ummu Sholihah
Mathematics Education Study Program, Universitas Islam Negeri Sayyid Ali Rahmatullah Tulungagung
Mayor Sujadi Timur Street No. 46 Tulungagung, East Java, Indonesia
Email: ummu_sholihah@uinsatu.ac.id

1. INTRODUCTION

Calculus is a fundamental mathematical tool that underpins various fields of study, with the concept of the derivative serving as a central representation of the dynamics of change through function limits. A deep understanding of derivatives is an essential prerequisite for analyzing variation phenomena in science and engineering [1], making the role of calculus in higher education curricula crucial. The derivative enables for precise measurement of the speed and direction of change in variables, which is fundamental to engineering and economic modeling [2]. In physics, the application of derivatives facilitates calculations in kinematics, such as velocity and acceleration [3], while in engineering, this concept is critical for analyzing material elasticity and deformation [4]. However, despite its importance, many students experience difficulty in truly understanding the definition of the first derivative beyond procedural manipulation, indicating a gap between technical skill and conceptual comprehension.

A profound understanding of the first derivative has been shown to significantly enhance students' critical thinking and mathematical competences in tackling advanced topics [2], [5]. Yet, conventional instruction often emphasizes procedural fluency over conceptual meaning, leaving students capable of performing algebraic operations but struggling to articulate or interpret the underlying principles of the derivative [6]. This gap highlights a specific educational problem that needs to be addressed.

Mathematical understanding is fundamentally semiotic, requiring students to coordinate multiple representational systems and transition from embodied to symbolic thinking while interpreting rather than merely perceiving mathematical symbols. Semiotics is the study of signs and meaning-making processes, in which meaning is constituted through the relationship between the sign, the object, and the interpreter's interpretation [7]. Understanding students' semiotic processes is therefore crucial not only for theoretical insight but also for educational practice, as it enables educators to diagnose learning difficulties, design more effective instruction, and develop assessments that capture students' conceptual meaning-making rather than mere procedural performance. Within Peirce's triadic semiotic framework [8], the first derivative in calculus can be conceptualized as a complex representamen (sign) that represents an object, namely the instantaneous rate of change of a function. The success of mathematical meaning-making relies heavily on students' ability to construct accurate interpretants that connect algebraic representations with the intended functional reality [9]. This sign connects the algebraic representamen with the object through the interpretant, a mental concept that integrates mathematical symbols into functional reality [10]. Students' cognitive success in mastering calculus largely depends on their ability to form the right interpretants, ensuring that the learning process extends beyond mechanical procedures to real-world applications [11].

Furthermore, ideal meaning-making in calculus occurs when students actively engage with the object represented by the sign, which is crucial for applying concepts across disciplines [12]. An instructional approach that integrates Peirce's semiotic theory is projected to have a positive impact on foundational calculus understanding and problem-solving effectiveness in everyday life. From this perspective, therefore, the first derivative should be viewed as a complex entity requiring critical analysis and deep thinking to enhance students' ability to solve problems in various fields [13].

However, in practice, a disconnect often occurs in students' mathematical semiosis processes. Many students demonstrate high procedural ability in manipulating algebraic symbols but struggle to construct a coherent interpretation when asked to express the meaning of the definition. This phenomenon indicates a primary obstacle in mastering the first derivative definition, reflected in students' preference for symbolic manipulation over intrinsic meaning understanding [14], creating a cognitive disconnection from the underlying mathematical concepts [15]. An overemphasis on procedural steps without geometric context understanding leads to misconceptions in the application of complex calculus concepts [16], [17]. Consequently, it is imperative to investigate how instructional approaches can better support students in bridging procedural knowledge and conceptual understanding through semiotic processes [18].

Students' difficulties in interpreting the relationship between differential ratio limits and graphic representations reflect translational barriers between semiotic systems, especially when transitioning from the symbolic realm to the visual representations. This impedes the assimilation of abstract concepts, as students are unable to navigate representations flexibly [19]. Deficits in constructing consistent representations often stem from insufficient instructional scaffolding, leading students to become trapped in symbolic formalism without integrating numeric or geometric meaning [20].

Although calculus is recognized as critical in various fields, there remains a gap in the literature regarding how students establish semiotic relationships between algebraic procedures and functional meaning. Most existing studies focus on students' procedural errors or general conceptual difficulties, but few explicitly examine how meaning is constructed through the interaction of representamen, object, and interpretant within Peirce's triadic framework. However, these studies do not explicitly examine how mathematical meaning is constructed through the interaction of representamen, object, and interpretant. As a result, detailed insights into students' semiotic processes remain limited. This study employs Peirce's triadic semiotic framework to analyze the formation of interpretants in students' understanding of the first derivative concept. This study advances existing research by providing a more detailed analysis of students' meaning-making processes beyond procedural and general conceptual perspectives. The novelty of this study lies in its systematic analysis of students' semiotic coherence across symbolic, visual, and verbal representations, providing a deeper explanation of how conceptual understanding emerges or fails to emerge in calculus learning.

Unlike previous studies that mainly examine procedural errors or general conceptual understanding, this study explicitly applies Peirce's triadic semiotic framework to systematically analyze students' meaning-making processes across symbolic, visual, and verbal representations. The focus of this study is not only on procedural errors but also on translational barriers between semiotic systems that lead to cognitive disconnection. This study is crucial as a theoretical foundation for pedagogical reorientation that aligns symbolism with integrative conceptual understanding, supporting the development of scientific literacy in higher education. Accordingly, this study addresses the following research question: how do students interpret or express the meaning of the first derivative definition based on the triadic relationship among representamen, object, and interpretant? Using Peirce's semiotic theory, this study aims to uncover the depth of students' understanding in interpreting or expressing the meaning of the first derivative definition.

Understanding the first derivative conceptually requires integrating symbolic and visual meanings within a coherent semiotic process. Bakker and Hoffmann [21] showed that diagrams serve as semiotic systems for reasoning about change, while Radford [22] highlighted the role of multiple representational systems in constructing mathematical meaning. However, few studies examine the coherence of students' semiotic processes in understanding the formal definition of the first derivative within Peirce's framework. This study addresses this gap by analyzing how students construct representamen, object, and interpretant in developing this understanding.

2. METHOD

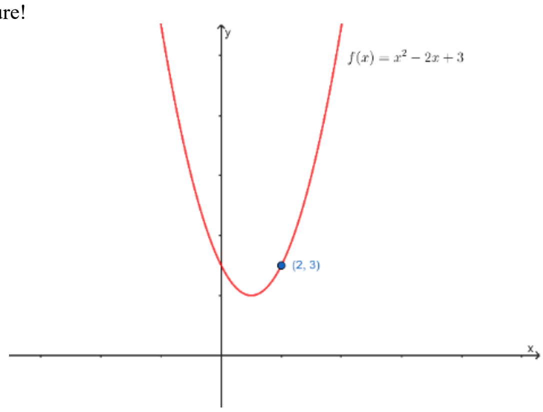
2.1. Research design

This research employs a qualitative approach with a case study method to examine the semiotic processes students undergo in interpreting or expressing the meaning of the first derivative definition. Through in-depth analysis of the symbolic, verbal, and graphical representations used by students, the research revealed how conceptual understanding is formed through interaction with formal mathematical notation. Data analysis is grounded in Peirce's semiotic theory to provide a more comprehensive understanding of students' conceptual understanding of the first derivative definition.

2.2. Participants

This study involves 21 fourth-semester students from the Mathematics Education at Universitas Islam Negeri Sayyid Ali Rahmatullah Tulungagung who have completed a calculus course and possess a basic understanding of derivatives. The participants were selected using purposive sampling, with the criteria that they had completed a calculus course and had prior exposure to the formal definition of the derivative. This sampling approach was chosen to ensure that participants possessed sufficient foundational knowledge to engage in semiotic interpretation processes. The students completed a written test consisting of two questions on the first derivative definition, as shown in Figure 1, individually and simultaneously. Students' responses were analyzed for completeness, consistency, and the presence of semiotic elements. Subsequently, three students were selected for in-depth interviews based on specific criteria: i) completeness and consistency of written test responses; ii) presence of all semiotic elements (representamen, object, and interpretant) in their answers; and iii) good written and oral communication skills. This purposive selection ensured that participants could provide rich data reflecting diverse semiotic processes while maintaining reliability of interpretation.

1. Write the definition of the first derivative of the function f that represents the rate of change of the function's value with respect to the variable x, or geometrically, the slope of the tangent line to the curve $y = f(x)$ at a specific point. Then, provide an explanation in sentence form along with a visual illustration!
2. Please refer to the following figure!



Based on the definition of the first derivative in question 1, find the equation of the tangent line to the function f at the point (2,3)!

Figure 1. Test sheet on the definition of the first derivative

2.3. Instruments

The research instruments include the researcher as the primary instrument, supported by a test sheet and interview guidelines. To ensure content validity, the test items and interview guidelines were reviewed by two experts (mathematics and mathematics education) and revised based on their feedback. The test sheet contains two questions on the first derivative definition and is used to assess students' abilities to interpret or express its meaning, as shown in Figure 1. The interview guidelines are used to further explore students' semiotic processes after the written test.

The instruments were designed to capture multiple semiotic dimensions of students' understanding. The questions on the test sheet, as shown in Figure 1, are designed to uncover students' semiotic processes. The first question assesses students' ability to write the notation and definition of the first derivative and to explain it through symbolic, visual, and verbal representations. The second question assesses students' ability to apply the definition of the first derivative to determine the derivative and the equation of the tangent line. The two questions refer to the research indicators outlined in Table 1, which are based on Peirce's semiotic theory.

Table 1. Research indicators based on Peirce's semiotic theory

Semiotic elements	Indicator
Representamen	Students are able to: i) write the notation for the first derivative; ii) write and read (according to its meaning) the definition of the first derivative; and iii) draw a graph sketch and explain the illustration of the first derivative definition.
Object	Students are able to: i) explain the notation for the first derivative; ii) explain the symbols of the first derivative definition; iii) explain the concept of the secant line or chord; iv) explain the concept of the tangent line; and v) explain the concept of the gradient or slope of the line.
Interpretant	Students are able to: i) explain the meaning (interpret) of the first derivative definition both analytically and graphically; ii) apply the definition of the first derivative to compute the first derivative; and iii) explain the algebraic process of calculating the first derivative.

2.4. Data collection procedures

Data were collected through written tests and in-depth interviews [23]. The data collection process was conducted in two stages. First, students completed a written test individually under controlled classroom conditions. Second, selected participants were invited for semi-structured interviews guided by probing questions designed to explore their reasoning processes in depth. All interviews were audio-recorded and transcribed to ensure data accuracy. Written test responses were classified based on completeness, consistency, and the presence of Peirce's semiotic elements. The 21 students completed a written test on the formal definition of derivative for 120 minutes, which was conducted in the classroom. Interview participants were selected following the criteria described in section 2.2. For each selected participant, the interview was conducted in one session lasting 90 minutes in a private interview room. The interview focused on exploring students' semiotic processes in interpreting or expressing the first derivative definition, with prompts designed to elicit symbolic, visual, and verbal representations systematically.

2.5. Data analysis

Data analysis began with transcribing interview results, followed by a coding process to identify major themes [24]–[26]. The coding procedure involved several steps: i) organizing data according to each semiotic element (representamen, object, interpretant); ii) labeling segments of text, drawings, and verbal explanations; iii) grouping codes into categories reflecting patterns in students' semiotic processes; and iv) identifying themes that describe the formation of conceptual understanding. The coding scheme was developed deductively based on Peirce's semiotic elements and inductively refined during analysis. To ensure reliability, the researchers conducted independent coding, and any discrepancies were discussed until consensus was reached. Inter-coder agreement was used to strengthen the validity of the findings. An example of the coding process is provided in Appendix to illustrate how students' responses were categorized into semiotic elements and levels of understanding.

Indicators of semiotic elements as shown in Table 1 were applied at each stage of coding to ensure systematic interpretation. To enhance reliability and validity, data were cross-checked through peer review of coding, triangulation between written and interview data, and member checking with participants to confirm interpretations. The final thematic analysis focused on how students coordinate symbolic, visual, and verbal representations to construct coherent understanding of the first derivative definition, highlighting the interplay of representamen, object, and interpretant in line with Peirce's triadic semiotic framework [24]–[26].

3. RESULTS AND DISCUSSION

Based on the written test results from 21 students, 10 students (47.62%) were able to answer all questions, while 11 others (52.38%) only answered one question. Further analysis showed that only six students (28.57%) provided complete and consistent responses. Among these six students, only three (14.29%) exhibited complete semiotic elements, not only writing symbols and mathematical procedures (representamen), but also linking them to the referenced concepts (objects) and explaining the conceptual meaning of these symbols (interpretants). These three students were selected for interviews, as shown in Figure 2, to explore their semiotic processes in more depth.

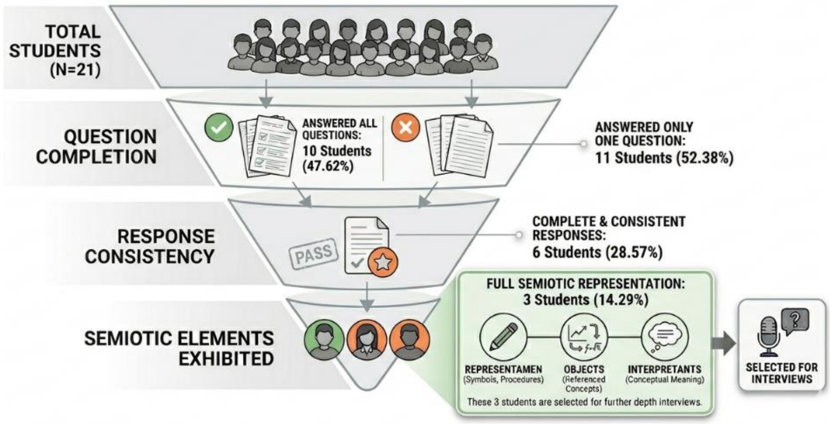


Figure 2. Visualization and analysis of students' semiotic performance

3.1. Results

Based on the written test results from 21 students, six demonstrated consistent and complete responses in the written test. Among them, three students exhibited full semiotic elements and were selected for follow-up interviews. This selection allowed the study to focus on variations in semiotic understanding of the first derivative. Each subject also demonstrates a different approach in interpreting or expressing their understanding of the first derivative definition. There are three categories of semiotic ability in interpreting or expressing the meaning of the first derivative definition, as presented in Table 2.

Table 2. Categories of semiotic ability regarding the meaning of the first derivative definition

No.	Category of ability	Description	Example response
1	Comprehensive	Students are able to correctly express the formal definition of the first derivative symbolically and consistently interpret it as the instantaneous rate of change or the slope of the tangent line.	"The first derivative is defined as the limit of the difference quotient and represents the slope of the tangent line at a point, indicating the instantaneous rate of change."
2	Partial	Students are able to express the formal definition of the first derivative symbolically, but their interpretation of its meaning is still limited and does not show integration between the symbols and the concept.	"The derivative is the limit of $\frac{f(x+h)-f(x)}{h}$ and is related to the concept of the tangent line; however, understanding of the relationship between the secant line and the tangent line remains limited"
3	Pseudo	Students are able to write the formal definition of the first derivative symbolically, but do not demonstrate conceptual understanding of the meaning of the instantaneous rate of change contained within it.	"The derivative is expressed as a lim $\frac{f(x+h)-f(x)}{h}$ without further conceptual explanation."

3.1.1. Semiosis at the representamen stage

There is a clear variation in students' ability to express the definition of the first derivative through symbolic, visual, and verbal representations. Subject 1 (S1), representing comprehensive ability, is able to write the symbol f' as the derivative of the function f. S1 write the definition of the first derivative as the slope of the tangent line symbolically, and explain the meaning of each symbol used, as shown in Figure 3.

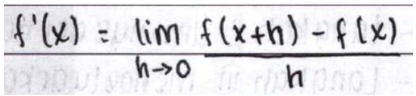


Figure 3. S1's response regarding the notation of the first derivative definition

S1 is also able to represent the concept visually through a Cartesian plane, the function curve, the secant line or chord, and the tangent line. However, the slope triangle is not explicitly depicted and some hesitation appears when explaining the process of drawing the function curve. This visual representation of the concept is shown in Figure 4.

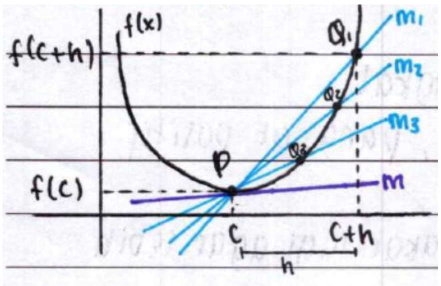


Figure 4. Illustration of the first derivative definition by S1

Subject 2 (S2) representing partial ability, is also able to write the derivative symbol and the definition of the first derivative symbolically, as shown in Figure 5. However, S2 has not yet been able to explain the meaning of these symbols adequately. Visually, S2 can depict the curve, secant line, and tangent line, but most elements are not labeled, as shown in Figure 6, and hesitation and confusion are evident during the explanation.

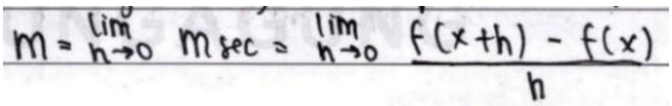


Figure 5. S2's response regarding the notation of the first derivative definition

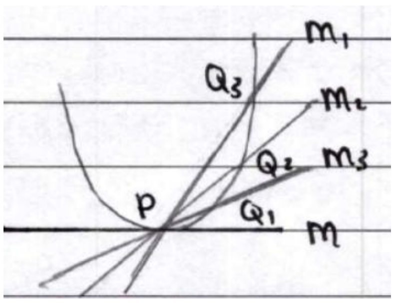


Figure 6. Illustration of the first derivative definition by S2

Meanwhile, subject 3 (S3), representing pseudo ability, demonstrates only limited skill in basic symbolic representation. S3 only writes the derivative symbol and the formal definition of the derivative, as shown in Figure 7, but is unable to represent the secant line, tangent line, or slope triangle, as shown in Figure 8. S3 also appears confused and forgets the basic concepts when explaining the function curve diagram.

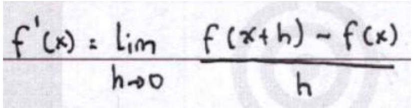


Figure 7. S3's response regarding the notation of the first derivative definition

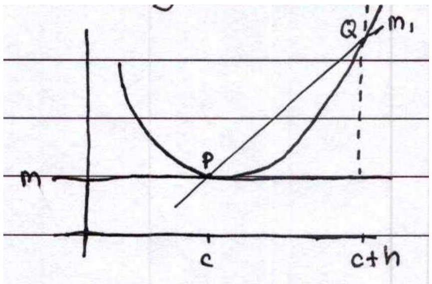


Figure 8. Illustration of the first derivative definition by S3

The research analysis shows that students' understanding of the first derivative definition is strongly influenced by the completeness and coherence of the semiotic process in coordinating symbolic, visual, and verbal representations. Students with comprehensive ability are able to consistently integrate these three representations, forming stable interpretants that reflect a conceptual understanding of the first derivative. In contrast, students with partial and pseudo abilities exhibit fragmented semiosis, characterized by weak connections between representations, which leads to an incomplete understanding of the first derivative concept and a tendency towards a procedural approach.

3.1.2. Semiosis at the object stage

In the object element, the differences in students' conceptual understanding become more apparent. S1 is able to connect the representations used with the mathematical objects referred to, such as the meaning of $f(x)$ as the original function and $f'(x)$ as its derivative, as shown in Figure 3, the explanation of the points $P = (c, f(c))$ and $Q = (c + h, f(c + h))$, and the role of the secant line as an approximation to the tangent line, as shown in Figure 4. S1 is also able to explain how to determine the slope of the line by forming a right triangle through these points. S2 demonstrates partial understanding of the object, being able to explain the function and its derivative, as well as recognizing the existence of two points on the curve. However, S2 is unable to explain these points clearly and often hesitates when explaining the meaning of the secant and tangent lines. Meanwhile, S3 can only explain the function and its derivative in general terms but does not understand the existence of points on the curve, cannot explain the meaning of the secant and tangent lines, and is unable to explain how to determine the slope of the line, both in the written test and the interview.

The research analysis indicates differences in the level of mastery of the derivative concept, reflected in students' ability to relate symbolic, visual, and procedural representations to the mathematical objects. Students with comprehensive understanding are able to establish a consistent connection between derivative notation, points on the curve, secant lines, and tangent lines, and explain the procedure for determining the slope of the line accurately, thus demonstrating deep conceptual integration. In contrast, students with partial understanding can recognize only a few key elements but still struggle to explain the relationship between representations and objects fully. Meanwhile, students with limited (pseudo) understanding grasp the concept only in general terms, without connecting the representations to conceptual meaning, resulting in a fragmented understanding.

3.1.3. Semiosis at the interpretant stage

The differences in the quality of meaning-making of the first derivative are most significant in the interpretant element. S1 is able to interpret the transition of the secant line or chord into the tangent line as a representation of the concept of limit, and to interpret the slope of the tangent line as the core of the first derivative definition (Figure 4). This interpretation is also reflected in the second question, as shown in Figure 9, where S1 can explain the process of determining the slope of the tangent line and formulating the equation of the tangent line, although some hesitation remains at the algebraic manipulation stage.

Meanwhile, S2 has not yet been able to build a complete interpretant, as S2 cannot explain the meaning of the transition of the secant line to the tangent line as a concept of limit, and have not been able to link the slope of the tangent line to the derivative definition conceptually. Although S2 has started to illustrate the definition of the first derivative in Figure 6. In the second question, as shown in Figure 10, S2 can explain the procedure for determining the slope and the equation of the tangent line, but the explanation proceeds slowly, is often interrupted, and is accompanied by confusion.

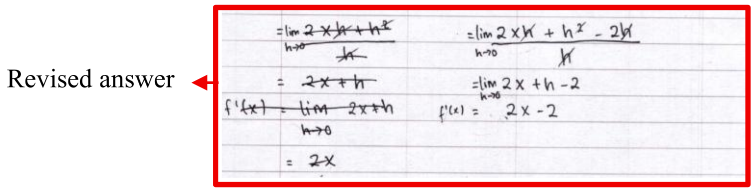


Figure 9. S1's response regarding the application of the first derivative definition

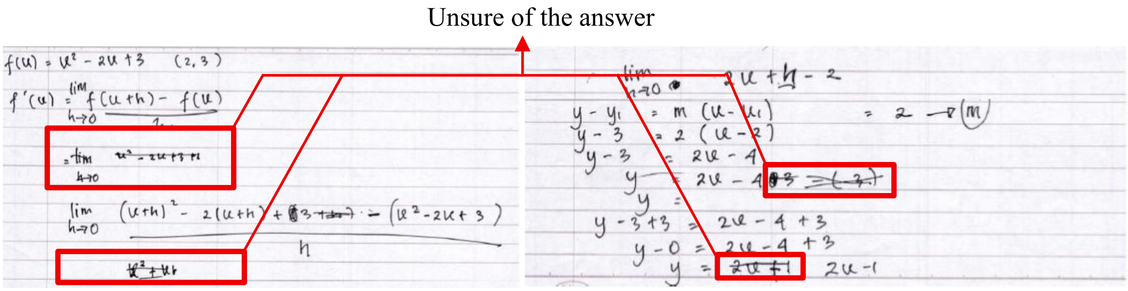


Figure 10. S2's response regarding the application of the first derivative definition

On the other hand, S3 has not been able to build a meaningful interpretant in either the first or second question. S3 cannot explain the concept of limit in the context of the secant and tangent lines, nor understand the meaning of the slope of the tangent line, even though S3 has successfully created an illustration of the first derivative definition in Figure 8. Additionally, S3 remains uncertain and depends on the scaffolding provided by the researchers when explaining the process of determining the derivative and the equation of the tangent line, as shown in Figure 11.

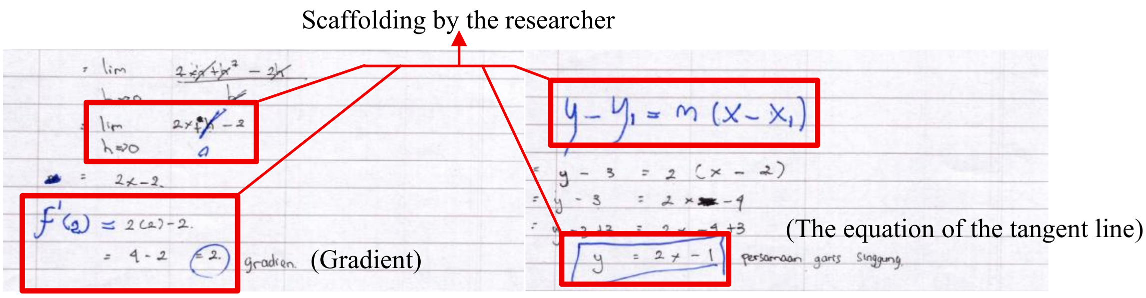


Figure 11. S3's response regarding the application of the first derivative definition

The research analysis shows that the subjects' understanding of the first derivative is strongly influenced by their ability to build an interpretant, that is, to integrate visual, symbolic, and procedural representations with conceptual meaning. Subjects with comprehensive ability can interpret the transition from the secant line to the tangent line as a representation of the limit and understand the slope of the tangent line as the core of the derivative definition. In contrast, subjects with partial ability build only partial interpretants, while subjects with pseudo ability fail to build meaningful interpretants and remain highly dependent on the researchers' scaffolding. Overall, the subjects' semiotic abilities in interpreting or expressing the meaning of the first derivative definition can be visualized in Figure 12.

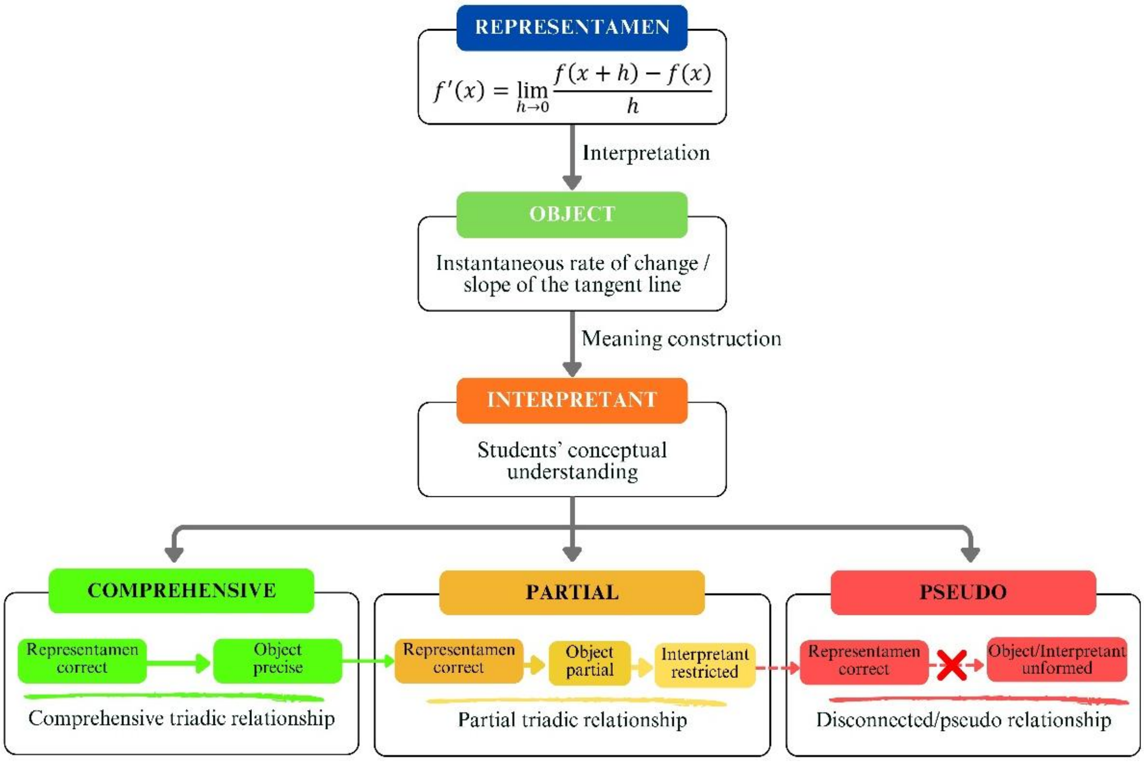


Figure 12. Semiotic process of meaning-making the first derivative definition

3.2. Discussion

The purpose of this study is to analyze students' conceptual understanding of the first derivative definition beyond procedural symbolic mastery. By using Peirce's triadic semiotic framework—representamen, object, and interpretant [8], these findings show that three participants who provided complete semiotic elements could express the formal definition of the first derivative symbolically. However, their ability to represent the definition visually and explain it verbally varies, which classifies them into comprehensive, partial, and pseudo semiotic abilities.

First, students with comprehensive ability not only express the first derivative symbolically but also integrate it with conceptual meaning, such as interpreting the slope of the tangent line. These students not only write the mathematical notation correctly but also connect the symbol to its conceptual meaning in geometric and physical contexts, such as understanding the slope of the tangent line on a curve or the rate of change of a function at a particular point. This indicates that these students have a comprehensive understanding, not only procedural, but also conceptual, of the first derivative concept.

This finding aligns with the research of Borji *et al*. [27], which shows that the use of technology-based approaches, such as the ACE cycle with Maple software, can enhance students' understanding of the derivative concept, particularly in relation to rate of change and mathematical symbolism. The relational approach, which emphasizes the connection between the derivative and the slope of the tangent line, as highlighted by Sahin *et al*. [28], also supports the finding that geometric interpretation can strengthen students' conceptual understanding. Furthermore, the rigorous mathematical thinking (RMT) approach applied by Hidayat *et al*. [29] demonstrates that students engaged in deep mathematical thinking activities tend to internalize abstract concepts such as derivatives more effectively. The use of multiple semiotic representations—including graphical, symbolic, and verbal modes—has been shown to facilitate this integration of conceptual understanding [30]–[32]. Dynamic semiosis, in which the interpretation of one sign becomes the representamen for the next, also supports students' ability to link symbols to conceptual meaning [33], [34]. The use of interactive technology, such as GeoGebra, as noted by Illanes *et al*. [35], is also consistent with this finding, as graphic and symbolic representations help students build connections between symbols and the mathematical objects they represent. Overall, these findings suggest that symbolic mastery, coupled with conceptual interpretation, is not a coincidence but rather the result of a learning approach that integrates technology, mental models, and active teaching strategies, which strengthen students' understanding of derivatives.

Second, students with partial ability can write the first derivative symbolically but fail to fully integrate it with conceptual meaning, reflecting a partial interpretant. This condition aligns with previous research reporting that students are generally able to perform formal tasks related to derivatives through symbolic, graphic, or contextual representations, but still face difficulties in coordinating and linking these representations coherently [35]. This difficulty is consistent with the challenges in coordinating multiple semiotic representations, as students struggle to integrate graphical, symbolic, numerical, and verbal modes meaningfully [30], [31]. This difficulty is also reflected in students' inability to conceptually differentiate between differentiation and integration, which indicates a weak integration of symbolic understanding into a broader conceptual framework [36]. This phenomenon is closely related to the symbol-grounding problem, in which students fail to connect symbolic or numerical representations with the underlying conceptual meaning [37], [38].

Additionally, students' performance often shows a clear gap between symbolic proficiency and conceptual understanding. While students can represent problem situations visually, they are not yet able to use symbolic representations meaningfully and effectively in problem-solving processes [39]. This indicates that success in producing mathematical symbols does not automatically reflect an understanding of their meaning in both theoretical and applied contexts. In line with this finding, Oliveira and Lopes [40] reveal that the mismatch between conceptual images and the operational definition of functions often triggers misconceptions in students' understanding of derivatives and their characteristics. Semiotic scaffolding and the use of bridging representations are strategies suggested in the literature to address this gap [31], [41].

Based on these findings, it can be concluded that a systematic learning approach is needed to bridge the gap between symbolic representation and deep conceptual understanding. Efforts to address this conceptual gap require an emphasis on building a conceptual foundation before introducing formal symbolism, allowing students to internalize the meaning of mathematical symbols comprehensively [42]. This approach is supported by various studies showing that learning strategies integrating definitions, symbols, graphic representations, and real-world contexts, including the use of dynamic media such as GeoGebra, can enhance retention, coherence of understanding, and the quality of students' mathematical concept mastery in a holistic and meaningful way [35], [36], [40].

Third, students with pseudo ability can write the derivative symbolically but fail to construct meaningful interpretants, demonstrating procedural competence without conceptual understanding. This condition indicates a cognitive gap between mastery of symbolic notation and conceptual interpretation, causing students to display procedural competence without fully understanding the essence of the derivative concept. This gap emphasizes the need for further investigation into the learning approaches that can bridge the relationship between mathematical symbols and the conceptual meaning they represent.

This phenomenon is consistent with the concept of pseudo-understanding in mathematics education, where students can reproduce definitions and algorithms accurately but fail to grasp the underlying meaning. For example, students may state the formal definition of the first derivative as the limit of the ratio of the change in the function's value to the change in the independent variable as the change approaches zero [43], but often fail to interpret this formalism in an applied context, such as the meaning of the instantaneous rate of change [44]. This limitation is consistent with literature emphasizing procedural dominance and pseudo-understanding, where semiosis halts at the symbolic level, preventing conceptual connections [30], [34], [45]. This limitation indicates weak conceptual understanding because students are unable to relate the formal definition to the concrete idea of change. Previous studies also emphasize that difficulties in linking the symbolic representation of derivatives to real-world contexts affect students' ability to solve conceptual problems and can even create epistemological barriers in the learning process [43], [44]. Using Peirce's triadic framework to analyze representamen, object, and interpretant can guide interventions that bridge symbolic mastery with conceptual understanding [34], [45].

These findings suggest that educational strategies should leverage students' semiotic processes to strengthen derivative learning. By integrating visual, symbolic, and applied contexts, including technology-based tools such as dynamic geometry software and online platforms [46], [47], teachers can facilitate conceptual integration, reduce pseudo-understanding, and promote meaningful comprehension of derivatives. Explicit scaffolding that connects symbolic notation to conceptual meaning is critical in designing effective calculus instruction. For instructional practice, teachers are encouraged to design learning activities that explicitly connect symbolic, graphical, and verbal representations, for example through tasks that require students to explain the meaning of derivative concepts using multiple representations. In terms of curriculum design, it is recommended that calculus curricula incorporate semiotic-oriented learning sequences that progressively develop students' ability to construct interpretants, rather than focusing solely on procedural fluency. With regard to assessment, educators should include tasks that evaluate students' ability to interpret and explain mathematical meaning, not only compute results, in order to capture deeper conceptual understanding.

The limitation of this study concerns the time gap between the written test and the follow-up interviews. The data collection for the written test and the interviews was conducted with an interval of approximately one-week. This time lag may have introduced potential information bias in the responses provided by the interview participants. It is possible that the participants were no longer able to recall accurately the reasoning behind their written answers due to the elapsed time. Therefore, future studies are recommended to collect data simultaneously for both the written responses and the participants' underlying reasoning. One possible approach is to implement a think-aloud strategy, in which students are asked to complete the written test while verbalizing their thought processes. These verbalizations can be audio-recorded to capture more immediate and authentic data. By employing the think-aloud technique during the written test, the potential for data bias may be reduced. In addition, this study is limited by the fact that the participants were drawn from a single university, which may restrict the generalizability of the findings. The qualitative case study design also emphasizes depth rather than breadth; therefore, the results should be interpreted within the specific context of the participants involved.

4. CONCLUSION

This study concludes that students' understanding of the first derivative definition heavily depends on the completeness of the semiotic process within Peirce's triadic framework, which includes representamen, object, and interpretant. The study highlights that using Peirce's framework provides a theoretical lens to analyze students' conceptual understanding beyond procedural mastery, revealing how semiotic integration can contribute to more meaningful learning in calculus. The results reveal three categories of students' semiotic abilities: comprehensive, partial, and pseudo, reflecting differences in the quality of coordinating symbolic, visual, and verbal representations in building conceptual meaning of the first derivative concept. Students with comprehensive ability are able to correctly express the formal definition of the first derivative symbolically, consistently link it with the referenced mathematical object, and build stable interpretants, understanding the derivative as the instantaneous rate of change or the slope of the tangent line. The completeness of the semiotic process in this category demonstrates a strong conceptual integration among symbols, objects, and meaning, resulting in a conceptual and relational understanding, rather than merely procedural knowledge. In contrast, students with partial and pseudo abilities show fragmented semiosis processes. Students in the partial category are only able to build limited connections between representations and mathematical objects, resulting in partial interpretants. Meanwhile, students in the pseudo category tend to master formal symbolic representations without adequate conceptual understanding, failing to build meaningful interpretants and relying heavily on external assistance.

These findings imply that Peirce's triadic semiotic theory provides an effective analytical framework for uncovering the depth and quality of students' understanding calculus concepts, particularly in distinguishing comprehensive, partial, and pseudo understandings of the first derivative definition. This study also implies the need for calculus instruction that is explicitly designed to facilitate the integration of symbolic, visual, and conceptual representations, as well as to encourage the formation of meaningful interpretants. The use of dynamic learning technologies, such as GeoGebra, has the potential to support a more complete and coherent semiotic process, thereby reducing the tendency toward procedural understanding and pseudo-understanding, while improving the quality of students' conceptual understanding of the first derivative concept. From a practical perspective, this study emphasizes the importance of instructional design that explicitly facilitates semiotic integration through the use of multiple representations and guided meaning-making processes. Lecturers and curriculum developers are encouraged to focus not merely on procedural aspects but also on students' interpretive abilities and conceptual understanding.

This study is limited by the approximately one-week interval between the written test and the follow-up interviews, which may have introduced recall bias in participants' explanations. Future studies are recommended not only to collect written responses and reasoning simultaneously through a think-aloud strategy, but also to explore students' semiotic processes across different learning contexts, instructional designs, and mathematical topics. Such investigations could provide deeper insights into how semiotic integration develops in diverse educational settings and how teaching strategies can be optimized accordingly. In addition, this study is limited by the fact that the participants were drawn from a single university. Therefore, future research should involve a larger and more diverse group of participants, employ experimental or design-based research approaches, and examine semiotic processes across various mathematical topics and different educational levels.

FUNDING INFORMATION

The authors state no funding involved.

AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Beni Asyhar	✓	✓	✓		✓	✓	✓	✓	✓		✓		✓	
Ummu Sholihah	✓	✓	✓		✓	✓	✓	✓	✓		✓		✓	
Elva Yohana		✓			✓		✓			✓				
Anwaril Hamidy		✓			✓		✓			✓				

C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

The authors declare that there is no conflict of interest regarding the publication of this paper.

DATA AVAILABILITY

The data that support the findings of this study are available from the first author, [US], upon reasonable request.

REFERENCES

[1] C. Fuentealba, E. Badillo, G. Sánchez-Matamoros, and A. Cárcamo, "The understanding of the derivative concept in higher education," *EURASIA Journal of Mathematics, Science and Technology Education*, vol. 15, no. 2, p. em1662, Dec. 2018, doi: 10.29333/ejmste/100640.

[2] A. M. Al-Zoubi and L. M. Suleiman, "Flipped classroom strategy based on critical thinking skills: helping fresh female students acquiring derivative concept," *International Journal of Instruction*, vol. 14, no. 2, pp. 791–810, Apr. 2021, doi: 10.29333/iji.2021.14244a.

[3] M. Frodyma, "Using elementary calculus and dimensional analysis to prepare students for physical chemistry," *Journal of Chemical Education*, vol. 97, no. 3, pp. 741–748, Mar. 2020, doi: 10.1021/acs.jchemed.9b00807.

[4] J. Lai *et al.*, "Investigation progresses and applications of fractional derivative model in geotechnical engineering," *Mathematical Problems in Engineering*, vol. 2016, no. 1, pp. 1–15, 2016, doi: 10.1155/2016/9183296.

[5] R. Neville, P. D. Shipman, N. E. Levinger, and A. T. Krummel, "ConfChem conference on mathematics in undergraduate chemistry instruction: applied mathematics for chemistry majors," *Journal of Chemical Education*, vol. 95, no. 8, pp. 1438–1439, Aug. 2018, doi: 10.1021/acs.jchemed.8b00107.

[6] M. T. López-Díaz and M. Peña, "Improving calculus curriculum in engineering degrees: implementation of technological applications," *Mathematics*, vol. 10, no. 3, p. 341, Jan. 2022, doi: 10.3390/math10030341.

[7] R. G. Cook, "Can dialogic narratives and discourse engage online learners?" *Participatory Educational Research*, vol. 10, no. 5, pp. 1–18, Sep. 2023, doi: 10.17275/per.23.72.10.5.

[8] A. Sáenz-Ludlow and G. Kadunz, "Constructing knowledge seen as a semiotic activity," in *Semiotics as a Tool for Learning Mathematics*, A. Sáenz-Ludlow and G. Kadunz, Eds., Rotterdam: SensePublishers, 2016, pp. 1–21, doi: 10.1007/978-94-6300-337-7_1.

[9] N. Presmeg, *Semiotics in mathematics education*. Cham: Springer International Publishing, 2020, doi: 10.1007/978-3-030-15789-0_137.

[10] Y. Pradita, E. J. K. R. Bumen, S. Lelunu, J. A. Manuputty, and E. Wardiannata, "What is an academic journey [marks, interpretations, and objects]?: Reflections by the theories of Charles Sanders Peirce," *Lingua: Journal of Linguistics and Language*, vol. 1, no. 1, pp. 45–61, Dec. 2023, doi: 10.61978/lingua.v1i1.163.

[11] H. A. Eshaq, "The effect of using STEM education on students' mathematics achievement," *Journal of Pedagogical Research*, vol. 8, no. 1, pp. 75–82, Dec. 2024, doi: 10.33902/JPR.202423476.

[12] R. Santos, D. Anderson, and M. Milner-Bolotin, "Research trends in international science, technology, engineering, and mathematics education conference series: an analysis of a decade of proceedings," *Frontiers in Education*, vol. 7, p. 1099658, Jan. 2023, doi: 10.3389/feduc.2022.1099658.

[13] M. G. Eastman, J. Christman, G. H. Zion, and R. Yerrick, "To educate engineers or to engineer educators?: exploring access to engineering careers," *Journal of Research in Science Teaching*, vol. 54, no. 7, pp. 884–913, Sep. 2017, doi: 10.1002/tea.21389.

[14] K. Klingbeil and F. Moons, "The omnipresence of the letter-as-object misconception: a multilevel latent transition analysis on understanding variables," *Educational Studies in Mathematics*, vol. 120, no. 3, pp. 455–476, Nov. 2025, doi: 10.1007/s10649-025-10428-7.

[15] M. Khalid, F. Yakop, and H. Ibrahim, "Year 7 students' interpretation of letters and symbols in solving routine algebraic problems," *Qualitative Report*, vol. 25, no. 11, pp. 4167–4181, Nov. 2020, doi: 10.46743/2160-3715/2020.4062.

[16] S. Aleksić, "Exploring student understanding of limits and derivatives through examples and pedagogical commentary," *Journal of Educational Studies in Mathematics and Computer Science*, vol. 2, no. 1, pp. 5–14, 2025, doi: 10.5937/JESMAC2501005A.

[17] P. N. Mathaba and A. Bayaga, "Analysis of types, sources of errors and misconceptions in South African algebra cognition," *Universal Journal of Educational Research*, vol. 9, no. 5, pp. 928–937, May 2021, doi: 10.13189/ujer.2021.090505.

[18] F. Feudel and R. Biehler, "Students' understanding of the derivative concept in the context of mathematics for economics," *Journal für Mathematik-Didaktik*, vol. 42, no. 1, pp. 273–305, Mar. 2021, doi: 10.1007/s13138-020-00174-z.

[19] N. Arefaine, K. Michael, and S. Assefa, "GeoGebra assisted multiple representations for enhancing students' representation translation abilities in calculus," *Asian Journal of Education and Training*, vol. 8, no. 4, pp. 121–130, Nov. 2022, doi: 10.20448/edu.v8i4.4309.

[20] A. Maries, S.-Y. Lin, and C. Singh, "Challenges in designing appropriate scaffolding to improve students' representational consistency: the case of a Gauss's law problem," *Physical Review Physics Education Research*, vol. 13, no. 2, p. 020103, Aug. 2017, doi: 10.1103/PhysRevPhysEducRes.13.020103.

[21] A. Bakker and M. H. G. Hoffmann, "Diagrammatic reasoning as the basis for developing concepts: a semiotic analysis of students' learning about statistical distribution," *Educational Studies in Mathematics*, vol. 60, no. 3, pp. 333–358, Nov. 2005, doi: 10.1007/s10649-005-5536-8.

[22] L. Radford, "Signs and meanings in students' emergent algebraic thinking: a semiotic analysis," *Educational Studies in Mathematics*, vol. 42, no. 3, pp. 237–268, May 2000, doi: 10.1023/A:1017530828058.

[23] R. K. Yin, *Case study research and applications: design and methods*, 6th ed. Thousand Oaks: SAGE Publications, Inc., 2018.

[24] X. Gu *et al.*, "A qualitative thematic analysis exploring Chinese young adults' experiences in decision making on the management of low-risk papillary thyroid cancer," *Thyroid*, vol. 34, no. 12, pp. 1486–1494, Dec. 2024, doi: 10.1089/thy.2024.0210.

[25] L. Kogen, "Qualitative thematic analysis of transcripts in social change research: reflections on common misconceptions and recommendations for reporting results," *International Journal of Qualitative Methods*, vol. 23, pp. 1–11, Jan. 2024, doi: 10.1177/16094069231225919.

[26] V. V. Lee, S. C. C. van der Lubbe, L. H. Goh, and J. M. Valderas, "Harnessing ChatGPT for thematic analysis: are we ready?" *Journal of Medical Internet Research*, vol. 26, p. e54974, May 2024, doi: 10.2196/54974.

[27] V. Borji, H. Alamolhodaei, and F. Radmehr, "Application of the APOS-ACE Theory to improve students' graphical understanding of derivative," *EURASIA Journal of Mathematics, Science and Technology Education*, vol. 14, no. 7, pp. 2947–2967, May 2018, doi: 10.29333/ejmste/91451.

[28] Z. Sahin, A. A. Yenmez, and A. K. Erbas, "Relational understanding of the derivative concept through mathematical modeling: a case study," *EURASIA Journal of Mathematics, Science and Technology Education*, vol. 11, no. 1, pp. 177–188, Dec. 2015, doi: 10.12973/eurasia.2015.1149a.

[29] D. Hidayat, A. W. Kohar, N. R. Prihartiwi, H. Mubarok, and A. Yohannes, "Design of learning activities using rigorous mathematical thinking (RMT) approach in application of derivatives," *IJORER: International Journal of Recent Educational Research*, vol. 2, no. 1, pp. 111–120, Jan. 2021, doi: 10.46245/ijorer.v2i1.75.

[30] C. M. Krause and A. M. Wille, "Sign language in light of mathematics education: an exploration within semiotic and embodiment theories of learning mathematics," *American Annals of the Deaf*, vol. 166, no. 3, pp. 352–377, 2021, doi: 10.1353/aad.2021.0025.

[31] L. Pham and R. Tytler, "The semiotic function of a bridging representation to support students' meaning-making in solution chemistry," *Research in Science Education*, vol. 52, no. 3, pp. 853–869, Jun. 2022, doi: 10.1007/s11165-021-10022-w.

[32] L. Moreira and J. Queiroz, "Revealing Alice in wonderland through intersemiotic translation," *Sign Systems Studies*, vol. 50, no. 2–3, pp. 346–377, Dec. 2022, doi: 10.12697/SSS.2022.50.2-3.04.

[33] T. Strand, "A semiotic model of learning," *Chinese Semiotic Studies*, vol. 17, no. 1, pp. 153–162, Feb. 2021, doi: 10.1515/css-2021-0008.

[34] J. P. Baron, "Peirce and archaeology: recent approaches," *Annual Review of Anthropology*, vol. 50, no. 1, pp. 187–202, Oct. 2021, doi: 10.1146/annurev-anthro-101819-110112.

[35] M. K. G. Illanes, D. C. Manríquez, A. Breda, and G. Sala-Sebastià, "Instructional process for the construction of the derivative function: modelling and simulation in GeoGebra from the Ontosemiotic approach," *Journal on Mathematics Education*, vol. 16, no. 3, pp. 1001–1022, Sep. 2025, doi: 10.22342/jme.v16i3.pp1001-1022.

[36] R. A. Funny, "Analysis of engineering students' understanding in differentiate derivative and integral," *Journal of Physics: Conference Series*, vol. 1957, no. 1, p. 012002, Jul. 2021, doi: 10.1088/1742-6596/1957/1/012002.

[37] T. Leibovich and D. Ansari, "The symbol-grounding problem in numerical cognition: a review of theory, evidence, and outstanding questions," *Canadian Journal of Experimental Psychology/Revue Canadienne de Psychologie Expérimentale*, vol. 70, no. 1, pp. 12–23, Mar. 2016, doi: 10.1037/cep0000070.

[38] T. Tikhomirova, Y. Kuzmina, I. Lysenkova, and S. Malykh, "The relationship between non-symbolic and symbolic numerosity representations in elementary school: the role of intelligence," *Frontiers in Psychology*, vol. 10, p. 2724, Dec. 2019, doi: 10.3389/fpsyg.2019.02724.

[39] L. L. Prayitno, Purwanto, Subanji, Susiswo, and A. As'ari, "Exploring student's representation process in solving ill-structured problems geometry," *Participatory Educational Research*, vol. 7, no. 2, pp. 183–202, Aug. 2020, doi: 10.17275/per.20.28.7.2.

[40] S. M. de Oliveira and R. Lopes, "Exponential functions and their derivatives in the light of conceptual image and conceptual definition," *Revemop*, vol. 6, p. e2024007, Jul. 2024, doi: 10.33532/revemop.e2024007.

[41] A. Olteanu and S. Ongstad, "Utterance-genre-lifeworld and sign-habit-umwelt compared as Phenomenologies. Integrating socio- and biosemiotic concepts?" *Biosemiotics*, vol. 17, no. 2, pp. 523–546, Aug. 2024, doi: 10.1007/s12304-024-09561-2.

[42] P. Mutodi and M. Mosimege, "Learning mathematical symbolization: conceptual challenges and instructional strategies in secondary schools," *Bolema: Boletim de Educação Matemática*, vol. 35, no. 70, pp. 1180–1199, May 2021, doi: 10.1590/1980-4415v35n70a29.

[43] A. Prihandhika and N. Azizah, "Rethinking how we teach derivatives representation: the framework of hypothetical learning trajectory," *Plusminus: Jurnal Pendidikan Matematika*, vol. 5, no. 2, pp. 315–326, Jul. 2025, doi: 10.31980/plusminus.v5i2.2816.

[44] E. Kurniadi, Zulkardi, R. I. I. Putri, and D. Darmawijoyo, "Beyond the formula: a systematic review of how senior high students make sense of derivatives," *Mosharafa: Jurnal Pendidikan Matematika*, vol. 14, no. 1, pp. 131–150, Jan. 2025, doi: 10.31980/mosharafa.v14i1.2922.

[45] R. J. Ormerod, "The logic of semiotics applied to mathematical and social interaction in operational research consulting practice: towards a foundational view," *Systems Research and Behavioral Science*, vol. 40, no. 1, pp. 16–42, Jan. 2023, doi: 10.1002/sres.2828.

[46] M. S. Alabdulaziz, "The effect of using PDEODE teaching strategy supported by the e-learning environment in teaching mathematics for developing the conceptual understanding and problem-solving skills among primary stage students," *EURASIA Journal of Mathematics, Science and Technology Education*, vol. 18, no. 5, p. em2109, Apr. 2022, doi: 10.29333/ejmste/12019.

[47] B. Chimmalee and A. Anupan, "Enhancement of mathematical conceptual understanding in a cloud learning environment for undergraduate students," *International Journal of Engineering Pedagogy (iJEP)*, vol. 12, no. 6, pp. 50–69, Dec. 2022, doi: 10.3991/ijep.v12i6.33775.

APPENDIX

Detailed coding scheme and sample data

I. Coding framework

The coding process in this study follows a two-stage procedure:

1. Open coding: identifying segments of students' written and verbal responses related to semiotic elements (representamen, object, and interpretant).
2. Axial coding: categorizing these segments into levels of understanding (comprehensive, partial, and pseudo) based on the coherence and integration of semiotic elements.

II. Sample verbatim responses and coding

Data source	Verbatim response	Semiotic element	Open code	Axial code	Category
Written test (S1)	"The first derivative is the limit of $\frac{f(x+h)-f(x)}{h}$ as h approaches zero."	Representamen	Correct use of derivative definition	Accurate symbolic representation	Comprehensive
Written test (S1)	"It represents the slope of the tangent line at a specific point on the curve."	Object	Interpretation as tangent slope	Conceptual meaning of derivative	Comprehensive
Interview (S1)	"When the secant line keeps getting closer to a point, it becomes the tangent line, which shows instantaneous change."	Interpretant	Linking limit to tangent concept	Coherent integration of representations	Comprehensive
Written test (S2)	"The derivative is lim $\frac{f(x+h)-f(x)}{h}$."	Representamen	Correct formula without explanation	Procedural knowledge only	Partial
Interview (S2)	"It is related to the tangent line, but I still do not understand how the process works."	Interpretant	Incomplete explanation	Partial conceptual connection	Partial
Written test (S3)	"lim $\frac{f(x+h)-f(x)}{h}$"	Representamen	Symbol only	No conceptual meaning	Pseudo
Interview (S3)	"I do not really understand what it means."	Interpretant	No interpretation	Absence of meaning-making	Pseudo

III. Coding interpretation criteria

1. Comprehensive: responses demonstrate coherent integration of representamen, object, and interpretant.
2. Partial: responses show correct symbolic representation but incomplete or inconsistent interpretation.
3. Pseudo: responses are limited to symbolic expressions without meaningful interpretation.

BIOGRAPHIES OF AUTHORS

Beni Asyhar is a lecturer in the Mathematics Education Study Program, Universitas Islam Negeri Sayyid Ali Rahmatullah Tulungagung, Indonesia. He completed his undergraduate education in mathematics and continued his studies at the master's and doctoral degree in mathematics education. His research interests span various areas of mathematics education. He can be contacted at email: asyhar_beni@uinsatu.ac.id.

Ummu Sholihah is a lecturer in the Mathematics Education Study Program at Universitas Islam Negeri Sayyid Ali Rahmatullah Tulungagung, Tulungagung, East Java, Indonesia, where she also serves as the head of the Master's Program in Mathematics Education. She earned her Bachelor's degree in Mathematics Education, completed her Master's degree in Mathematics, and obtained her Doctoral degree in Mathematics Education. Her scholarly work focuses on mathematics education. She can be contacted at email: ummu_sholihah@uinsatu.ac.id.

Elva Yohana is a lecturer in the English Language Education Study Program at Universitas Islam Negeri Sayyid Ali Rahmatullah Tulungagung, Tulungagung, East Java, Indonesia, and currently serves as the Secretary of the Master's Program in English Language Education. She earned her Bachelor's degree in English Language and Literature, as well as her Master's and Doctoral degrees in English Language Education. Her research focuses on English language teaching. She can be contacted at email: elva.yoohana@uinsatu.ac.id.

Anwaril Hamidy is currently pursuing his doctoral degree in Mathematics Education at University of Bremen, Bremen, Germany. His research focuses on the psychology of mathematics learning, particularly mathematical resilience and assessment using quantitative approaches. He completed his Bachelor's and Master's degrees in Mathematics Education. His scholarly work examines factors influencing mathematics achievement, teachers' TPACK, online learning readiness, and students' mathematical process skills. He can be contacted at email: ahamidy@uni-bremen.de.