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Evaluating the geometric thinking levels of generation Z pre-service mathematics teachers in Indonesia

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ABSTRACT

Geometric thinking abilities remain crucial in mathematics education, particularly for pre-service teachers who will shape future generations' understanding of geometry. This study evaluates and compares the geometric thinking levels of generation Z pre-service mathematics teachers at Universitas Negeri Makassar (UNM) and Universitas Mulawarman (UNMUL) using the Van Hiele model. A quantitative comparative design was employed, involving 233 UNM and 227 UNMUL students selected through purposive sampling. The geometric thinking test (GTT) assessed students across five levels: visualization, analysis, informal deduction, formal deduction, and rigor. Results indicated that UNM students excelled in analysis and informal deduction, whereas UNMUL students displayed a broader distribution across all levels, with notable frequencies at visualization and formal deduction levels. A statistically significant difference in overall geometric thinking scores were identified, using the Mann-Whitney U test, with UNM students scoring higher. These findings emphasize the importance of adopting student-centered instructional strategies aligned with the Van Hiele model to enhance geometric thinking. Incorporating hands-on activities and technology is recommended to better prepare pre-service teachers for effective geometric instruction. The study provides insights for educators and policymakers to improve mathematics education by fostering higher geometric thinking levels in future teachers.

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1. INTRODUCTION

Geometric thinking is fundamental in mathematics education, enhancing students' reasoning, problem-solving skills, and comprehension of spatial relationships and shapes, which are essential for advanced mathematical concepts [1]–[5]. Research indicates that engaging students in geometric thinking significantly improves their ability to visualize and mentally manipulate objects, critical for understanding abstract mathematical ideas [6]–[8]. For generation Z pre-service mathematics teachers, mastering geometric thinking is vital as it directly impacts their ability to teach these concepts effectively to future students [9], [10].

Generation Z, defined as individuals born between 1997 and 2012, has unique learning preferences and a deep familiarity with technology, necessitating innovative teaching approaches [11]. The digital era in which generation Z has grown up significantly shapes their cultural identity and social interactions, making technology an integral part of their lives [2]. Consequently, teaching strategies for this generation need to incorporate technological tools and interactive methods that align with their digital experiences.

Establishing a strong foundation in geometric thinking during pre-tertiary education is crucial. According to research by Hourigan and Leavy [12], a solid understanding of geometric concepts is essential for pre-service primary teachers to enhance their teaching proficiency and ensure their students' success in learning geometry. This foundational knowledge enables teachers to foster a deeper understanding of geometry among their students, cultivating the analytical skills necessary for complex mathematical problems [13], [14].

The Van Hiele model, developed by Dina Van Hiele-Geldof and Pierre Van Hiele, delineates five hierarchical levels of geometric thinking: visualization, analysis, informal deduction, formal deduction, and rigor [15], [16]. This model emphasizes structured and sequential learning experiences, which are crucial for progressing through these levels [7]. Implementing Van Hiele phase-based instruction has been shown to significantly improve pre-service teachers' geometric thinking skills by aligning instruction with these phases [17]–[20]. Such instructional strategies promote active learning and encourage students to develop a deeper understanding of geometric concepts through hands-on activities and real-life applications [21], [22].

Despite the benefits of the Van Hiele model, many educational systems, particularly in Indonesia, continue to rely on traditional methods that fail to develop higher-order geometric thinking skills. In contexts such as Sokoto State, Nigeria, these conventional strategies have been criticized for their limited effectiveness in fostering higher-order geometric thinking skills [23], [24]. Traditional teaching methods often fail to engage students in meaningful learning experiences, preventing them from appreciating the practical applications of geometry in daily life. This reliance on rote learning and lack of interactive, student-centered activities contribute to the gap in students' geometric understanding and their ability to achieve higher levels of geometric thinking [25], [26].

In Indonesia, conventional teaching methods have been inadequate in improving students' understanding of geometry, as evidenced by the performance of pre-service teachers in college who demonstrate deficiencies in geometric thinking [27], [28]. Many elementary and junior high school students in Indonesia struggle with basic geometric concepts such as shapes and planes [29], [30]. This is undoubtedly connected to the performance of pre-service teachers in college who demonstrate a deficiency in geometric thinking. However, comparative investigations of geometric thinking levels among universities remain relatively understudied.

One of the primary institutions for training pre-service mathematics teachers in Indonesia is Universitas Negeri Makassar (UNM) and Universitas Mulawarman (UNMUL). By examining these two institutions, the study seeks to provide a foundation for recommending appropriate geometry learning strategies and enhancing the geometric thinking levels of students in educational institutions. This study aims to evaluate and compare the geometric thinking levels of generation Z pre-service mathematics teachers at UNM and UNMUL, providing valuable insights for educators and policymakers.

2. METHOD

This study used a quantitative comparative design to examine the geometric thinking level of pre-service mathematics teachers at Universitas Negeri Makassar (UNM) and Universitas Mulawarman (UNMUL). The population encompassed all generation Z pre-service mathematics teachers enrolled at both institutions. A total of 233 students from UNM and 227 students from UNMUL was selected using purposive sampling, which involves deliberately choosing participants who had successfully completed the basic geometry course. The instrument employed was the geometric thinking test (GTT), based on Van Hiele's model, consisted of 25 multiple-choice questions assessing five levels of geometric thinking: visualization, analysis, informal deduction, formal deduction, and rigor [31]. Each level had five questions, and the students had to answer at least three questions correctly in each level to be categorized into that level. Additionally, Usiskin [31] established a weighted geometric thinking score to determine the thinking level of students. For example, if a student answered three questions correctly from levels one, three, and four, their score would be 13 (1+4+8). Usiskin [31] summarized all potential scores obtained by students at each level in Table 1.

The data were collected through the GTT. Participants from both universities completed the test within the same timeframe. Descriptive and inferential statistics were used to analyze the data, with the results presented in tables and charts, to summarize the distribution of geometric thinking levels and to compare the geometric thinking level between universities through mean ranks and sum of scores. The Mann-Whitney U test, a non-parametric test, was used to compare differences in geometric thinking levels and total scores between UNM and UNMUL students where the data is in ordinal [32]. The level of significance was set at $p<0.05$. A statisty.app was used to visually display and process the data.

Table 1. Weighted sum of score of geometric thinking level

Geometric thinking level	Score	Sum of score
0	0	0, 2, 4, 8, 16, 18, 20, 24
1	1	1, 5, 9, 17, 21, 25
2	2	3, 11, 19, 27
3	4	6, 7, 22, 23
4	8	13, 14, 15, 29, 30
5	16	31
Not fit	-	10, 12, 26, 28

The GTT, developed by Usiskin [31] through the cognitive development and achievement in secondary school geometry (CDASSG) project, has a reported reliability coefficient of r=0.64, indicating a high level of dependability and trustworthiness in measuring geometric thinking levels. The Indonesian version of the GTT, adapted from Zainal [33], was used to eliminate any potential language barriers faced by students. Ethical considerations were addressed by obtaining informed consent from all participants, ensuring the confidentiality and anonymity of their responses, and adhering to responsible data usage practices.

3. RESULTS AND DISCUSSION

3.1. Analysis of the distribution of students' geometric thinking levels

This section evaluates the geometric thinking levels of students from UNM and UNMUL, following the established criteria of the Van Hiele model. The Van Hiele model categorizes geometric thinking into five distinct levels, each representing a progressively deeper understanding of geometric concepts. Table 2 and Figure 1 provide an overview of the distribution of students across different levels of geometric thinking.

Table 2 shows that both universities have students distributed across various levels, with notable differences in the patterns observed. At the lowest level (level 0), 27 students from UNM (11.6%) and 25 students from UNMUL (11.0%) struggled with basic shape recognition. Many students were at level 1 (visualization), where 65 students from UNM (27.9%) and 87 students from UNMUL (38.3%) were able to recognize shapes based on appearance but had not yet progressed to understanding their properties. Level 2 (analysis) saw a significant number of students, with 89 from UNM (38.2%) and 74 from UNMUL (32.6%) demonstrating a stronger grasp of geometric properties.

Table 2. The overview of the levels of geometric thinking among students in UNM and UNMUL

Geometric thinking level	Sum of score	Criteria of three out of five correct answer			
		UNM		UNMUL	
		Frequency	Total	Frequency	Total
0	0	7	27 (11.6%)	17	25 (11.0%)
	2	11		2	
	8	0		2	
	16	5		2	
	18	4		2	
1	1	47	65 (27.9%)	64	87 (38.3%)
	5	3		10	
	9	2		4	
	17	9		8	
	21	2		1	
	25	2		0	
2	3	46	89 (38.2%)	53	74 (32.6%)
	11	14		4	
	19	25		16	
	27	4		1	
3	6	2	34 (14.6%)	2	27 (11.9%)
	7	17		23	
	22	1		0	
	23	14		2	
4	13	1	4 (1.7%)	0	10 (4.4%)
	15	3		10	
5	31	6	6 (2.6%)	3	3 (1.3%)
Not fit	10	7	8 (3.4%)	1	1 (0.4%)
	26	1		0	
Total		233	100	227	100

As the complexity of the levels increased, fewer students advanced to higher levels. At level 3 (informal deduction), 34 students from UNM (14.6%) and 27 students from UNMUL (11.9%) understood relationships between geometric properties. Level 4 (formal deduction) had very few students: four from UNM (1.7%) and 10 from UNMUL (4.4%), indicating a limited capacity for developing and understanding formal proofs. At level 5 (rigor), only a small number of students were present, with six from UNM (2.6%) and three from UNMUL (1.3%), suggesting that rigorous geometric thinking is rare among the participants.

Additionally, some students did not fit into any specific level, with eight students from UNM (3.4%) and one from UNMUL (0.4%). Overall, while a substantial proportion of students possess foundational geometric knowledge, a smaller number have achieved higher proficiency in geometric cognition. Figure 1 illustrates the comparison of geometric thinking levels among the participants from UNM and UNMUL.

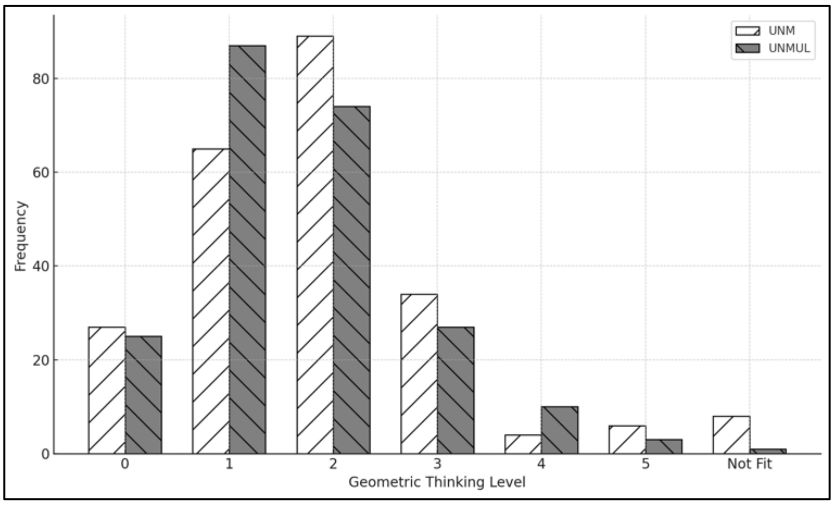

Figure 1. Comparison of students' geometric thinking level

3.2. Analysis of the students' geometric thinking levels

Table 3 presents the mean ranks for geometric thinking levels and overall scores between the two universities. It offers a quantitative comparison of student performance. The mean ranks for both geometric thinking levels and sum scores highlight a performance disparity between UNM and UNMUL students. It suggests differences in how effectively each institution fosters geometric thinking.

UNM students demonstrated higher mean ranks in both geometric thinking levels (240.88) and sum scores (254.9) compared to UNMUL students (219.84 and 205.46, respectively). This suggests that UNM students not only exhibit better geometric thinking but also achieve higher overall scores in geometric tasks. The higher sum scores at UNM indicate a more robust understanding and application of geometric concepts.

The results of the Mann-Whitney U test, as shown in Table 4, provide a deeper understanding of these differences. The difference in geometric thinking levels between UNM and UNMUL was not statistically significant (U=24026, p=0.09), indicating similar performance levels in geometric thinking. However, the difference in sum scores was statistically significant (U=20760.5, p<0.001), confirming that UNM students outperformed their UNMUL counterparts overall.

Table 3. Mean ranks of two universities

	University	N	Mean rank	Sum of ranks
Geometric thinking level	UNM	233	240.88	56126
	UNMUL	227	219.84	49904
	Total	460		
Sum of score	UNM	233	254.9	59391.5
	UNMUL	227	205.46	46638.5
	Total	460		

Table 4. Result of Mann-Whitney U test of two universities

	Mann-Whitney U	z	asymptotic p	p-value
Geometric thinking level	24026	-1.77	0.076	0.09
Sum of score	20760.5	-4.04	0.000	0.000

3.3. Discussion

The comparative analysis of geometric thinking among generation Z pre-service mathematics teachers from UNM and UNMUL provides critical insights into the effectiveness of current educational strategies at these institutions. The findings reveal disparities in geometric thinking levels and overall performance, with significant implications for improving teaching methodologies. These disparities suggest that while both institutions are developing students' geometric thinking, the approaches and outcomes vary, indicating a need for tailored instructional strategies.

UNM students generally performed better, particularly in intermediate geometric thinking levels (analysis and informal deduction). This higher performance may be attributed to the instructional strategies employed at UNM, which appear to be more effective in fostering a deeper understanding of geometric properties and logical relationships. On the other hand, UNMUL students exhibited a broader distribution across all levels, with a notable presence at both the basic visualization level and the advanced formal deduction level. This distribution indicates a less focused but broader range of geometric thinking skills among UNMUL students, suggesting that while some students excel in basic recognition and advanced deduction, others may struggle to progress beyond initial levels. This discrepancy underscores a divergence in the foundational and advanced geometric thinking abilities between the two academic institutions. The prevalence of UNM students at intermediate levels indicates the efficacy of pedagogical approaches that facilitate the advancement of analytical and informal deductive level.

These results align with previous studies that criticize traditional teacher-centered instructional approaches for their limited effectiveness in developing geometric thinking. Conventional teaching methods, which are prevalent in many educational contexts, including Nigeria, have been shown to inadequately address the real-life application of geometric concepts and fail to engage students in hands-on, student-centered learning activities [23]. Such approaches are not aligned with the Van Hiele model's phases of learning geometry, which emphasize discovery-based learning and incremental development of geometric understanding [17], [34], [35]. This is also consistent with findings from Hassan *et al.* [23] who reported poor performance in geometry among Nigerian secondary school students due to inadequate geometric skills and ineffective teaching strategies. Moreover, studies have confirmed that the use of traditional instructional strategies contributes to a significant gap in students' geometric understanding and their acquisition of advanced geometric thinking skills[36]–[38].

The predominant use of rote learning, as highlighted in the findings, hinders students from reaching higher levels of geometric thinking. Hourigan and Leavy [12] emphasize the necessity for pre-service primary teachers to possess a comprehensive understanding of geometric concepts to enhance their teaching proficiency. The efficacy of Van Hiele phase-based instruction in enhancing the geometric thinking abilities of pre-service teachers has been substantiated by empirical evidence. Armah *et al.* [36] have demonstrated that structured instructional approaches aligned with the Van Hiele phases can markedly enhance geometric thinking skills.

The findings underscore the need for educational reforms that adopt more effective student-centered instructional strategies aligned with Van Hiele's phases of learning. Van Hiele's model proposes that students progress through different levels of geometric thinking, from basic recognition of shapes to constructing formal proofs [39]. By implementing instruction based on Van Hiele's phases, students can achieve a better grasp of geometric concepts compared to traditional methods [18], [34], [40]. The use of Van Hiele's phase-based teaching strategies has been found to help students overcome challenges in geometry learning [20]. Furthermore, studies have indicated that interventions based on Van Hiele's phases can effectively address students' difficulties in geometry, leading to improved achievement and attitudes towards the subject [21], [24]. Educational institutions should focus on interactive and practical learning experiences, encouraging students to explore and understand geometric concepts deeply, thereby preparing them more effectively for future teaching roles. These insights are critical for educators and policymakers aiming to elevate the quality of mathematics education and geometric instruction in universities.

4. CONCLUSION

The study revealed notable differences in the geometric thinking levels between students at UNM and UNMUL, with UNM students generally outperforming their UNMUL counterparts, particularly in intermediate levels of geometric thinking. This suggests that the instructional strategies at UNM may be more effective in fostering a deeper understanding of geometric concepts. To effectively enhance geometric thinking levels, it is recommended that both institutions implement specific student-centered learning strategies. These include incorporating interactive geometry software like GeoGebra to facilitate hands-on learning, organizing collaborative problem-solving sessions to foster deeper understanding, and integrating project-based learning activities that require students to apply geometric concepts to real-world problems.

Additionally, regular assessments using Van Hiele's phase-based model should be conducted to monitor progress and tailor instruction to meet the diverse learning needs of students. Moreover, professional development for instructors is essential to ensure they are well-equipped with effective teaching strategies that align with the Van Hiele model. Cross-institutional collaboration between UNM and UNMUL could foster the exchange of best practices, leading to improved outcomes in teaching geometry. Recognizing the varied distribution of students across the Van Hiele levels, instruction should be tailored to individual learning needs, offering additional support for those struggling and advanced challenges for those ready to progress. Longitudinal studies tracking students' progress could provide valuable insights into the effectiveness of these strategies and inform ongoing curriculum improvements. These recommendations highlight the need for educational reforms that prioritize student-centered, phase-based instructional strategies to elevate the quality of mathematics education and geometric instruction in universities.

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