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Team assisted individualization: enhancing students' conjecturing skills in statistics and probability

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ABSTRACT

Many students struggle to understand mathematical concepts, leading to inadequate conjecturing skills. This study investigates the effectiveness of team assisted individualization (TAI), a cooperative learning approach, in improving these skills in mathematics. Utilizing a pre-test-post-test control group design, the research involved matched groups from two senior high school sections. Data were collected through tests and interviews conducted before and after the intervention, and analyzed using means, standard deviations, paired samples t-tests, analysis of covariance (ANCOVA), and content analysis. The findings indicate a significant enhancement in students' conjecturing abilities following the implementation of TAI, even when controlling for their numerical reasoning skills. These results demonstrate that TAI is more effective than traditional lecture-discussion methods (LDM) in promoting conjecturing skills. Consequently, this study encourages educators to explore and adopt TAI strategies to better facilitate the development of students' mathematical reasoning and conjecturing capabilities.

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1. INTRODUCTION

Conjecturing remains a vital yet often overlooked component of students' mathematical proficiency [1], particularly when recognizing patterns, forming hypotheses, and establishing foundational principles. Although many learners can follow procedural lessons, they frequently encounter difficulties grasping the underlying logic and justifications behind mathematical concepts [2]. This struggle may stem from the abstract nature of mathematical reasoning, where intuitive thinking plays a significant role [3]. Studies suggest that incorporating realistic mathematics approaches can significantly improve students' ability to communicate their mathematical thinking [4], allowing them to express reasoning more clearly and engage with problem-solving tasks more effectively [5]. By prioritizing the development of conjecturing skills, educators can bridge understanding gaps to foster a deeper appreciation for mathematics.

However, being able to comprehend a problem does not necessarily mean students can formulate meaningful conjectures [6]. In the Philippine context, many students encounter challenges in making inferences, generating conjectures [7], and analyzing data in the context of statistics [8]. This difficulty may be attributed to underdeveloped reasoning skills, which are crucial in understanding and evaluating statistical problems [8]. As such, there is a pressing need to strengthen students' abilities in conjecturing, particularly in the areas of statistics and probability.

Given the current inadequacies in students' conjecturing skills [9], collaborative learning strategies may offer a pathway for improvement [10]. One such strategy is team assisted individualization (TAI), a cooperative learning model that organizes students into heterogeneous groups. TAI encourages peer support, allowing students to leverage each other's diverse knowledge to enhance their mathematical skills [11]. TAI is posited to be effective [12], [13], as it improves mathematics achievement [14], mathematical skills such as mathematical problem-solving abilities [15], conceptual understanding [16], and mathematical communication [17]. TAI also fosters critical thinking capabilities when exposed to higher-order thinking skills (HOTS) questions [18] and connection skills among students [19]. Moreover, when TAI was used to teach Statistics, aside from scores, the mathematical communication ability of students was improved [20], [21]. Henceforth, TAI is among the cooperative learning approaches that may improve the conjecturing ability of the students.

Cooperative learning has proven effective in enhancing various mathematical abilities, including problem-solving and conceptual understanding. However, research on the use of TAI specifically for improving conjecturing abilities in statistics and probability is limited. Given that cooperative learning is linked to improved numerical reasoning [21], [22], an essential component of conjecturing [23], this study aims to evaluate TAI's effectiveness in enhancing the students' conjecturing abilities in statistics and probability while controlling for their numerical reasoning. Specifically, the study sought to describe the conjecturing abilities, compare the conjecturing capabilities of groups taught through traditional lecture-discussion methods (LDM) with those using TAI, and assess differences while accounting for numerical skills.

2. METHOD

2.1. Research design

The researchers used a pre-test-post-test control group design using matched group subjects. The main idea behind the pre-test-post-test design is to collect a pre-test measure of the desired outcome before the actual administration of the test, then collect a post-test on the same measure after treatment has been applied. The design was used to measure the conjecturing scores of the TAI and LDM groups before and after the treatment and compare the conjecturing abilities of the two groups while controlling their numerical reasoning. Also, an interview was done to give more depth to the study since the researchers would also like to know the perceptions of students on the effectiveness of TAI.

2.2. Participants

The study participants are students from a standalone senior high school in the Philippines. Two science, technology, engineering, and mathematics (STEM) standalone sections were selected for their comparable abilities, as statistics and probability are part of the STEM curriculum. A total of 54 students were chosen, with 27 students from each section, exceeding the minimum requirement of 15 cases per group [24]. To ensure comparability, the pretest scores were matched to form 27 pairs of respondents.

2.3. Research instruments

2.3.1. Statistics conjecturing test

This is a 30-item test developed by the authors to assess the conjecturing abilities of the senior high school students in the topics language of hypothesis testing and hypothesis testing for means. Each item on the test is scored at one point. The statistics conjecturing test was validated by three senior high school teachers with a minimum of three years' experience teaching statistics and probability. Additionally, three university instructors, each with at least three years of teaching experience, were consulted during the validation process. Furthermore, the above .9 Cronbach's alpha coefficient of the researcher-made instrument (Cronbach's alpha=.98) indicates acceptable internal consistency [25].

2.3.2. Numerical reasoning test

A 23-item mathematical reasoning test developed by David Faulkner from the University of Hertfordshire. It is a material that is ready to use since has a Creative Commons license. The scoring in this test is one point per item.

2.3.3. Interview guide

An interview guide was employed to capture students' insights during the implementation of both the LDM and TAI. The guide featured thoughtfully designed open-ended questions intended to draw out meaningful reflections on their learning experiences. To ensure that the questions were clear, appropriate, and aligned with the study's goals, three experienced and published researchers reviewed and validated the guide. This validation process contributed to the overall credibility and trustworthiness of the data gathered.

2.4. Data gathering procedure
2.4.1. Pre-experimental phase
The researchers first obtained permission from the principal and the participants' advisers to conduct the study, along with signed waivers from involved faculty members. After receiving approval, they validated the statistics conjecturing test, interview guide, and lesson plans through consultations with subject matter experts. Reliability was established through a pilot test conducted at various national high schools, targeting grade 12 students who had recently completed the general education course in statistics and probability.

The researchers obtained informed consent from all participants and provided an orientation on the study's procedures. Participants were instructed on completing the assessments and informed they could decline without repercussions. The researchers then administered the numerical reasoning test and statistics conjecturing pre-test to both groups, allowing 60 minutes for each group. To ensure comparability, students in both the LDM and TAI groups were matched based on pre-test results, which showed equivalent performance levels, establishing a solid foundation for comparison in subsequent analyses.

2.4.2. Experimental phase
Both groups were taught by one of the researchers over nine consecutive 40-minute sessions using two sets of lesson plans: one for the LDM group, and another for the TAI group. The TAI group engaged in teacher-designed activities, including group discussions, problem-solving, and hands-on tasks. Moreover, the students in the TAI group were divided into seven groups consisting of six members per group. In contrast, the LDM group received instruction through a traditional LDM.

Two topics were covered in the nine consecutive sessions: the language of hypothesis testing and hypothesis testing concerning means. The learning outcomes included: i) illustrating null hypothesis, alternative hypothesis, level of significance, rejection region, and types of error in hypothesis testing; ii) calculating the possibilities of committing type I and type II error; iii) identifying the parameter to be tested given a real-life problem hypothesis; iv) formulating the appropriate null and alternative hypotheses on a population mean; v) identifying the appropriate form of the test statistic when: the population variance is known, the population variance is unknown, and the Central Limit Theorem is to be used; vi) determining the rejection region under different conditions; vii) computing the test statistic value for population mean; viii) drawing conclusions about the population mean based on the test statistic value and the rejection region; and ix) solving problems involving test of hypothesis on the population mean.

2.4.3. Post-experimental phase
The researchers administered the post-test, ensuring items were parallel in difficulty to those in the pre-test. Following this, they interviewed ten participants - five from the TAI group and five from the LDM group-to assess their acquired knowledge of the topics covered using the two different teaching approaches. Subsequently, the quantitative data were tallied, summarized, and analyzed. Additionally, thematic analysis was used to analyze the participants' responses.

2.5. Data analysis procedure
To analyze the data quantitatively, frequencies, percentages, means and standard deviations were used to describe the conjecturing abilities of senior high school students in statistics and probability during the pre-test and post-test. The level of conjecturing ability was described as highly proficient (25–30), proficient (19–24), nearly proficient (12–18), low proficient (6–11), and not proficient (0–5). Meanwhile, paired-samples t-test was also utilized to test the significant difference between the conjecturing abilities of each group before and after the treatment. The measurement between two group means was computed using Cohen's d. Cohen's d was interpreted as trivial effect (.00-.19), small effect (.20), medium effect (.50), and large effect (.80 or higher). Then, the analysis of covariance (ANCOVA) was used to test the significant difference between the conjecturing abilities of the TAI and the direct instruction group after treatment while controlling their numerical reasoning.

Moreover, content analysis was employed to systematically examine the interview transcripts, enabling the extraction of qualitative insights into students' experiences and perceptions regarding their conjecturing abilities before and after the experiment. This method provided a structured framework for analyzing both quantitative and qualitative data, ensuring a comprehensive understanding of the findings. Consequently, it illuminated the effects of the instructional methods used, offering valuable perspectives on their impact on student learning outcomes.

3. RESULTS AND DISCUSSION

3.1. Level of conjecturing ability

The level of conjecturing ability was administered to the participants to evaluate their capacity to draw conclusions based on presumptive evidence related to concepts in statistics and probability. Table 1 illustrates the level of conjecturing ability of both LDM and TAI groups. The post-test scores were used to identify the level of conjecturing ability of the participants after the intervention.

Table 1 shows that most participants in the LDM group (N=10, 37%) have a low proficiency in conjecturing ability in statistics and probability, while only a small proportion (N=8, 29.60%) demonstrated proficient ability after implementation. Furthermore, the mean score of the LDM students was nearly proficient (M=14.00, SD=4.88). This aligns with previous findings that students' conjecturing abilities range from low to moderately high, even after instruction [26].

Table 1. Level of conjecturing ability based on post-test conjecturing score of the lecture-discussion and TAI groups

Level of conjecturing ability	Lecture-discussion		TAI	
	N	%	N	%
Highly proficient (25–30)	0	0	8	29.60
Proficient (19–24)	8	29.60	7	25.90
Nearly proficient (12–18)	9	33.30	10	37.00
Low proficiency (6–11)	10	37.00	2	7.40
Not proficient (0-5)	0	0	0	0

Note: N=52 (N=27 for each group). Participants in the lecture-discussion group were on average nearly proficient (M=14.00, SD=4.88); while those in the TAI group were proficient (M=19.78, SD=5.73).

In contrast, the majority of the TAI group exhibited proficient to highly proficient levels of conjecturing ability (N=15, 55.5%) after the intervention. Their overall mean score was proficient (M=19.78, SD=5.73), suggesting a strong ability to develop conjectures in testing hypotheses. This supports findings that students' abilities to develop conjectures and draw conclusions improved significantly after instruction [27].

Generally, both groups achieved low proficiency as their lowest level of conjecturing ability, but they differed in their highest levels: the TAI group reached highly proficient while the LDM group reached proficient. With a mean difference of -5.78, the TAI group's post-test score was slightly higher than that of the LDM group. These results suggest that while both methods have limitations in addressing low-level conjecturing skills, TAI is significantly more effective in developing high-level conjecturing abilities in statistics and probability. The TAI group's performance, characterized by higher mean scores and broader range of proficiency levels, underscores the benefits of collaborative learning and active student engagement. From the perspective of TAI intervention, group activities benefit the students since aside from being student-centered, they also promote collaboration and brainstorming where students discuss and share ideas that help them learn in a more engaging way [28].

The success of TAI approach highlights the importance of active learning environments where students collaborate and share ideas. This method not only enhances conjecturing skills but also prepares students for real-world applications of statistical reasoning. Therefore, educators may consider integrating TAI methodologies into their teaching practices to boost student engagement and improve learning outcomes in complex subjects like statistics and probability, ultimately equipping students with the critical thinking skills necessary for academic and professional success.

3.2. Comparison between the pre-test and post-test scores of the lecture-discussion and team assisted individualization groups

The difference in conjecturing ability among senior high school students before and after their exposure to the LDM is presented in Table 2. Meanwhile, Table 3 displays the results of the paired samples t-test, which assessed the significant difference in pre-test and post-test scores for students who participated in TAI. Finally, Table 4 compares the conjecturing abilities of the two groups while controlling for their numerical ability.

It is apparent in Table 2 that there is a significant increase in the LDM groups' conjecturing scores from pre-test (M=7.56, SD=2.93) to post-test (M=14.00, SD=4.88); t (26)=-7.94, $p<.005$, with a large effect size (Cohen's d=-1.53). This indicates that the LDM effectively improved the students' conjecturing abilities, despite some gains being attributed to general learning effects. These findings align with previous research indicating the post-tests are administered after the intervention implying that students already learned the concepts of the lesson which is why students' scores during the post-test are higher compared to their pre-test scores [29], [30]. Moreover, the fact that the LDM has shown its effectiveness in improving the conjecturing skills of the students could be because it improves students' motivation and communication abilities [31]. Interviews conducted before and after the implementation indicated that many students initially had little knowledge of hypothesis testing (LD1), (LD4). This may indicate the need for targeted instructional strategies to enhance students' comprehension of hypothesis testing.

"*I have no any idea about hypothesis testing, sir.*" (LD1)
"*I have only heard about null hypothesis, but I am clueless about it, sir.*" (LD4)

Table 2. Paired samples t-test results for the difference in the pre-test and post-test conjecturing scores of lecture-discussion groups

Test	M	SD	t (26)	p	Cohen's d
Pre-test	7.56	2.93	-7.94**	.00	-1.53
Post-test	14.00	4.88			

**p<.001. Note: N=27. Participants in the lecture-discussion group's pre-test scores were low proficient (M=7.56, SD=2.93); while their post-test scores were on average nearly proficient (M=14.00, SD=4.88).

After the intervention, students demonstrated varying levels of understanding based on their cognitive abilities. With some demonstrated a foundational understanding of hypothesis testing (LD4), others were able to distinguish between one-tailed and two-tailed tests (LD23), reflective a mix of basic and deeper understanding of the concept. This implies that while the LDM was effective in developing some aspects of students' understanding of hypothesis testing, there is a clear variation in the depth of learning achieved. These results highlight the need for diversified instructional strategies that cater to different learning styles and cognitive levels to ensure that all students develop a comprehensive understanding of hypothesis testing.

"*What I learned about hypothesis testing is determining whether to accept or reject the null hypothesis by following the steps in hypothesis testing, then after that, we get to decide if we are going to accept the null hypothesis, and also give a conclusion.*" (LD4)
"*Sir, I learned when to use the one-tailed test and the two-tailed test. We use a one-tailed test if it is directional while it is a two-tailed test if it is non-directional. I remember that the area of rejection is larger when it is one-tailed test. However, I find it hard to draw a conclusion because it is confusing but it is fun to learn because it is connected to our practical research subject.*" (LD23)

Overall, while the LDM shows promise in improving conjecturing abilities but underscores the need for diversified instructional strategies to accommodate different learning styles and cognitive levels. Integrating this method into a broader instructional toolkit can yield to positive outcomes in student learning and engagement. Regardless of whether the students find the LDM less motivating [32] or more engaging [33], they can still make learning meaningful as long as they can connect the subject matter to the real world [34]. Lectures are effectives for teaching because they allow instructors to present theories clearly and efficiently [35]. Research indicates that flipped courses can yield similar content knowledge improvements as traditional lectures [36]. While some studies suggest that LDM effectively establish foundational skills [37], others reveal that many students prefer this approach for its relevance [38] and interactive nature [39], [40].

Table 3 displays the results of a paired samples t-test comparing the pre-test and post-test scores of the students who participated in the TAI intervention. The results indicate a significant improvement in the conjecturing scores, with pre-test scores (M=7.56, SD=2.92) rising to post-test scores (M=19.78, SD=5.73); t(26)=-11.39, p<.005. This substantial increase suggests that TAI is an effective method in enhancing students' HOTS in statistics and probability, supported by a large effect size (Cohen's d=-2.19). The effectiveness of TAI in improving the conjecturing ability could be attributed to the fact that it is a methodology that can increase the student's understanding of mathematical [41], HOTS [42], and learning motivation [43], [44].

Table 3. Paired samples t-test results for the differences on the pre-test and post-test scores of TAI group

Test	M	SD	t (26)	p	Cohen's d
Pre-test	7.56	2.92	-11.39**	.00	-2.19
Post-test	19.78	5.73			

**p<.001. Note. N=27. Participants in the TAI group's pre-test scores were low proficient (M=7.56, SD=2.92); while their post-test scores were proficient (M=19.78, SD=5.73).

Interviews conducted before and after the TAI intervention revealed a gap in students' prior knowledge of hypothesis testing. Many students had little familiarity with the topic hypothesis testing (T10 and T12) and some have the slightest knowledge of hypothesis testing such as the null hypothesis (T3), indicating confusion about basic concepts like the null hypothesis.

> "*None, sir. I never heard or read anything about it.*" (T10)
> "*Sir, based on what I have read, is it related to null hypothesis, sir? Like, is it connected with Ha? I am not quite sure, sir, but I still do not have enough knowledge about hypothesis testing, sir*" (T3)

After the intervention, some students reported learning how to solve *z* and *t* and apply hypothesis testing in the real world (T12, T16). Some students responded that they learned to formulate null and alternative hypotheses, find critical values, compute test value, make conclusion (T16). While many students demonstrated proficiency in applying these concepts, there was variability in the depth of knowledge acquired.

> "*At first, sir, I thought it's going to be challenging but as time goes by, I realized that it is not that hard. I learned how to locate the critical values using the level of significance. I also learned how to compute for the value of z and t using the formula. I still find it hard to identify the given but once you understand it, it is easier to solve. Plus, it is not merely just about solving because we can apply hypothesis testing in real world.*" (T12)
> "*I learned the five steps of hypothesis testing. First, formulating the null and alternative hypotheses. The second step is finding the critical values. Step 3 is computing the test value. Then, step 4 is wait…when…when you decide whether to accept or reject Ho. Lastly, making the conclusion. I also learned that we can actually apply these stuffs about statistics in real life especially in practical research subject.*" (T16)

In general, TAI's success in improving conjecturing abilities can be linked to its emphasis on cooperative learning, which fosters problem-solving skills [14], [45], and enhances academic engagement [46]. The significant improvement in post-test scores and large effect size provide strong evidence for TAI's efficacy in developing HOTS [47]. As such, the study underscores the importance of integrating collaborative learning techniques like TAI into the curriculum, as they can significantly enhance student outcomes in complex subjects such as statistics and probability. By adopting TAI as a pedagogical strategy, educators can improve academic performance while fostering a deeper appreciation for statistical concepts among students, ultimately preparing them for more advanced applications.

3.3. Comparison between the conjecturing abilities of the lecture-discussion and team assisted individualization groups while controlling their numerical reasoning

Table 4 shows the ANCOVA results for the difference between the conjecturing abilities of the TAI group and the LDM group after the intervention while controlling their numerical reasoning ability. As revealed in the table, the TAI group achieved a higher mean post-test score (M=19.78, SD=5.73) compared to the LDM group (M=14.00, SD=4.88); F=15.34, p<0.005, covariate=4.72, with notable difference of 5.78 points. Although the effect size is small (Cohen's d=0.23), these findings suggest that TAI is more effective than LDM in enhancing students' conjecturing abilities, even when controlling for numerical reasoning ability.

Table 4. ANCOVA results for the difference between the conjecturing abilities of the lecture-discussion and TAI groups while controlling their numerical reasoning ability

Group	M	SD	F	p	Cohen's d
TAI	19.78	5.73	15.34**	.00	.23
Lecture-discussion	14.00	4.88			

**p<.001. Covariate=4.72.

Interviews indicated that TAI students had a stronger grasp of hypothesis testing (T27) than their peers in LDM (LD1). TAI participants articulated key concepts, including null and alternative hypotheses, and outlined the five essential steps of hypothesis testing. This suggests that TAI promotes both theoretical understanding and practical application of statistical concepts. While students in the LDM group acknowledged the importance of hypothesis testing and its applications–such as z-tests and t-tests–their understanding lacked depth and clarity. This disparity highlights how the interactive nature of TAI enhances student engagement and retention of complex ideas. Group work allows students to analyze not only the problem but also their peers' reasoning, improving metacognitive and conjecturing skills [48].

> "*I remember the meaning of hypothesis and definition of hypothesis testing, most importantly the null and alternative hypothesis and their importance in real life as well as in applying them in research purposes. I also learned the five essential steps in hypothesis testing namely formulating hypotheses, identifying the critical value using level of significance, computing the value of test-statistic, deciding whether to reject or not the null hypothesis, and concluding based on the problem.*" (T27)
>
> "*I learned that hypothesis testing is important because it is a valid method that can help us to decide if a null hypothesis will be rejected or not. With the use of the 2 test statistics that are used for hypothesis testing, which are the z-test and t-test, we are able to make statements about the population parameters given in the problem. Hence, it proves if something 'is' or 'is not'. I figured out that hypothesis testing is very relevant to us, the students, at the moment as well as in the future because it plays a crucial part in our research. With the help of the lessons we had, we learned how to properly test the hypothesis given in our studies. On the other hand, hypothesis testing is also applied not just in research, but also in scientific experiments, trials, and processes.*" (LD1)

The significant improvement in the TAI group's post-test underscores its effectiveness in fostering deeper engagement with complex material. The structured articulation of their understanding indicates enhanced cognitive processing. This could be because cooperative learning effectively promotes mathematical reasoning skills by engaging students in the active and collaborative creation of understanding [49]. These findings align with previous research that cooperative learning outperforms traditional LDM [50]–[52]. Although both groups demonstrated comprehension of essential concepts, TAI students exhibited superior argumentation and critical thinking skills, particularly when confronted with higher-order thinking questions [19]. Therefore, the TAI approach is a better teaching approach in honing the ability of the students to develop conjectures in hypothesis testing. The evidence suggests that educators may prioritize collaborative instructional methods like TAI to improve learning outcomes. By adopting TAI, educators can create a classroom environment that enhances students' conjecturing abilities and prepares them for real-world applications of statistical reasoning and hypothesis testing.

4. CONCLUSION

While both LDM and TAI approaches are effective in enhancing students' conjecturing abilities in statistics and probability, TAI proves to be significantly more impactful. Participants exposed to TAI demonstrated a higher level of proficiency in developing conjectures during hypothesis testing compared to those in LDM. This indicates that the collaborative and interactive nature of TAI fosters deeper engagement with the material, allowing students to better understand and apply complex concepts. To further maximize the benefits of TAI, educators may consider implementing it in a controlled learning environment, such as during fully face-to-face classes, where the interactive components can be fully realized.

While the LDM also effectively improved participants' conjecturing abilities, it may be more suitable for crowded classrooms as it does not rely heavily on group activities. To further develop students' conjecturing skills through LDM, educators may provide more challenging word problems and employ creative visual aids to boost engagement. However, the consistent evidence supporting TAI's superior effectiveness across various studies reinforces the need for its broader implementation in the curriculum. By adopting TAI, educators can cultivate a classroom environment that not only improves students' conjecturing abilities but also prepares them for real-world applications of statistical reasoning and hypothesis testing.

Future research should explore the long-term effects of TAI on students' conjecturing abilities across different educational settings and subject areas. Additionally, studies could investigate how variations in group composition within TAI influence learning outcomes. Furthermore, examining the integration of technology in TAI could provide insights into enhancing collaborative learning experiences.

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AUTHOR CONTRIBUTIONS STATEMENT

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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

The authors declare that they have no known financial, personal, or professional conflicts of interest that could have influenced the work reported in this paper. There are no political, religious, ideological, academic, or intellectual competing interests to disclose. All authors confirm that they have no competing interests related to this manuscript. Authors state no conflict of interest.

INFORMED CONSENT

Before the study began, the researchers secured informed consent from all participants. During an orientation session, participants were clearly briefed on the study's objectives, procedures, potential risks, and anticipated benefits. They were explicitly informed that their involvement was voluntary and that they could withdraw at any point without facing any penalties or consequences. To ensure participant confidentiality, all personal data were treated with strict discretion, and identifying information was protected throughout the research process. Each participant provided written consent, affirming their agreement to take part in accordance with ethical guidelines and legal standards. In line with these protocols, all individuals included in the study provided their informed consent.

ETHICAL APPROVAL

This study involving human participants was conducted in accordance with applicable national guidelines and institutional policies, with careful adherence to the ethical principles set forth in the Declaration of Helsinki. Prior to implementation, the research protocol underwent review and received approval from the Institutional Ethics Review Board of Isabela State University, confirming that all procedures complied with standards for the ethical treatment of participants. Throughout the study, every effort was made to uphold participants' rights, prioritize their safety, and safeguard their overall well-being.

DATA AVAILABILITY

The data underpinning the results of this study can be obtained from the corresponding author [KARGF], upon reasonable request. To protect the privacy of the student participants, the dataset is not openly accessible. Researchers who wish to access the data may do so by contacting the corresponding author and agreeing to the necessary confidentiality and ethical terms.

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