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Didactic experience based on mathematics competencies and soft competencies for teacher education

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ABSTRACT

The aim of this research was to implement a didactic experience to facilitate the study of linear algebra. This article shows the potential of working the curriculum based on both disciplinary competencies and soft competencies, prioritizing collaborative learning, in future secondary school teachers of mathematics. The nature of the study is exploratory with a mixed methodology. In this article, we show the phases that constituted in its entirety the way of working during a semester in higher education and the forms of evaluation that are part of the qualitative-interpretative methodology, which include the elaboration, validation and application of a questionnaire of students' opinion on mathematical and soft skills, in addition to the analysis by categories of open answers, considering the similarities and differences found in the opinions written by the students, regarding the strengths and weaknesses of the didactic proposal, validating the usefulness of this successful teaching methodology.

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1. INTRODUCTION

In the Latin American context competency-based education was successfully introduced in many universities in the hemisphere to improve the learning process and transform higher education [1]–[3]. This began in the European education area with the development of the Tuning Europe project, it crossed borders, establishing the Tuning project for Latin America adapted to the continent in universities and for universities [4], [5]. For Bredekamp [6], the focus of education evolved towards learning so that teachers are no longer the center of the process but the students.

With the emergence of the competency-based educational model, education must be oriented towards training integral individuals for an increasingly complex society, where technical or hard competencies are developed equally with those that favor personal growth, ethics and values for life, i.e., the so-called generic or soft competencies. The latter include collaborative learning and are becoming important requirements as a product of competitiveness, graduates who can think critically, work in teams, solve problems and communicate, to name a few, are highly sought after by employers [7]–[11]. They are also known by terms such as 21st century competencies, transversal or non-cognitive skills, socioemotional skills. Regardless of their denomination, these skills include social and interpersonal skills or metacompetencies [12]–[14]. Collaborative learning in higher education has been incorporated into curricula over the past two decades [15]–[17] and is presented as a successful methodological alternative to traditional models [18]–[23]. However, there is very little literature regarding the role of collaborative learning instructors [24], [25].

In the real world, the demands go beyond the knowledge acquired in mathematics, since in order to achieve success, it is necessary to have developed and integrated skills. With technical or hard skills to achieve excellence at work, this is when the need to combine the concepts of life skills in educational programs is born [26]. In practice many institutions still approach teaching and learning processes with methods very similar to those used in the times of the industrial revolution [27]. The teaching of linear algebra and its results are a case in point.

In Chile, linear algebra constitutes a content of the national curriculum of higher education by competencies, and also appears in the curricula of international training institutions. It is an important academic course due to its wide application in both science, technology and economics [28], [29]. Students often consider linear algebra courses at the college level as difficult mathematics courses [30]. For Stuhlmann [31], most students have great difficulty in making sense of algebraic structures and axiomatic algebraic methods [32] and in selecting appropriate methods for system of linear equations [33]. Research by Ferryansyah *et al*. [34] analyzed the difficulty of students in learning linear algebra in mathematics education course and according to their results, the difficulty of students in learning linear algebra is very high: 88.63% of students are not able to represent the symbol or notation, 88.11% of students have difficulties in using mathematical ideas and logical reasoning. Added to this, teachers are also not optimal enough in the use of learning media and methods to teach linear algebra in the classroom [35]–[37]. Therefore, it seems clear that teaching in a traditional way may not significantly improve student learning and becomes a relevant issue for the continuing and initial training of mathematics teachers. This article is related to these aspects and shows the experience with students who are being trained as future mathematics teachers in a state university in Chile, where linear algebra is one of the required courses, however, the performance of students in this subject, has not been in accordance with the expected academic performance.

Considering the dizzying changes that have strained the work of higher education in the context of the COVID-19 pandemic which in the case of Chile implied the transfer of a large part of the face-to-face educational work to the remote mode during the year 2020, which had a national impact on the learning of more than 3.5 million children and young people [38], [39] to which is added the current explosion of artificial intelligence tools, mostly used by students, which is challenging teaching practices maintained for centuries [40], [41], the following question was formulated and served as a guide for the research: is it possible to successfully implement a didactic strategy that promotes the development of mathematical competencies and soft competencies that are part of the curriculum in mathematics teacher training? To answer this question, a didactic strategy was designed in a linear algebra course of the first semester, which began in March and concluded in August, whose general objective was to know the incidence of a didactic strategy based on the execution of mathematical competencies and soft competencies in mathematics pedagogy students. The specific objectives: i) to describe the didactic strategy for the teaching of linear algebra about the incorporation of collaborative methodology; ii) to elaborate, validate and apply an evaluation instrument based on mathematical competencies and soft competencies; and iii) to describe and analyze the students' opinion regarding the strengths and weaknesses of the development of the didactic strategy.

2. RESEARCH BACKGROUND

To enrich our teaching, we used a didactic strategy that has as its research framework the competencies defined in our university's own curriculum as accredited in pedagogical quality, which includes the mathematical or hard competencies based on the "Guiding Standards for Pedagogy Careers in Secondary Education: Pedagogical and Disciplinary Standards" [42], in which standard number 4 states that the future mathematics teacher must demonstrate disciplinary competency in linear algebra and, in addition, must demonstrate the ability to lead the learning of linear algebra applications in school mathematics.

With respect to the soft competencies, those declared by the institution were considered, and represent what is expected of the student. According to the curriculum, the soft competencies to be achieved in the students are: to carry out academic activities by continuously seeking to improve oneself, in a persevering manner, complying with established norms and standards and maintaining ethical and collaborative actions in the process to obtain optimal results; to manage one's own learning, individually and collaboratively, activating cognitive, meta-cognitive and socio-affective strategies that contribute to a favorable experience of the academic processes and professional life; to understand the ethical responsibility of their functions, behaving with honesty and personal integrity, acting with responsibility and social commitment. In response to these soft competences, collaborative learning was included as a priority in our research. The use of groups in instruction is based on the principle of constructivism, with special attention to the contribution that social interaction can make. In essence, constructivism is, in turn, based on the idea that individuals learn by constructing their own knowledge, connecting new ideas and experiences with existing knowledge and experiences to form a new or improved understanding [43], [44].

2.1. Teaching linear algebra

Since linear algebra today is useful to a wide variety of students, the question arises: how do students learn linear algebra and what teaching methods are most effective? The availability of computers and software has forced mathematicians to rethink the way they teach mathematics. When a computer can perform an algorithm quickly and successfully, it begs the question: what does a student really need to learn and what is the most appropriate teaching strategy for a future secondary school teacher? Roughly two main traditions can be distinguished in the teaching of linear algebra: one tradition focuses on the study of formal vector spaces, while the other proposes a more analytical approach based on the study of IR^n and matrix calculus [45].

For the approach based on matrix calculus, it has been proposed the use of software designed from a didactic point of view, to help students in learning the contents, giving examples, testing with matrices of different sizes to see what happens or even to solve exercises. The latter is not a problem, in the sense that no matter how much a result is obtained, the important thing is not to obtain the correct result, but to understand why what is obtained is obtained and to understand the internal processes of the algorithms [46]. Regarding the study focused on vector spaces, both its learning and teaching represents a challenge due to its high level of abstraction and, in addition, because this content was not created to solve problems, but was created to unify and generalize both existing methods and concepts [47]. In recent years, research has focused on making innovations for learning content related to vector spaces [48], [49], identifying difficulties that students have with them [50], [51], and errors [52].

2.2. Collaborative learning

Both in school, academic and professional careers, we are destined to face problems individually, but also - increasingly and collaboratively. Thus, collaborative problem solving became a new domain in the PISA 2015 study. Collaborative problem solving is an advantage due to the division of labor, variety of knowledge, different perspectives and experiences, reciprocal motivation, all aimed at a better solution of a given problem or academic work [53]. In general, collaborative learning is a construct that identifies a field of strong and growing relevance and is part of a soft or cross-cutting competency. It refers to instructional arrangements that involve a minimum of two or more students working together on a shared learning objective [54], [55].

A large number of studies demonstrate the positive effects of collaborative learning on cognitive, metacognitive, affective-motivational, and social aspects of learning [15], [17], [56], [57]. Several previous researches [58]–[60] have shown that students working in small groups have better learning outcomes than students working on an individual assignment, such as greater learning gains, higher performance on standardized tests or on self-reported assessments during the course. However, these positive outcomes can only be achieved when teachers make appropriate pedagogical decisions [61], [62].

While students work collaboratively, teachers should monitor what problems students encounter and intervene when necessary [63]. Therefore, the teacher's role is crucial in fostering positive interaction among learners [61]. The teacher diagnoses the progress of the group and intervenes when necessary [62]. Without proper guidance, positive interaction among students is at risk. When teachers fail to identify problems early and intervene appropriately, the quality of the collaborative process and the resulting learning outcomes can be hindered. Guiding groups of students in collaborative work is a demanding task for teachers [64]. For example, it requires teachers to master multiple advanced competencies [65]. This is because, during the learning process in which it takes place, teachers must supervise several groups at the same time, provide support regarding the content of the task as well as strategies for collaboration, and must decide whether intervention is necessary and, if so, what type of intervention is most appropriate. Therefore, collaborative learning requires much more from the teacher than simply placing students in groups and instructing them to work together [20], [61].

3. METHOD

3.1. Research design

Due to the exploratory nature of the research and its nature, where both the process and the product are of interest, a quantitative-descriptive and qualitative-interpretative methodology was used. In the following, we describe the details of the methodological process, focusing on the context in which the research was carried out, the data that was collected and how the analysis of this data was carried out in order to respond to the research objectives which are related to the knowledge of the impact of a didactic intervention based of the achievement of disciplinary competences in mathematics and soft or transversal competences and applied to students in initial training.

3.2. Sample of research

The unit of analysis of the research corresponded to 25 students, of which 10 are women and 15 are men between the ages of 18 and 20, who made up the totality of those who took the compulsory subject of linear algebra in their second year, belonging to a degree course in mathematics teaching at a university in the south of Chile. It should be noted that all participants received detailed information on the objectives and methodologies of the study prior to their participation.

3.3. Research procedure

Linear algebra is a compulsory course (semester) of 6 hours per week and is located in the second semester of the second year of the curriculum. The previous mathematical background of the students of the linear algebra course is an algebra course and a geometry course. The main objective of the linear algebra course is for students to develop skills such as mathematical logical reasoning and problem solving. It is expected that this course will provide students with key elements for the development of subsequent courses. It is divided into five units: i) matrices and systems of linear equations; ii) determinants; iii) vectors in the plane and in space; iv) vector spaces; and v) linear transformations.

The explicit mathematical competencies to be achieved in the syllabus for the first two units were: developing matrix algebra; identifying different types of matrices and their properties; operating with matrices; relating systems of linear equations to matrices; classifying and solving systems of linear equations using matrix techniques; defining determinant of matrices; identifying its properties, and using it to calculate the inverse matrix and to solve systems of linear equations where the number of unknowns is equal to the number of equations. For the teaching of these units, each student was provided with a “Linear Algebra Manual” created by the teacher-researcher, with the contents of both units, which began with an introduction to the topic of study, with examples of problems from everyday life, and with work guides as the units progressed. All of this was incorporated into the student's portfolio of evidence. At the end of each unit, educational YouTube tutorials oriented to the study of mathematics were suggested to them to broaden their individual knowledge, indicating the addresses of the websites.

According to the curriculum, the soft competencies to be achieved in students involve acting ethically, honestly, with personal integrity, acting with social commitment and personal integrity, and collaboratively in the process to obtain optimal results. Take charge of their own learning, both individually and collaboratively, activating not only cognitive and meta-cognitive strategies, but also socio-affective strategies that contribute to a favorable experience of academic processes and professional life. As a result of these expectations in the training of future secondary school teachers in mathematics, students were asked to form working groups, to provide and promote an active learning environment by involving them in their learning and getting them to participate actively through the various activities created with mathematical tasks that were designed so that students could experience mathematical processes for themselves. For this purpose, the following three units were distributed by the teacher to the three formal groups formed by the students on a voluntary basis. In this way, the aim was to encourage cooperative learning or group work, written and oral mathematical communication through discussion and the exchange of ideas among the students, prior to the individual presentations of each mathematical sub-unit.

3.3.1. Working in groups

The following three units were curricularly defined mathematical content: vectors in the plane and in space, vector spaces, and linear transformations. The explicit mathematical competencies to be achieved according to the syllabus were: identifying vectors in the plane and space; algebraic structure of the set of vectors, their properties and representation; identifying vector spaces, their properties and elements; identifying linear transformations between vector spaces, their properties and elements, and calculating eigenvalues and eigenvectors of linear transformations.

For the presentations, they could make use of technological resources. In addition to the oral presentations, each group had to carry out group workshops. At the end of the learning unit, they had to write a summative test on the topic to be applied to their peers, which was known and thoroughly reviewed by the teacher researcher. The knowledge that they would be assessed greatly increased the attention of each student to the presentations of the corresponding contents and to the continuous consultations with the lecturers. In general, the level of demand in these tests was high, which forced the rest of the students to study much more independently.

Throughout, students were actively supported to discuss, verbalize and write down their understanding of mathematical ideas and concepts. The various activities had motivated the students and provided them with opportunities to take charge of their learning. At first, students were uncomfortable with the activities, as they were different from their usual learning experiences, even though the stated curriculum is supposed to be competency-based. However, after some individual exposures as part of group work, they gradually adapted to the new environment, showing particular enthusiasm for working in groups, sharing ideas and working out the mathematics for themselves. Therefore, the environment created had facilitated thinking and communication skills among the students, which made the classes much more lively, where their peers during the presentations asked them for further clarification of a concept presented, to which some students responded with less or more clarification skills, as can be seen in Figure 1.

The evaluation of the students' performance included summative written tests, portfolio of evidence per student, oral presentations, co-evaluations of the groups formed. In addition, following the curricular requirements, at the end of the semester, the teacher-researcher conducted a written exam integrating the five units covered throughout the semester. The university also allows an integrative proficiency exam for those who fail the first exam, which was not necessary.

Figure 1. Moment of exposure and interaction with students

3.4. Evaluation instrument

3.4.1. Opinion questionnaire

In general, opinion questionnaire are a great resource for finding out what people think and allow you to gather data an opinions on a wide range of topics. In order to estimate the students' satisfaction with the experience of the didactic strategy and to gather their opinion on the process experienced in all its phases, an opinion questionnaire was designed, prepared, validated, and applied aimed at self-assessment on mathematical competencies and soft competencies.

3.5. Data analysis

3.5.1. Content validity

Content validity based on expert judgement was carried out. For this purpose, the collaboration of 8 professionals with 10 or more years of experience in teaching and research in mathematics, in the competency construct and in the evaluation of measurement instruments was requested. Validation was based on the evaluation of each item, using the Expert Judgement template proposed by Escobar-Pérez and Martínez [66], whose rating establishes four levels: 1=does not meet the criterion, 2=low level, 3=medium level, 4=high level, which constitutes the basis for the characteristics to be evaluated: coherence, relevance, adequacy and clarity.

The collected data were statistically processed in STATA version 14.0. Subsequently, the degree of agreement between experts was determined with the Fleiss kappa coefficient as an analysis statistic, used to assess agreement between evaluators who independently judge measurement criteria, in a number of categories of an ordinal nature [67]. The minimum value assumed by the coefficient is 0 and the maximum is 1 [68]. For the interpretation of this coefficient, the scale established by Landis and Koch [69] was taken into account, which qualitatively expresses the degree of agreement between the evaluators detailed in Table 1.

Items that met all of the established requirements were considered adequate, items that required some changes were considered adequate, and items that expressed total inconsistency with the established criteria were considered inadequate. Likewise, the characteristics of the instrument were also estimated for the categorical indicators of coherence, relevance, sufficiency and clarity, based on the ordinal measurement scale. The statistical performance of the Fleiss kappa coefficient and the p-value as the most informative value for each characteristic is shown in Table 2.

The degree of agreement found fluctuates between substantial and almost perfect, with the consistency characteristic being the highest (0.87) according to the degree of overall agreement between the judges. The statistical significance of these characteristics had a behavior of 96% security, implying a $p<0.05$, in which the relevance characteristic $p=0.026$ was relevant. With the results described, and considering the observations and recommendations issued qualitatively by the experts, whose proposed changes were modifications in the wording of some of the questions and the elimination of others, the content of the opinion questionnaire was validated and the final version was obtained with 40 questions measured on a Likert scale of four response instances: 4=strongly agree (SA), 3=agree (A), 2=disagree (D), and 1=strongly disagree (SD), and two open-ended questions on the strengths and weaknesses of the didactic intervention as presented in Table 3. Once the validation process was completed, a pilot application of the opinion questionnaire was carried out and its reliability was calculated using Cronbach's alpha, which was 0.75.

Table 1. Fleiss' kappa scores

Fleiss kappa coefficient	Degree of agreement
<0.0	Poor agreement
0.0-0.20	Slight agreement
0.21-0.40	Fair agreement
0.41-0.60	Moderate agreement
0.61-0.80	Substantial agreement
>0.80	Almost perfect agreement

Table 2. Fleiss' kappa coefficient and statistical significance of the characteristics

Characteristics	Fleiss' kappa coefficient	p
Coherence	0.87	0.028
Relevance	0.85	0.026
Adequacy	0.76	0.031
Clarity	0.83	0.025

Table 3. Final version of the opinion questionnaire

N°	Assertion	SA	A	D	SD
1	Receiving a portfolio for my personal use at the beginning of the Algebra II course pleased me				
2	Did you included in your portfolio the material you received at the beginning of the course and augmented it with your own exercises?				
3	You were pleased that at the end of each unit you were given a web address for feedback on your learning				
4	Did you use websites with educational channels as support videos to enhance your learning?				
5	You think these educational channels were useful for a better understanding of the course contents				
6	Voluntarily decided to solve exercises on the blackboard				
7	The feedback he received after each evaluation carried out by the course teacher was useful to understand his errors				
8	Becoming a teacher to his fellow Algebra II students motivated him				
9	The change of role from student to teacher of his classmates allowed him to demonstrate mathematical abilities				
10	Taking on the role of teacher reinforced his vocation				
11	The work assigned in groups made him doubt his ability to learn the subject linked to the contents of the subject				
12	You understood how to act in accordance with the ethical sense that education demands				
13	The preparation of the content assigned to him as part of his individual presentation was time-consuming and demanding				
14	Did you carry out recent research in different sources for the preparation of your topic?				
15	He considers that he was actively involved in his learning process				
16	He considers that he demonstrated ability in mastering mathematical content				
17	He considers that he achieved the didactic transposition of his topic to his classmates				
18	Generated connections between the academic requirements and the work competencies that will be demanded in the future, as a mathematics teacher?				
19	Worked collaboratively and with pleasure				
20	He considers that it developed behaviors favorable to social coexistence				
21	He considers that with this way of working his socio-affective relationships improved				
22	He considers that he demonstrated skill in the individual presentation of his topic				
23	He felt that exposing a mathematical subject unknown to his peers gave him greater responsibility				
24	The exposure allowed him to develop communication skills				
25	Questions from their peers were able to challenge their knowledge				
26	The didactic indications of the course teacher were useful to him				
27	Did you contribute to the development of your group's unit assessment?				
28	Did you contribute to the correction of your group's unit test?				
29	The experience as an evaluator satisfied him				
30	He considers that his self-esteem improved, as he felt he was able to meet his challenges				
31	He felt motivated in his role as a teacher				
32	He felt responsible for his own learning				
33	Acquired knowledge and enquiry skills to innovate in their future work as a teacher				
34	He considers that he managed to build his own learning				
35	This more student-centered methodology is preferred in mathematics				
36	He considers that this experience allowed him to combine his mathematical knowledge with attitudes, skills and values that he will require in his future work				
37	The use of software, in their opinion, made the contents of the units clearer to their peers				
38	The use of software, in your opinion, better clarified the contents to yourself				
39	This work experience increased their learning in linear algebra in particular and algebra in general				
40	Did this didactic experience increase your academic performance averages in algebra?				
41	Indicate the strengths of these teaching experiences				
42	Indicate the weaknesses of these teaching experiences				

Once the evaluation process had been completed, the students were asked to complete this opinion questionnaire, which gave an account of the process they had undergone and which was answered anonymously and voluntarily by all the students on the course. The 40 questions, graded from 1 to 4 points, were related to the guiding standards for secondary education teaching degrees: pedagogical and disciplinary standards [42], which include mathematical competencies such as disciplinary competency in linear algebra, use of technological resources, research capacity, evaluation capacity, adequate oral and written communication skills, and conduction of learning in linear algebra. In addition to soft skills such as teamwork/communication, collaboration skills, autonomy, work ethic, interest/vocation. The last two open-ended questions of the opinion questionnaire (41 and 42), allowed analysis by categories, based on the similarities and differences found in the opinions written by the students. Soft competencies were measured through items 1, 3, 5, 6, 7, 8, 10, 12, 19, 20, 21, 29, 30, 31, and 35, while the remaining items correspond to mathematical competencies.

4. RESULTS

4.1. Questionnaire on mathematical competences and soft competences

The descriptive statistics of the scores obtained by the students are presented in Table 4. According to the table, it can be seen that, on average, the students in their evaluations state that they "Agree" with regard to their competencies (complete instrument). The same is true when considering each competency separately. On the other hand, there is greater relative variability in the responses for the soft competencies (CV=11.28).

Figure 2 shows the greater dispersion in the ratings given to the soft competences compared to the full instrument and the mathematical competencies, with the full instrument and the mathematical competencies. It can be seen that in mathematical competencies one student (A14) rates this competency lower compared to the rest. Figure 3 shows the percentage distribution of the students' responses to the mathematical and soft skills questionnaire, based on the Likert scale used. According to the responses recorded in Figure 3, the 96% of the study subjects "strongly agreed" and "agreed" with the questions asked in relation to the didactic experience. While 4% disagreed.

Table 4. Descriptive statistics for each competency and the full instrument

Questionnaire	Mean	Standard deviation	Coefficient variation (%)
Mathematical competencies	4.22	0.40	9.27
Soft competencies	4.27	0.45	11.28
Complete instrument	4.22	0.41	9.29

4.2. Reliability analysis

Reliability was calculated using Cronbach's alpha coefficient (α) and McDonald's omega (W), being the most commonly used indices to calculate the reliability of the instruments. The overall reliability of the instrument was acceptable (α=0.90; W=0.91). On the other hand, for each competency the reliability was: mathematical competency (α=0.86; W=0.87); soft competency (α=0.78; W=0.79).

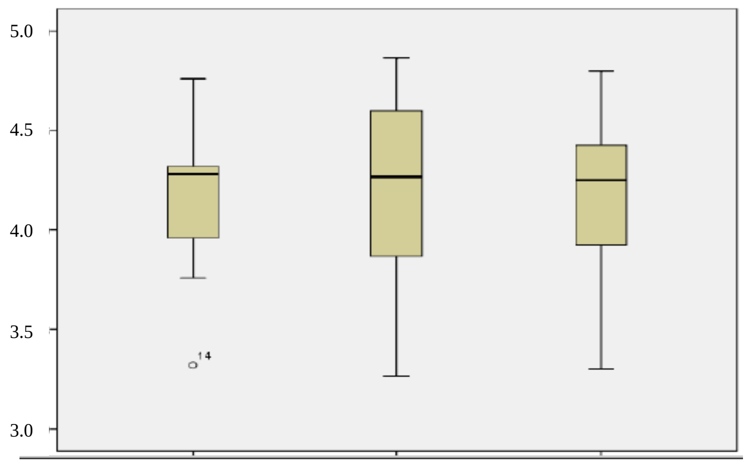


Figure 2. Box-plot graph of mathematical competency, soft and full instrument

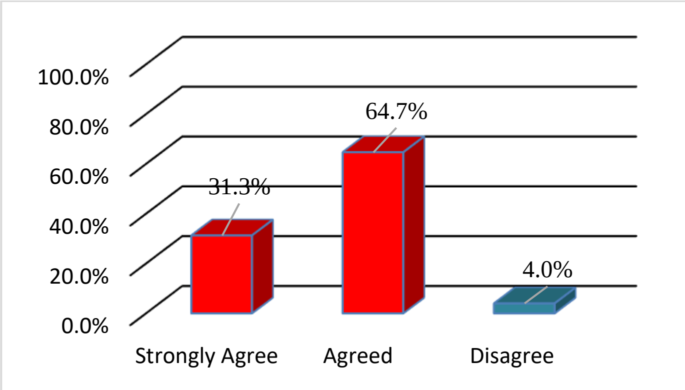


Figure 3. Distribution of students' responses in the questionnaire

4.3. Qualitative evaluation of the teaching strategy

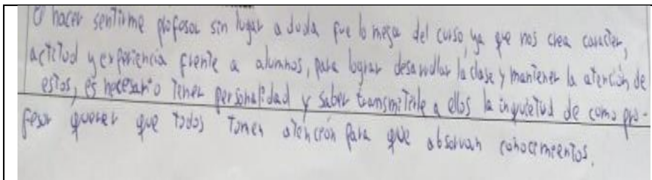
Qualitative assessment is a pedagogical activity that can provide a suitable opportunity for learners to practice evaluative judgement. The following is an analysis by categories, based on the similarities and differences found in the opinions written by the pupils in open questions 41 and 42 of the mathematical and soft competences questionnaire, regarding the strengths and weaknesses of the teaching strategy implemented and experienced by them during a school semester of which evidence is presented based on the student's responses.

4.3.1. Strengths: similarities

- **Assume the teaching role**

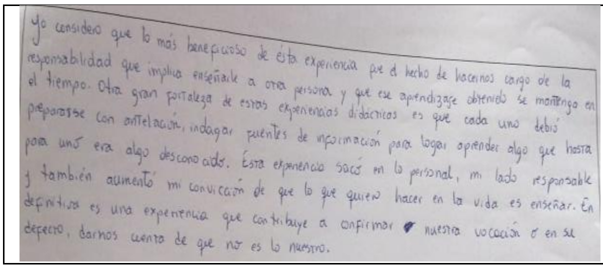
Students stressed the importance of the concept of the teaching role. Assuming this responsibility early in their training, as a first approach to their future profession, since thanks to this experience, students lived what it is to be a teacher, highlighting the experience of designing a class, and its planning. They were able to verify if their ideas about the realization of a class were effective or not, also had a small approach to their future responsibilities, and again experience what a teacher does every day in the exercise of the profession.

In addition, they were responsible for the learning of their classmates, and all that this represents. They also had to be responsible in the first instance, to deliver the knowledge in the best possible way for their classmates and then apply an evaluative instrument to verify the knowledge delivered in that class, and this activity was very important for the students, because they could experience it for themselves and with their classmates. Two responses from the students are presented in Figures 4 and 5.


El hacer sentirme profesor sin lugar a duda fue lo mejor del curso, ya que nos crea carácter, actitud y experiencia frente a alumnos, para lograr desarrollar la clase y mantener la atención de estos, es necesario tener personalidad y saber transmitirle a ellos la inquietud de como profesor querer que todos tomen atención para que absorban conocimientos.

"*Making me feel like a teacher was undoubtedly the best part of the course, since it gave us character, attitude and experience in front of students. To be able to develop the class and maintain their attention, it is necessary to have personality and know how to transmit to them the concern of how to teach, wanting everyone to take attention so that they can absorb knowledge*."

Figure 4. A student's response to the similarities of the teaching experience


Yo considero que lo más beneficioso de esta experiencia fue el hecho de hacernos cargo de la responsabilidad que implica enseñarle a otra persona y que ese aprendizaje obtenido se mantenga en el tiempo. Otra gran fortaleza de estas experiencias didácticas es que cada uno debió prepararse con antelación, indagar fuentes de información para lograr aprender algo que hasta para uno era algo desconocido. Esta experiencia sacó en lo personal, mi lado responsable y también aumentó mi convicción de que lo que quiero hacer en la vida es enseñar. En definitiva es una experiencia que contribuye a confirmar nuestra vocación o en su defecto, darnos cuenta de que no es lo nuestro.

"*I think that the most beneficial thing about this experience was that we took on the responsibility of teaching someone else and that this learning was maintained over time. Another great strength of these teaching experiences is that everyone had to prepare themselves in advance, researching sources of information in order to learn something that was unknown to them. This experience brought out my personal responsible side and increased my conviction that what I want to do in life is to teach. In short, it is an experience that helps to affirm our vocation or, otherwise, to realize that it is not for us*."

Figure 5. A student's response to the similarities of the teaching experience

– **Research capacity**

As similarity arises the capacity of investigation, the students at the moment of assuming the educational role did not have the knowledge of the content to work with their companions. That is why they had to investigate in depth the concept to work, under that premise the students emphasized like great strength the development of their own capacity of investigation, and all the abilities that entails to make a true investigation. Figure 6 shows one student's response.

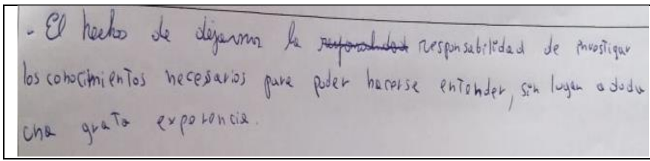
 - El hecho de dejarnos la ~~responsabilidad~~ responsabilidad de investigar los conocimientos necesarios para poder hacerse entender, sin lugar a duda una grata experiencia.	"*The fact of leaving us the responsibility of investigating the necessary knowledge to be able to be understood, without a doubt a pleasant experience*."

Figure 6. A student's response

– **Teamwork**

As a result of the analysis of the students' answers to the question about the strengths of the experience, teamwork emerges as a similarity. The students were divided into teams in their role as student-teachers (students in training), with the possibility of taking classes on a given mathematical content. As a result of this work and its demand, the students were able to value teamwork within the teaching profession, noting the importance of this in order to be a good teacher. Figures 7 and 8 show the responses of two students.

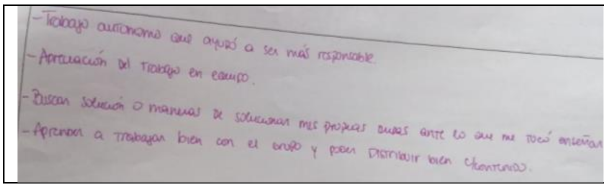
 - Trabajo autónomo que ayudó a ser más responsable - Apreciación del trabajo en equipo. - Buscar solución o maneras de solucionar mis propias dudas ante lo que me tocó enseñar - Aprender a trabajar bien con el grupo y poder distribuir bien contenidos.	– "*Self-employment that helped make me more responsible*." – "*Appreciation of field work. Searching for solutions to my own doubts among what I had to teach*." – "*Learn to work well with the group and be able to distribute the content well*."

Figure 7. A student's response to the similarities of the teaching experience

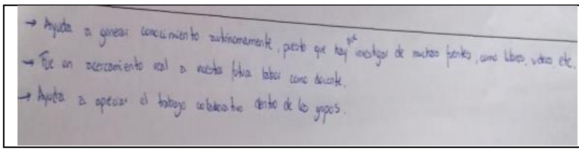
 → Ayuda a generar conocimiento autónomamente, puesto que hay que investigar de muchas fuentes, como libro, video, etc. → Fue un acercamiento real a nuestra futura labor como docente. → Ayuda a apreciar el trabajo colaborativo dentro de los grupos.	"*It helps to generate knowledge autonomously, since it is necessary to investigate from.* *-Many parts, and like book, video.* *-It was a real approach to our future work as teachers.* *-It helps to appreciate collaborative work within groups*."

Figure 8. A student's response to the similarities of the teaching experience

4.3.2. Strengths: differences

– **Reaffirm the teaching vocation**

Vocation is a concept that in its genesis carries the word "courage", the courage to give value to what one chooses. Through this experience, some students highlighted as a strength the personal reaffirmation that this was the right career to pursue in the future, which in their opinion, they achieved through first-hand experience of the teaching work they faced as seen in Figure 9.

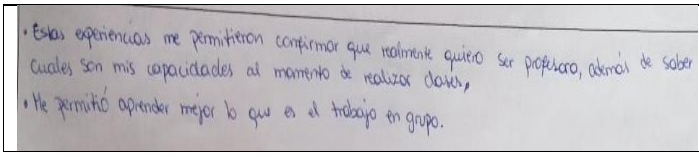
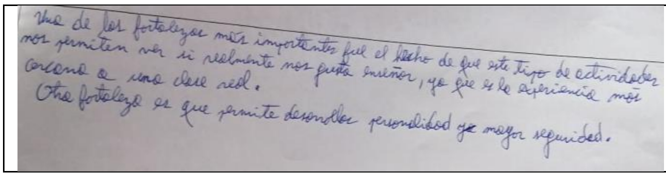
 • Estas experiencias me permitieron confirmar que realmente quiero ser profesora, además de saber cuales son mis capacidades al momento de realizar clases. • Me permitió aprender mejor lo que es el trabajo en grupo.	"*These experiences allowed me to confirm that I really want to be a teacher, in addition to knowing what my abilities are at the time of doing classes. It allowed me to learn better what group work is*."

Figure 9. A student's response to differences in teaching experience

– **Develop personality and security in front of the course group**

An important aspect for a teacher is to have the personality to be in front of a course. However, within this aspect also highlights the security of what is being done as a teacher. One student stressed that the experience helped him to start working on these essential skills for a teacher as shown in Figure 10.

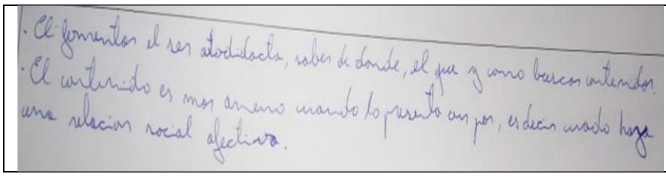


"*One of the most important strengths was the fact that this type of activity allows us to see if we really like teaching, as it is the closest experience to a real class. Another strength is that it allows us to develop personality and greater security*."

Figure 10. A student's response to differences in teaching experience

– **Peer learning**

Whit regard to the forms of organization of the activity by the students, peer collaboration was identified in the responses occurs when two or more students of similar levels work collaboratively in a constant way towards the resolution problem. For some students, the experience was outstanding in terms of peer learning as seen in Figure 11.



"*Encouraging self-education, knowing where from, what and how to search for content. The content is more enjoyable when presented by a couple, i.e. when there is a social affective relationship*."

Figure 11. A student's response to the differences in the teaching experience

4.3.3. Weaknesses: similarities

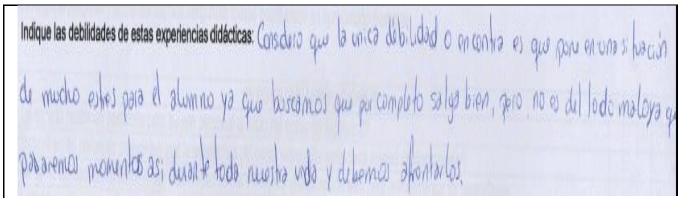
– **Difficulty in mathematical content**

The weakness according to the open answers of the students, has relation with the worked contents, and the difficulty associated to it, which was increased by the scarce or null experience that the students in training have, on the accomplishment of classes. The contents worked on were not completely clear to some, and they also made mistakes when working on the classes, which, although they were corrected by the teacher of the subject. Some students did not manage to master the mathematical content worked on in its entirety.

4.3.4. Weaknesses: differences

– **Emotional level**

Stress is a condition that human beings have suffered from for centuries and that refers to an automatic response of the organism to any event that is imposed on it, causing the nervous system to stimulate and react by producing changes at al psychological (mental) or psychological (physical) level, and that occurs in a particular way between the person and the situation. For some students, having to deal with a different modality and assuming the role of teacher, caused them stress because of the responsibilities of learning of their peers. Another was frustrated by his own expectations as shown in Figure 12.



"*Indicate the weaknesses of these teaching experiences: I consider that the only weakness or against is that it puts in a situation of much stress for the student since we look for that it completely goes well, but it is not completely bad since we go through moments like this during all our life and we must face them*."

Figure 12. A student's response in relation to the emotional level

– **Inequity of teamwork and lack of depth in the units treated**

In the analysis of the weaknesses and specifically in the different perceptions given by the mathematics pedagogy students, another answer is about teamwork, indicating that not all members of the group actively participated in the task assigned by the work team. This student also refers to the depth he would have liked in the treatment of the units of study as seen in Figure 13.

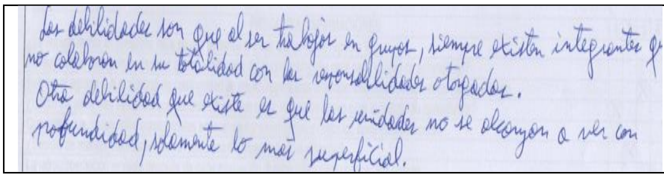
	"*The weaknesses are that when working in groups, there are always members who do not fully cooperate with the responsibilities given. Another weakness is that the units do not see themselves in depth, only in the most superficial aspects*."

Figure 13. A student's response in relation teamwork and the treatment of the study units

– **Time**

The development of the entire didactic experience was adjusted to a defined time of the lineal algebra course. For some students, the class time was too short to reach the depth of the assigned topics as presented in Figure 14.

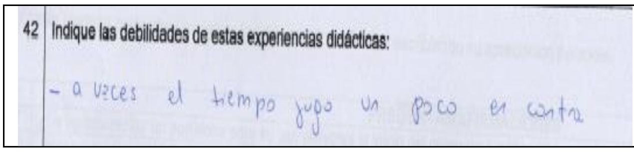
	"*Indicate the weaknesses of these didactics experiences:* *Sometimes time played a little against*."

Figure 14. A student's response in relation the short class time

5. DISCUSSION

In this research, the importance of teaching linear algebra with the development of soft skills, considered by the literature as high-level and fundamental abilities in the appropriation of knowledge in mathematics, was confirmed [70]. From the curricular level, the processes that involve soft skills must be planned in the curriculum or class plan so that when they are executed they fulfil the indicated function in the development of students' intellectual skills, facilitating the learning of mathematics [71], [72]. The importance of this study lies in the fact that it constitutes a fundamental advance in the knowledge of the educational phenomenon through the application of collaborative learning in groups and its relevance in the teaching and learning process, as stated by several researchers [73]–[75] and requires much more from the teacher than simply placing students in groups and instructing them to work together [61]. For Duran and Monereo [76], collaborative learning in university education is presented as a methodological alternative to the individualistic models that are not very creative and reflective, typical of traditional methodologies, while according to study by Boud *et al*. [77], it corresponds to a peer learning model. After the study carried out, we can affirm that the students expressed greater agreement with the soft skills achieved and of all those who responded to the opinion questionnaire, the teaching strategy used favors collaborative work, coinciding with the results of previous studies [78], [79].

The results indicate that students presented more strengths than weaknesses. Strengths that for most of them are related to assuming the teaching role as a first approach to their future profession, the development of their own research capacity and all the skills involved in carrying out real research, valuing teamwork within the teaching profession, pointing out its importance in order to be a good future mathematics teacher. This coincides with several studies [80]–[83] who also affirm that students who use collaborative methodologies in their learning process show more strengths than those who do not use them, including better retention of knowledge, better communication of ideas, increased respect and tolerance for the opinions of their peers, improved analytical skills, development of decision-making skills and increased interest in learning.

In turn, and according to the results, the weaknesses of the students show the difficulties with the contents worked on in linear algebra, as expressed by previous studies [32], [52], [84]–[86], it was verified that learning linear algebra is difficult for most students at university. As it is a cognitively demanding discipline, which requires the student to be able to move between different languages (matrix theory and vector space theory), Cartesian and parametric points of view. This is also explained by the structural complexity of the subject and the students' prior knowledge.

With regard to the evaluation of the didactic strategy, we can generally affirm that the students perceive this learning model positively. We believe that when students are offered a work model, where they interact with each other, teaching and learning in a reciprocal way, they try to demonstrate their learning by showing greater motivation and commitment. This should be the learning model to follow in university classrooms, where students learn by teaching and sharing knowledge with each other. In the specific case of university teaching, research has been reporting a progressive shift towards a learner-centered style, with an increase in teachers' dissatisfaction with the lecture as the sole mode of teaching [87], [88].

6. CONCLUSION

A didactic strategy with active learning was used to integrate the disciplinary learning of linear algebra, but also, fundamentally, to prioritize the soft skills in which students could be active participants in the construction and development of their mathematical knowledge through the use of collaborative learning, which required a division of tasks between the components of the groups, each one taking responsibility for the solution of a part of the thematic units of the linear algebra course. They worked together to distributively realize a goal as part of the teaching and learning process. The qualitative evaluation method used made it possible to learn from the direct opinion of the students involved, both the strengths and weaknesses of the process experienced, validating the usefulness of this successful teaching methodology. After having analyzed the results of the research, previous research and the contribution of the theoretical framework, it can be concluded that, at present, the process of teaching and learning by competencies is not an option in mathematics education, but a curricular requirement, which implies incorporating didactic strategies that guarantee not only the prioritization of algorithmic disciplines, but also the installation of both mathematical and transversal skills.

REFERENCES

[1] L. Q. Ramírez, G. Zárate-Ruiz, E. Miranda-Aburto, and P. S. Celi, "Competency-based approach (CB) and formative evaluation (FE). Case: rural school," (in Spanish), *Propósitos y Representaciones*, vol. 9, no. 1, 2021, doi: 10.20511/pyr2021.v9n1.1036.

[2] P. C. Marchena and M. L. Recinos, "Pedagogical strategies based on the competency-based approach: an experience in the Diplomacy course," (in Spanish), *Revista Ensayos Pedagógicos*, vol. 15, no. 1, pp. 149–168, May 2020, doi: 10.15359/rep.15-1.8.

[3] P. Christian, "Theoretical review of the competency-based approach and its application in the Bolivian University," (in Spanish), *Revista Ciencia, Tecnología e Innovación*, vol. 16, no. 18, pp. 57–74, 2018.

[4] P. Beneitone, J. González, and R. Wagenaar, *Meta-profiles and profiles: an approach for degrees in Latin America*. Bilbao, España: Universidad de Deusto (in Spanish), 2014.

[5] M. V. Moruno, C*urriculum planning: professional competencies approach*. Madrid, España: Editorial Académica Española (in Spanish), 2013.

[6] S. Bredekamp, *Effective practices in early childhood education: Building a foundation*, 2nd. Pearson, 2014.

[7] World Economic Forum, *Skills of the future: assessing the demand for high-value skills in the global labour market*. Geneva: WEFORUM, 2022.

[8] T. Chaudhari, "Soft skills: types, characteristics and importance," *Journal of Commerce & Trade*, vol. 16, no. 2, pp. 9–12, Oct. 2021, doi: 10.26703/JCT.v16i2-2.

[9] T. Tuononen, H. Hyytinen, K. Kleemola, T. Hailikari, I. Männikkö, and A. Toom, "Systematic review of learning generic skills in higher education-enhancing and impeding factors," *Frontiers in Education*, vol. 7, p. 885917, May 2022, doi: 10.3389/feduc.2022.885917.

[10] CEPAL, *Education, youth and work: Skills and competencies needed in a changing context*. Santiago, Chile: Publicación de las Naciones Unidas (in Spanish), 2020.

[11] J. Adriaensen, P. Bijsmans, and A. Groen, "Monitoring generic skills development in a bachelor European studies," *Journal of Contemporary European Research*, vol. 15, no. 1, pp. 110–127, Feb. 2019, doi: 10.30950/jcer.v15i1.1018.

[12] OEI, *Perspectives on education in Ibero-America 2020. Competencies for the XXI century in Ibero-America*. Madrid: Informe Miradas (in Spanish), 2020.

[13] J. Weller, S. Gontero, and S. Campbell, *Technological change and employment: a Latin American perspective. Macroeconomics of development series, No. 201 (LC/TS.2019/37)*. Santiago: Publicación de las Naciones Unidas (in Spanish), 2019.

[14] F. V. Millalén, "Infusion of soft skills into the higher education curriculum: key to the development of advanced human capital," (in Spanish), *Revista Akademia*, vol. 7, no. 1, pp. 53–73, 2016.

[15] H. Meijer, R. Hoekstra, J. Brouwer, and J.-W. Strijbos, "Unfolding collaborative learning assessment literacy: a reflection on current assessment methods in higher education," *Assessment & Evaluation in Higher Education*, vol. 45, no. 8, pp. 1222–1240, Nov. 2020, doi: 10.1080/02602938.2020.1729696.

[16] X. Yang, "Creating learning personas for collaborative learning in higher education: a Q methodology approach," *International Journal of Educational Research Open*, vol. 4, p. 100250, 2023, doi: 10.1016/j.ijedro.2023.100250.

[17] M. A. Flores, A. M. V. Simão, A. Barros, and D. Pereira, "Perceptions of effectiveness, fairness and feedback of assessment methods: a study in higher education," *Studies in Higher Education*, vol. 40, no. 9, pp. 1523–1534, Oct. 2015, doi: 10.1080/03075079.2014.881348.

[18] R. Awang-Hashim, N. Yusof, A. Benlahcene, A. Kaur, and S. K. S. Shanmugam, "Collaborative learning in tertiary education classrooms: what does it entail?" *Malaysian Journal of Learning and Instruction*, vol. 20, no. 2, pp. 205–232, 2023, doi: 10.32890/mjli2023.20.2.1.

[19] A. Yaacob, R. M. Asraf, R. M. R. Hussain, and S. N. Ismail, “Empowering learners’ reflective thinking through collaborative reflective learning,” *International Journal of Instruction*, vol. 14, no. 1, pp. 709–726, Jan. 2021, doi: 10.29333/iji.2021.14143a.

[20] H. Le, J. Janssen, and T. Wubbels, “Collaborative learning practices: teacher and student perceived obstacles to effective student collaboration,” *Cambridge Journal of Education*, vol. 48, no. 1, pp. 103–122, Jan. 2018, doi: 10.1080/0305764X.2016.1259389.

[21] K. A. A.-A.l Ibrahim, N. C. Carbajal, M. E. C. Zuta, and S. Bayat, “Collaborative learning, scaffolding-based instruction, and self-assessment: impacts on intermediate EFL learners’ reading comprehension, motivation, and anxiety,” *Language Testing in Asia*, vol. 13, no. 1, p. 16, Mar. 2023, doi: 10.1186/s40468-023-00229-1.

[22] G. S. de Borba, I. M. Alves, and P. D. B. Campagnolo, “How learning spaces can collaborate with student engagement and enhance student-faculty interaction in higher education,” *Innovative Higher Education*, vol. 45, no. 1, pp. 51–63, Feb. 2020, doi: 10.1007/s10755-019-09483-9.

[23] M. A. Qureshi, A. Khaskheli, J. A. Qureshi, S. A. Raza, and S. Q. Yousufi, “Factors affecting students’ learning performance through collaborative learning and engagement,” *Interactive Learning Environments*, vol. 31, no. 4, pp. 2371–2391, May 2023, doi: 10.1080/10494820.2021.1884886.

[24] N.-F. Liu and D. Carless, “Peer feedback: the learning element of peer assessment,” *Teaching in Higher Education*, vol. 11, no. 3, pp. 279–290, Jul. 2006, doi: 10.1080/13562510600680582.

[25] L. Song and J. R. Hill, “Exploring the role of the instructor in the implementation of collaborative learning in higher education,” *Journal of Educational Technology Development and Exchange*, vol. 10, no. 1, pp. 1–14, 2017.

[26] S. Kerckaert, R. Vanderlinde, and J. van Braak, “The role of ICT in early childhood education: scale development and research on ICT use and influencing factors,” *European Early Childhood Education Research Journal*, vol. 23, no. 2, pp. 183–199, Mar. 2015, doi: 10.1080/1350293X.2015.1016804.

[27] J. Hlouskova, “What makes autonomous learning objectives autonomous: a case study from higher education,” in *International Conference on Education and New Developments*, Jun. 2019, pp. 23–26. doi: 10.36315/2019v2end005.

[28] M. S. Abdurrahman, “Effect of peer tutoring strategy on students academic performance in a polytechnic linear algebra classroom,” *Journal of Advanced Research in Dynamical and Control Systems*, vol. 12, no. 3, pp. 415–422, Mar. 2020, doi: 10.5373/JARDCS/V12I3/20201209.

[29] G. Strang, *Introduction to linear algebra*, 5th. Wellesley, MA, USA: Cambridge Press, 2016.

[30] R. J. Rensaa, N. M. Hogstad, and J. Monaghan, “Perspectives and reflections on teaching linear algebra,” *Teaching Mathematics and its Applications: An International Journal of the IMA*, vol. 39, no. 4, pp. 296–309, Nov. 2020, doi: 10.1093/teamat/hraa002.

[31] A. S. Stuhlmann, “Mathematics students talking past each other: emergence of ambiguities in linear algebra proof constructions involving the uniqueness quantification,” *ZDM*, vol. 51, no. 7, pp. 1083–1095, Dec. 2019, doi: 10.1007/s11858-019-01099-9.

[32] J.-L. Dorier and A. Sierpinska, “Research into the teaching and learning of linear algebra,” in *The Teaching and Learning of Mathematics at University Level*, D. Holton, M. Artigue, U. Kirchgräber, J. Hillel, M. Niss, and A. Schoenfeld, Eds., Dordrecht: Kluwer Academic Publishers, 2001, pp. 255–273, doi: 10.1007/0-306-47231-7_24.

[33] I. M. Arnawa, Yerizon, and S. Nita, “Errors and misconceptions in learning elementary linear algebra,” *Journal of Physics: Conference Series*, vol. 1321, no. 2, p. 022095, Oct. 2019, doi: 10.1088/1742-6596/1321/2/022095.

[34] Ferryansyah, E. Widyawati, and S. W. Rahayu, “The analysis of students’ difficulty in learning linear algebra,” *Journal of Physics: Conference Series*, vol. 1028, p. 012152, Jun. 2018, doi: 10.1088/1742-6596/1028/1/012152.

[35] G. V. Ríos, G. Contreras, and J. C. R. Pabon, “Didactic strategies for the study of linear algebra at the university level,” (in Spanish), *Opción: Revista de Ciencias Humanas y Sociales*, vol. 34, no. 87, pp. 557–583, 2018.

[36] F. Montenegro, A. Gagliardo, S. Mangini, and A. Carrasco, “Learning vector spaces in linear algebra: a look from APOE/ESPAÇOS theory Vectorial spaces of learning in linear algebra: a look from the theoretical supports,” (in Spanish), *Brazilian Journal of Development*, vol. 6, no. 11, pp. 84339–84351, 2020, doi: 10.34117/bjdv6n11-009.

[37] V. G. O. Vidal, J. T. P. Alca, M. F. H. Prada, and I. G. López, “Teaching linear algebra to undergraduates,” (in Spanish), *Horizontes. Revista de Investigación en Ciencias de la Educación*, vol. 7, no. 27, pp. 380–387, Jan. 2023, doi: 10.33996/revistahorizontes.v7i27.522.

[38] M. P. Ruz, “Dimensions of teacher collaborative learning in professional learning communities in Chile,” (in Spanish), *Cuadernos de Investigación Educativa*, vol. 14, no. 1, pp. 1–20, Dec. 2022, doi: 10.18861/cied.2023.14.1.3280.

[39] S. Eyzaguirre, C. Le Foulon, and V. Salvatierra, “Education in times of pandemic: background and recommendations for discussion in Chile,” (in Spanish), *Estudios Públicos*, pp. 1–70, Aug. 2020, doi: 10.38178/07183089/1430200722.

[40] M. E. Auer, U. R. Cukierman, E. V. Vidal, and E. T. Caro, *Towards a hybrid, flexible and socially engaged higher education*. Cham: Springer Nature Switzerland, 2024, doi: 10.1007/978-3-031-51979-6.

[41] C. C. León, C. B. Coello, K. G. Goyes, J. C. Barzola, and L. C. León, “The teaching-learning process of linear algebra. Systematization and implication in the careers of the Faculty of Engineering Sciences of the State Technical University of Quevedo,” (in Spanish), *Ciencia Sociales y Económicas*, vol. 3, no. 2, pp. 134–155, Dec. 2019, doi: 10.18779/csye.v3i2.300.

[42] Ministerio de Educación de Chile (MINEDUC), *Guideline standards for careers in media education and media education*. Santiago, Chile: LOM Ediciones Ltda (in Spanish), 2012.

[43] D. G. Díaz, M. B. Moreno, E. M. Herrera, B. R. Garzón, and M. R. Torres, “Constructivism and connectivism as teaching and learning methods in university education today,” (in Spanish), *Ciencia Latina Revista Científica Multidisciplinar*, vol. 6, no. 6, pp. 14046–14058, Feb. 2023, doi: 10.37811/cl_rcm.v7i1.4672.

[44] J. M. M. Al-Jarrah, R. Talafhah, N. Mansor, and T. M. Al-Jarrah, “The application of metacognition, cognitivism, and constructivism in teaching writing skills,” *European Journal of Foreign Language Teaching*, vol. 3, no. 4, pp. 199–213, 2018, doi: 10.46827/ejfl.v0i0.2189.

[45] M. Lalaude-Labayle, “Teaching linear algebra at university level - Didactic and epistemological analysis,” (in French), Ph.D. dissertation, Université de Pau et des Pays de l’Adour, Pau, France, 2016.

[46] J. Jiménez, “Development of a didactic application in linear algebra: matrices, systems and determinants,” (in Spanish), M.S. thesis, Universidad Politécnica de Madrid, Madrid, España, 2023.

[47] J. Dorier, “State of the art in didactic research on the teaching of linear algebra,” (in French), *Recherches en Didactique des Mathématiques*, vol. 18, no. 2, pp. 191–230, 1998.

[48] M. Wawro, “Reasoning about solutions in linear algebra: the case of Abraham and the invertible matrix theorem,” *International Journal of Research in Undergraduate Mathematics Education*, vol. 1, no. 3, pp. 315–338, Oct. 2015, doi: 10.1007/s40753-015-0017-7.

[49] A. D. C. Bahamonde, J. M. F. Aymemí, and J. V. G. i Urgellés, “Mathematical modelling and the learning trajectory: tools to support the teaching of linear algebra,” *International Journal of Mathematical Education in Science and Technology*, vol. 48, no. 3, pp. 338–352, Apr. 2017, doi: 10.1080/0020739X.2016.1241436.

[50] N. Grenier-Boley, "Some issues about the introduction of first concepts in linear algebra during tutorial sessions at the beginning of university," *Educational Studies in Mathematics*, vol. 87, no. 3, pp. 439–461, Nov. 2014, doi: 10.1007/s10649-014-9564-0.

[51] L. H. N. Mutambara and S. Bansilal, "Dealing with the abstraction of vector space concepts," in *Challenges and Strategies in Teaching Linear Algebra*, Cham: Springer, 2018, pp. 147–173, doi: 10.1007/978-3-319-66811-6_7.

[52] A. D. Cárcamo, B. Barquero, E. R. Badillo, and C. E. Fuentealba, "Errors about vector subspaces: an exploratory study with first-year engineering students," (in Spanish), *Formación universitaria*, vol. 16, no. 2, pp. 13–24, Apr. 2023, doi: 10.4067/S0718-50062023000200013.

[53] Organisation for Economic Co-operation and Development (OECD), *Collaborative problem solving. PISA in focus*. OECD Publishing, 2017.

[54] C. S. C. Asterhan and B. B. Schwarz, "Argumentation for learning: well-trodden paths and unexplored territories," *Educational Psychologist*, vol. 51, no. 2, pp. 164–187, Apr. 2016, doi: 10.1080/00461520.2016.1155458.

[55] R. M. Gillies and A. Khan, "The effects of teacher discourse on students' discourse, problem-solving and reasoning during cooperative learning," *International Journal of Educational Research*, vol. 47, no. 6, pp. 323–340, Jan. 2008, doi: 10.1016/j.ijer.2008.06.001.

[56] E. Er, Y. Dimitriadis, and D. Gašević, "A collaborative learning approach to dialogic peer feedback: a theoretical framework," *Assessment & Evaluation in Higher Education*, vol. 46, no. 4, pp. 586–600, May 2021, doi: 10.1080/02602938.2020.1786497.

[57] D. W. Johnson and R. T. Johnson, "An educational psychology success story: social interdependence theory and cooperative learning," *Educational Researcher*, vol. 38, no. 5, pp. 365–379, Jun. 2009, doi: 10.3102/0013189X09339057.

[58] C. Chen, K. T. Jones, and S. Xu, "The association between students' style of learning preferences, social presence, collaborative learning and learning outcomes," *Journal of Educators Online*, vol. 15, no. 1, pp. 1–16, Jan. 2018, doi: 10.9743/JEO2018.15.1.3.

[59] E. Kyndt, E. Raes, B. Lismont, F. Timmers, E. Cascallar, and F. Dochy, "A meta-analysis of the effects of face-to-face cooperative learning. Do recent studies falsify or verify earlier findings?" *Educational Research Review*, vol. 10, pp. 133–149, Dec. 2013, doi: 10.1016/j.edurev.2013.02.002.

[60] C. J. Roseth, D. W. Johnson, and R. T. Johnson, "Promoting early adolescents' achievement and peer relationships: the effects of cooperative, competitive, and individualistic goal structures," *Psychological Bulletin*, vol. 134, no. 2, pp. 223–246, 2008, doi: 10.1037/0033-2909.134.2.223.

[61] C. Kaendler, M. Wiedmann, N. Rummel, and H. Spada, "Teacher competencies for the implementation of collaborative learning in the classroom: a framework and research review," *Educational Psychology Review*, vol. 27, no. 3, pp. 505–536, Sep. 2015, doi: 10.1007/s10648-014-9288-9.

[62] J. van de Pol, M. Volman, and J. Beishuizen, "Scaffolding in teacher–student interaction: a decade of research," *Educational Psychology Review*, vol. 22, no. 3, pp. 271–296, Sep. 2010, doi: 10.1007/s10648-010-9127-6.

[63] J. van de Pol, M. Volman, F. Oort, and J. Beishuizen, "The effects of scaffolding in the classroom: support contingency and student independent working time in relation to student achievement, task effort and appreciation of support," *Instructional Science*, vol. 43, no. 5, pp. 615–641, Sep. 2015, doi: 10.1007/s11251-015-9351-z.

[64] A. van Leeuwen, J. Janssen, G. Erkens, and M. Brekelmans, "Teacher interventions in a synchronous, co-located CSCL setting: analyzing focus, means, and temporality," *Computers in Human Behavior*, vol. 29, no. 4, pp. 1377–1386, Jul. 2013, doi: 10.1016/j.chb.2013.01.028.

[65] M. S. A. de Hei, E. Sjoer, W. Admiraal, and J.-W. Strijbos, "Teacher educators' design and implementation of group learning activities," *Educational Studies*, vol. 42, no. 4, pp. 394–409, Aug. 2016, doi: 10.1080/03055698.2016.1206461.

[66] J. Escobar-Pérez and A. C. Martínez, "Content validity and expert judgment: an approach to their use," (in Spanish), *Avances en Medición*, vol. 6, no. 1, pp. 27–36, 2008.

[67] M. L. McHugh, "Interrater reliability: the kappa statistic," *Biochemia Medica*, vol. 22, no. 3, pp. 276–282, 2012, doi: 10.11613/BM.2012.031.

[68] M. M. Pérez, N. P. Riba, J. C. O. Sagristà, A. Martinez, and V. G. García, "Human anatomy: a study of first-year medical students' reactions to the dissection room," (in Spanish), *Educación Médica*, vol. 10, no. 2, pp. 105–113, Jun. 2007, doi: 10.4321/S1575-18132007000300007.

[69] J. R. Landis and G. G. Koch, "The measurement of observer agreement for categorical data," *Biometrics*, vol. 33, no. 1, pp. 159–174, Mar. 1977, doi: 10.2307/2529310.

[70] M. A. L. Loaiza and J. R. L. Loaiza, "Learn linear algebra with innovative methodologies and interactive tools applied to problems of everyday life," (in Spanish), *LATAM Revista Latinoamericana de Ciencias Sociales y Humanidades*, vol. 4, no. 2, pp. 2555–2562, Jul. 2023, doi: 10.56712/latam.v4i2.777.

[71] W. L. V. Álava, A. D. V. Cedeño, W. O. R. Posligua, T. Y. F. Polanco, and J. R. L. Vera, "Collaborative learning in mathematics teaching," (in Spanish), *Revista Científica Arbitrada Multidisciplinaria PENTACIENCIAS*, vol. 5, no. 6, pp. 103–123, Oct. 2023, doi: 10.59169/pentaciencias.v5i6.842.

[72] C. Bolman and T. Deal, *The Fundamentals of leadership from education*. Atlante (in Spanish), 2016.

[73] J. López, *Motivational techniques for basic education: efficient tools for learning and teaching in the classroom and at home*. Portoviejo: Amazon KDP (in Spanish), 2020.

[74] A. E. C. Tello, H. C. G. Ramírez, P. C. H. Sosa, M. Á. F. Sánchez, and E. C. Paucar, "The use of problem-based learning (PBL) in higher education," (in Spanish), *Revista EDUCA UMCH*, vol. 27, no. 3, pp. 29–44, 2023.

[75] N. D. Roselli, "Collaborative learning: theoretical bases and strategies applicable to university teaching," (in Spanish), *Propósitos y Representaciones*, vol. 4, no. 1, pp. 219–280, Apr. 2016, doi: 10.20511/pyr2016.v4n1.90.

[76] D. Duran and C. Monereo, *Lattices: cooperative and collaborative learning methods*, 1st ed. Barcelona: Édebe (in Spanish), 2012.

[77] D. Boud, R. Cohen, and J. Sampson, Eds., *Peer learning in higher education: learning from and with each other*. Routledge, 2014, doi: 10.4324/9781315042565.

[78] K. C. B. Teixeira, "Pedagogical strategies to enhance learning in a linear algebra course," *PRIMUS*, vol. 33, no. 2, pp. 152–174, Feb. 2023, doi: 10.1080/10511970.2022.2048928.

[79] K. Durkin, B. Rittle-Johnson, J. R. Star, and A. Loehr, "Comparing and discussing multiple strategies: an approach to improving algebra instruction," *The Journal of Experimental Education*, vol. 91, no. 1, pp. 1–19, Jan. 2023, doi: 10.1080/00220973.2021.1903377.

[80] R. P. Bagozzi, "Structural equation models are modelling tools with many ambiguities: comments acknowledging the need for caution and humility in their use," *Journal of Consumer Psychology*, vol. 20, no. 2, pp. 208–214, Apr. 2010, doi: 10.1016/j.jcps.2010.03.001.

[81] M. Estrada, D. Monferrer, and M. A. Moliner, "Cooperative learning and socio-emotional skills: a teaching experience in sales techniques course," (in Spanish), *Formación universitaria*, vol. 9, no. 6, pp. 43–62, 2016, doi: 10.4067/S0718-50062016000600005.

[82] I. S. Godoy, "The development of autonomy through cooperative learning techniques in the L2 classroom," (in Spanish), *Porta Linguarum Revista Interuniversitaria de Didáctica de las Lenguas Extranjeras*, vol. 25, pp. 179–190, Jan. 2016, doi: 10.30827/Digibug.53915.

[83] M. G. Santana, J. R. Pulido, and J. A. Rodríguez, "Collaborative learning: innovative experience with university students," (in Spanish), *Revista de Estudios y Experiencias en Educación*, vol. 18, no. 36, p. 269, 2019, doi: 10.21703/rexe.20191836guerra5.

[84] J.-L. Dorier, "Duality between formalism and meaning in the learning of linear algebra," in *Didactics of Mathematics in Higher Education as a Scientific Discipline–Conference Proceedings*, 2017, pp. 101–105.

[85] L. Mutambara and S. Bansilal, "An exploratory study on the understanding of the vector subspace concept," *African Journal of Research in Mathematics, Science and Technology Education*, vol. 23, no. 1, p. 14, 2019, doi: 10.1080/18117295.2018.1564496.

[86] G. Harel, "Promoting linear algebraic reasoning among students: affordances and challenges," *PRIMUS*, pp. 1–21, Mar. 2024, doi: 10.1080/10511970.2024.2327325.

[87] S. Hurtado, K. Eagan, J. H. Pryor, H. Whang, and S. Tran, *Undergraduate teaching faculty: the 2010–2011 HERI faculty survey*. Los Angeles: Higher Education Research Institute, University of California, 2012.

[88] E. Johnson, R. Keller, and T. Fukawa-Connelly, "Results from a survey of abstract algebra instructors across the United States: understanding the choice to (not) lecture," *International Journal of Research in Undergraduate Mathematics Education*, vol. 4, no. 2, pp. 254–285, Jul. 2018, doi: 10.1007/s40753-017-0058-1.

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