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Optimizing prospective teachers' representational abilities through didactical design-based lesson study

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ABSTRACT

This research aimed to optimize the representational abilities of prospective mathematics teachers through the implementation of hypothetical lecture designs. The research design used was didactical design research combined with lesson study activities. Participants in this research were 29 prospective mathematics teacher students (18–21 years old) at a private university in Mataram, Indonesia. The researcher was the main instrument, with several additional instruments, one of which was the hypothetical lecture design. After conducting qualitative data analysis, it was concluded that there was an increase in the percentage of students' representation abilities through the implementation of hypothetical lecture designs. This was because the course design facilitated students to solve problems by exploring, using, and presenting ideas in various forms of representation. Apart from that, the lecture design integrated QR codes, Quizizz, and inspirational YouTube videos to attract students' interest or motivation during lectures, especially when solving problems.

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1. INTRODUCTION

Several previous studies reveal that representation ability is one of the important competencies that everyone should have [1]–[3]. This is because, by having this ability, it is easier for someone to solve problems using various forms of representation [4], [5]. In fact, this ability actually becomes a problem for students when learning mathematics [6]. Therefore, mathematics teachers, including prospective mathematics teachers, have a responsibility to optimize these representation abilities [7], [8]. Moreover, representation ability is one of the standard processes in mathematics learning itself [9]. Representation ability is a person's ability to use various methods or forms of representation to convey ideas or thoughts [3]. There are at least five modes of representation, namely visual, verbal, contextual, physical, and symbolic [4]. Visual representation is a person's ability to convey ideas using pictures, diagrams, number lines, and other forms of mathematical images. Verbal representation is conveying ideas using verbal language or descriptive words. Contextual representation is a person's ability to relate mathematical ideas or thoughts using the context of everyday life. Physical representation is a person's ability to use real objects when conveying ideas. Symbolic representation is conveying ideas using numbers, variables, or symbols [4], [7].

Quite a lot of research is studying student representation abilities and ways to optimize representation abilities in higher education [10]–[12]. However, not much is studied regarding the factors that cause the low representation ability of prospective mathematics teacher students and the solution for optimizing representation ability by using lesson study activities based on didactical design research. Research in Surabaya, Indonesia, has aimed to describe the representational abilities of prospective mathematics teacher students [13]. This research has used a qualitative-descriptive approach. The research has concluded that when solving calculus problems, prospective mathematics teachers have been able to carry out multiple representations. Research at a university in Ethiopia has aimed to identify the effect of the GeoGebra-assisted multiple representation approach on students' representational translation abilities in calculus courses [7]. The research has used a quasi-experimental research design. The results of this research have concluded that this approach has been able to optimize students' representational translation abilities after the intervention. Still with the same research design, research in Palembang, Indonesia, has concluded that the implementation of Maple in integral learning has been able to optimize the representation abilities of prospective mathematics teacher students [14].

In contrast to several studies previously described, this research tries to examine the representational abilities of prospective mathematics teachers for non-mathematics courses. The courses chosen are educational courses, namely didactical design research. Apart from that, this research uses lesson study activities based on didactical design research as a solution to optimize students' representational abilities. Even though it is in its initial steps, this research tries to examine the factors that cause students to experience learning obstacles. Lesson study activities [15], [16], based on didactical design research [17], [18], are used because they produce effective and efficient lecture designs. It is said to be effective because didactical design research is able to produce learning designs that suit the characteristics of students. The learning solutions offered in didactical design research are arranged based on factors that cause students to experience learning obstacles. Meanwhile, learning designs prepared based on lesson study activities have a variety of points of view or considerations, so learning designs tend to be presented in a simpler or more efficient form. Therefore, the aim of this research is to develop a lecture design that is able to optimize the representational abilities of prospective mathematics teachers. To achieve the research objectives, several research questions (RQ) are prepared, including:

i) What is the initial condition of prospective mathematics teachers' representation abilities? (RQ1)
ii) What does a hypothetical lecture design look like to optimize representation abilities? (RQ2)
iii) What is the description of the implementation of a hypothetical lecture design? (RQ3)
iv) What is the description of the representational abilities after implementing a hypothetical lecture design? (RQ4)
v) How are revisions made to hypothetical lecture designs after lecture implementation? (RQ5)

2. METHOD

This research employed didactical design research (DDR). This design was used because this research aimed to develop a lecture design that could optimize students' representational abilities [19]. Apart from that, this research used lesson study activities to produce lecture designs that could anticipate the various responses that students gave during lectures [16], [20]. The procedure in this research then integrated the DDR research steps with the steps in lesson study activities. The first step in DDR was a prospective analysis combined with a lesson study plan. The output of this step was a hypothetical lecture design that was implemented in actual lectures. The first step was used to answer RQ1 to RQ2. This activity was carried out for around two months, namely from September to October 2023. The second step was metapedadidactic analysis combined with do. The output of this step was a description of the lecture process during the implementation of the hypothetical lecture design and notes on the results of the observations. The second step was used to obtain answers to RQ3. This activity was carried out for one month, namely November 2023. Meanwhile, the third step, namely retrospective analysis combined with see, the output of this step was a revision or redesign of the hypothetical lecture design, referred to as the empirical lecture design [16]–[20]. Meanwhile, the third step was used to answer RQ4 to RQ5. This activity was carried out for one month in November 2023. The flow of research procedures can be seen in Figure 1.

This research involved 29 participants who were prospective mathematics teachers at a private university in Mataram, Indonesia. Of the 29 participants, seven were male, and the rest were female. Participants' ages ranged from 18 to 21 years. Regarding hobbies, most participants liked watching, drawing, writing, and reading. The prospective teachers referred to in this research were bachelor students in the mathematics education study program. The researcher was the main instrument in that research. This was because in qualitative research, there was no definite scope for the type or scope of data that had to be collected, including uncertainty about where the data source came from [21]. To make it easier to obtain data, researchers used several additional instruments, such as student-teacher biodata questionnaires, hypothetical lecture designs, student activity observation sheets, representation ability performance sheets, and documentation studies. The biodata questionnaire for prospective teacher students was used to obtain data about participants who were prospective teacher students. A hypothetical lecture design was used to obtain information regarding student responses during lectures. The course studied was one of the mandatory educational courses for prospective mathematics teacher students, namely didactical design research. The student activity observation sheet was an instrument used to obtain data related to students' unique activities during lectures. The representation ability performance sheet aimed to obtain information related to students' representation abilities. The representation ability performance sheet used in this research can be accessed at the following link: https://shorturl.at/bvKN6. Documentation studies were used to obtain information related to the lecture process in the form of video recordings or photos of student expressions.

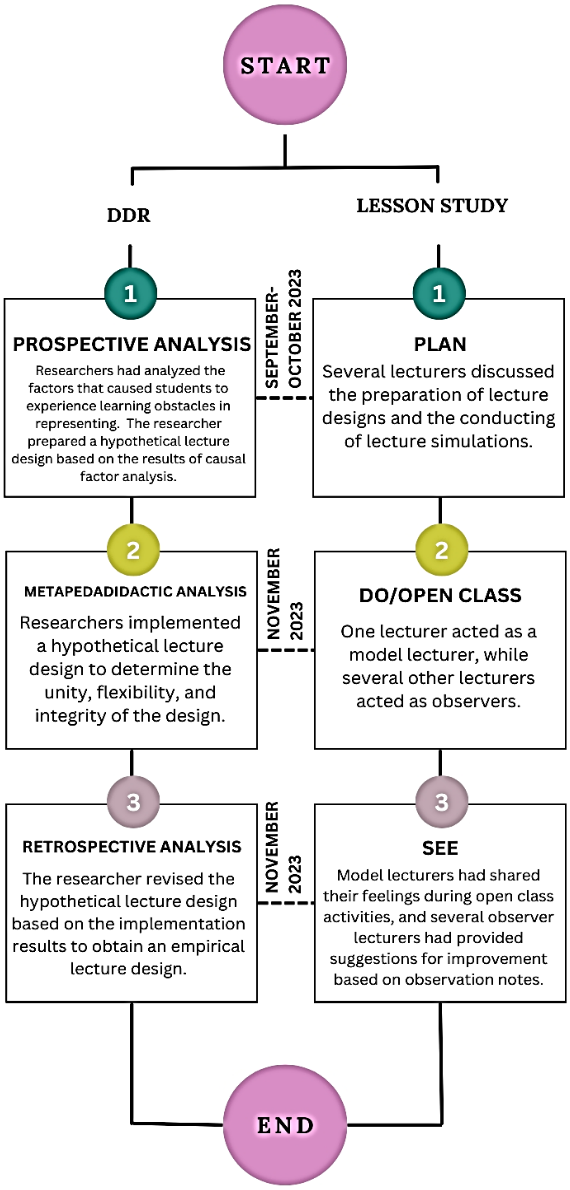


Figure 1. Research procedure

The data obtained were then analyzed using qualitative data analysis. This analysis consisted of several steps, namely reducing data, displaying data, and drawing conclusions [22]. Eliminating unnecessary data has been a method of data reduction. For example, not all learning video recording data had been analyzed, but researchers had only analyzed video recordings that showed unique activities carried out by students. Displaying data had been done by displaying data that had been reduced to various forms of representation, such as tables, diagrams, or qualitative descriptions. In this step, researchers grouped student answers according to the mode of representation presented. After that, the researcher had analyzed the grouping results and displayed the analysis results in the form of graphs, tables, or descriptions. For example, in this study, the researcher displayed data on the percentage of representation ability modes in the form of a line diagram, as shown in Figure 1. Finally, the data that had been displayed had been interpreted to obtain a conclusion regarding the answer related to the research question. Researchers in this step had looked for previous research results or theories related to representational abilities to confirm the research results. This confirmation had then been known as research findings.

3. RESULTS AND DISCUSSION

3.1. What is the initial condition of prospective mathematics teachers' representation abilities?

Based on the results of the qualitative analysis, information was obtained that students' representation abilities were still in the low category. This was because not all students were able to create representations in visual, physical, symbolic, verbal, and contextual forms. One proof of this was that only 20% of students were able to make visual and contextual representations during lectures. Of the five existing modes of representation, students' symbolic and verbal representation abilities had a higher percentage than the other three modes. This was due to the participants having good verbal language potential as prospective teachers and being accustomed to expressing ideas in the form of numbers, symbols, or variables as prospective mathematics teachers. Meanwhile, physical and contextual representation still tended to be low because students were in a low-level semester, so the ability to contextualize ideas or use concrete objects in learning was not optimal. The complete results regarding these percentages can be seen in Figure 2.

The results are then in line with several previous studies [4], [23], [24], which reveals that students tend to have problems making representations during lectures. Other research also reveals that students' visual and symbolic representation abilities are quite low [25]. This is because lecturers do not provide opportunities for students to explore ideas or understanding in solving problems, and students tend not to be used to conveying ideas visually but are limited to providing descriptions related to the ideas they have [26], [27].

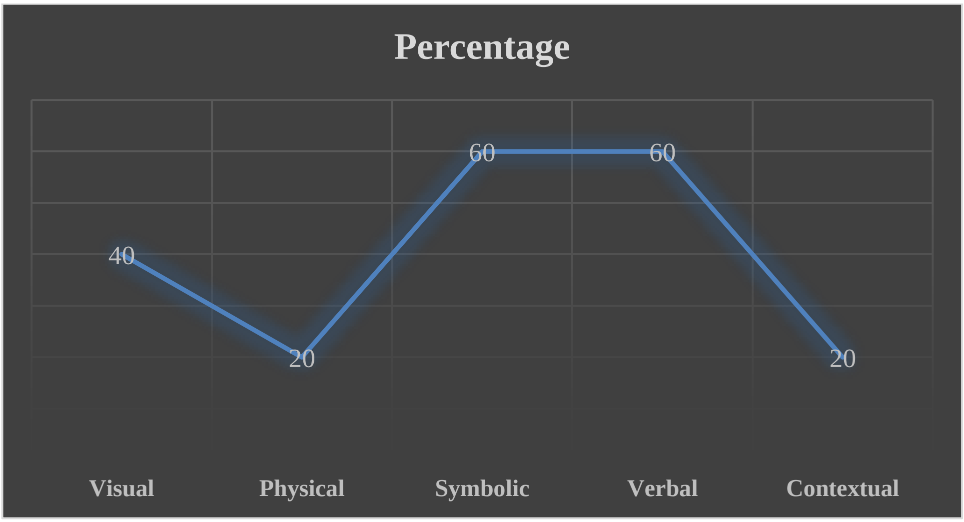


Figure 2. Percentage of students' representational abilities before implementing design

3.2. What does a hypothetical lecture design look like to optimize representation abilities?

As previously described, the representational abilities of prospective mathematics teachers were quite low. According to the findings of interviews, lecturer-led lectures tended to be conventional in nature. Lecturers typically used presentation slides containing material and assignments that students had to complete. This was because students were not given the opportunity to explore and convey ideas in various forms of representation. What's more, students were rarely faced with problems or cases to solve during lectures. An excerpt from one of the interview results can be seen in Table 1. These results were then in line with previous studies [26], [27], which revealed that one of the factors that caused students' representational abilities to be less than optimal was the lack of opportunities given to students to explore ideas when solving problems. When exploring ideas, students usually used various forms of representation, such as small notes, sketches, tables, or other forms of illustration, to make it easier to solve problems. However, when there had been no problem to be solved, there had been a tendency for students not to explore ideas or present these ideas in various forms of representation. This had then had an impact on the low representation ability of students in learning.

Table 1. Excerpt from an interview with one of the students

Researcher questions	Student answers
How is your teaching process currently going?	"*The learning process, right? Yes, as usual, sir.*"
How is it usual?	"*Yes, the lecturer comes into class, delivers the material, and assigns homework.*"
Is that all? Have you ever been asked to solve a problem or case?	"*I have, sir, but it felt like a task.*"
Have you ever been asked to create a poster or something similar during a lecture?	"*Rarely, sir. In fact, never.*"

Based on the results of interviews and studies of several theories, it was concluded that lectures conducted so far did not provide opportunities for students to solve problems by using ideas and conveying these ideas in various forms of representation. Therefore, this research offered a solution in the form of a hypothetical lecture design that facilitated students in solving problems or case studies. The design also facilitated students in using and conveying their ideas in the form of posters that were presented. A snapshot of this problem can be seen in Figure 3. The problem raised in Figure 3 was a case study that prospective teachers had to complete by first analyzing a video containing a learning design considered good by the participants. After that, prospective teachers were asked to create a good learning design, either by imitating or by designing the lesson. The learning design created was then expected to be presented in the form of a poster that combined visual, symbolic, and contextual representation.

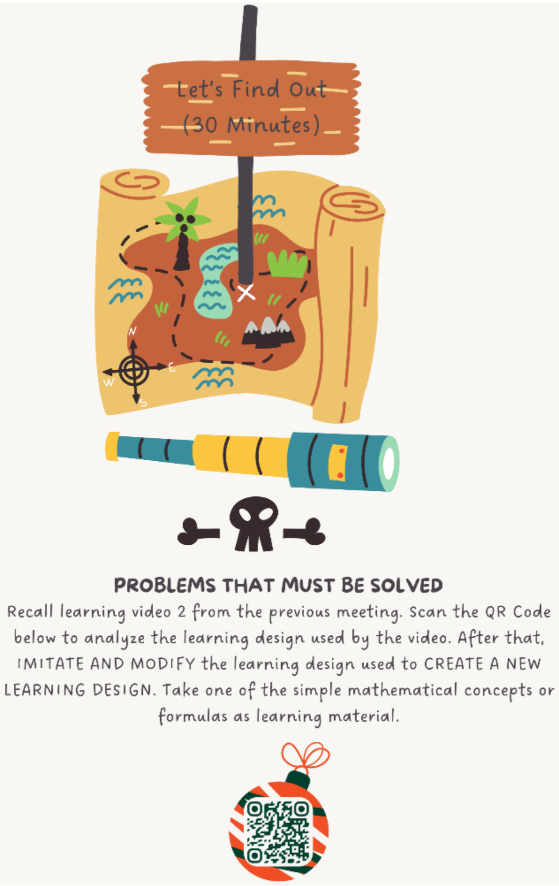


Figure 3. A snapshot of problems in hypothetical course design

In addition, a hypothetical lecture design was developed using didactic situation theory [28]. This theory revealed that learning consisted of at least several situations, namely action-formulation, validation, and institutionalization [17], [29]. Action-formulation situations were situations where students were asked to solve problems in various ways of representation in their respective groups [27]. In this activity, the focus of the representation that students carried out was visual, symbolic, and contextual representation [4] because students in this activity worked on cardboard, which required students' ability to express ideas in the form of images, numbers, or variables and contextualize ideas in classroom learning. This situation in the hypothetical lecture design was termed the “Let’s Find Out” activity. A validation situation was a situation where students presented solutions to previously solved problems. This activity was intended for other groups to provide responses and obtain validation of the concepts learned in lectures. The representation skills that students were expected to demonstrate in this activity were verbal and physical [4], because students in this activity presented problem solutions verbally using concrete objects as assistance. This activity in the hypothetical lecture design was termed the “Let’s Tell Stories” activity. An institutionalization situation was a situation where students used the concepts they had learned to solve other problems. The term “Let’s Practice” was used to represent these activities in hypothetical lecture designs [30], [31].

Furthermore, to optimize student activities during lectures, hypothetical lecture designs integrated the use of ICT. Some examples of integrated ICT were QR codes, inspirational YouTube videos, and Quizizz. QR codes were used to make it easier for students to use hypothetical lecture designs [32], [33]. QR codes were used as a container to store information, such as inspirational YouTube videos and Quizizz. This activity was expected to be able to train students’ visual representation skills because it related to displays. Inspirational YouTube videos were used as a means to arouse student motivation or interest during lectures [34]–[36]. It was hoped that this activity would also be able to optimize visual, contextual, and verbal representation skills because, after watching the video, students were asked to express their opinions verbally regarding the inspirational video. Quizizz also had almost the same role, namely to attract students’ interest during lectures. Although the main goal of Quizizz in hypothetical course design was to ascertain students’ prerequisite knowledge through playing games [37]–[39]. This activity also aimed to optimize students’ contextual representation abilities by answering questions on Quizizz. The term “Let’s Play” was used in hypothetical lecture designs that utilized Quizizz. Footage of “Let’s Play” activities can be seen in Figure 4.

In general, hypothetical lecture design consisted of three activities, namely preparatory, lecture, and evaluation [40]. Preparation was the initial activity in learning. The time allocation for that activity was fifteen minutes. This activity consisted of several activities, namely “Let’s Listen”, which contained the lecture objectives in the form of essential questions; “Let’s Watch”, which was used to attract students’ interest or motivation by watching inspirational YouTube videos; and “Let’s Play”. Lecture was the core activity in lectures, which consisted of the activities “Let’s Find Out”, “Let’s Tell Stories”, “Let’s Summarize”, and “Let’s Practice”. The time allocation for that activity was seventy minutes. In the meantime, students engaged in evaluation as a reflection activity on their learning experiences. The time allocation for that activity had been the same as for preparation activities, namely 15 minutes. The “My Reflection” activity served as a representation of the evaluation activity in the hypothetical lecture design. Basically, most of the activities in the hypothetical lecture design facilitated prospective mathematics teachers to use representation skills in various modes. The main activities that were expected to be able to optimize students’ representational abilities were “Let’s Find Out”, “Let’s Tell Stories”, and “Let’s Practice”, because in these activities students were asked to solve problems using various modes of representation and describe verbally with the help of concrete objects as a medium for learning mathematics. The complete hypothetical lecture design can be accessed on the following page: https://shorturl.at/cJPS5. Before being implemented in a real class, a hypothetical lecture design was simulated by one of the lecturers, who acted as a model lecturer. The model lecturer then received suggestions from other lecturers for the simulation. This activity was a form of lesson study activity. This activity aimed to ensure that model lecturers were truly ready to implement hypothetical lecture designs.

3.3. What is the description of the implementation of a hypothetical lecture design?

At the beginning of the lecture, the model lecturer opened the class with greetings and an opening prayer. After that, the model lecturer distributed the hypothetical lecture design to each group of students. Do not forget, model lecturers also checked student attendance before the lecture started. The first activity carried out by the model lecturer was to ask students to read the essential questions that would be answered at the meeting. The purpose of the lecture was contained in the hypothetical lecture design. One of the students with the initials MDAM then read out essential questions related to how to prepare a good lecture design. After confirming the essential questions, the next activity was “Let’s Watch”. In this activity, students were given the opportunity for three minutes to watch an inspirational YouTube video linked to a QR code. Students were seen scanning the QR code and starting the viewing activity. After that, students were asked to answer several questions related to the videos they had watched. Students in this activity seemed able to answer the questions given well. This activity was expected to optimize visual and verbal representation abilities. The next activity was “Let’s Play”. Students also did the same thing in this activity. Students scanned the QR code so they could connect to Quizizz. After a while, all the students were ready to start playing activities, and the model lecturer started playing activities. A summary of student answers can be seen in Figure 5. This activity was expected to be able to optimize students’ contextual representation abilities because several questions related to students’ understanding of actual classroom learning.

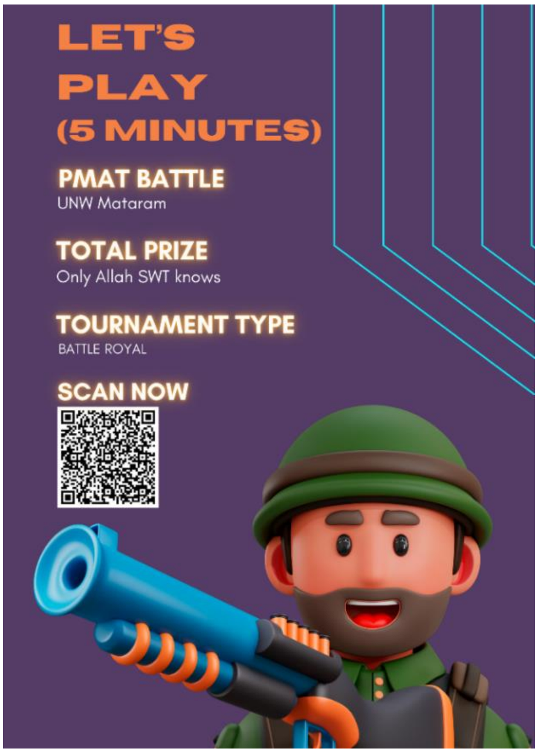


Figure 4. Footage of Let's Play activities

Quizizz

Quiz Name	Date	Hosted by
Developing a learning design	Fri Oct 13 2023 1:34 PM	Muhamad Isnawan

Average Accuracy	Total Questions	Number of Players	Participant Attempts
23%	5	19	19

ⓘ This report displays results derived from the students' all attempts.

Questions

No.	Question	Time	Accuracy	Responses: Correct	Responses: Incorrect	Responses: Unattempted
1	One example of an initial activity in learning is?	15 secs	47%	9	1	9
2	Providing related inspirational videos a topic aims to awaken what?	39 secs	11%	2	6	11
3	One example of core activities in learning is?	14 secs	11%	2	8	9
4	One example of a final activity in learning is ......si.	31 secs	21%	4	7	8
5	Which is not a core activity in learning is?	18 secs	26%	5	8	6

Figure 5. A summary of students' answers to the Let's Play activity

After completing the "Let's Play" activity, the next activity was "Let's Find Out". In this activity, students were asked to develop a good mathematics learning design. In this activity, students were seen to have a learning design. This is because, in the previous lecture, students were asked to prepare an example of a mathematics learning design. As a result, students carried out this activity by discussing the learning design they had created and then assembling it into a poster using cardboard. Excerpts of student answers can be seen in Figure 6. In this activity, students' visual, symbolic, and contextual representation skills were expected to be demonstrated through several pictures, mathematical symbols, and examples of daily life presented in poster form. Complete answers for all groups can be accessed on the following page: https://shorturl.at/fqsyE. In general, students could do this activity well. The results of this research are then in line with several previous studies, which reveal that problem-based learning or case studies tend to be able to optimize student performance during lectures [15], [27].

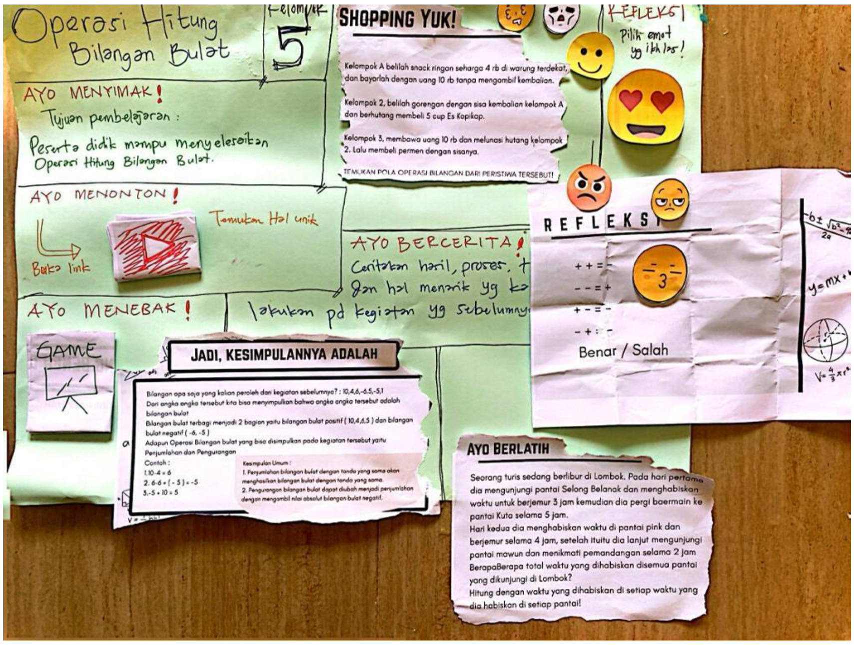


Figure 6. Excerpt from the answers of one of the student groups in the Let's Find Out activity

After completing the "Let's Find Out" activity, students presented their results from the "Let's Tell Stories" activity. In this activity, students from other groups were asked to provide responses. However, no students from other groups gave this response. This was due to the fact that students talked about a variety of topics, which made them less interested in what certain groups were saying. Students demonstrated verbal representation skills in this activity. Students could also see physical representation skills by using several concrete objects as media in the learning designs they created. In this activity, quite a lot of lecture time was used because one group tended to take longer than it should. In fact, not all groups presented their answers. This was because each group had to present the mathematical learning design they had created. The learning design contained quite a lot of activities, so it required more time to present.

The next activity was to come to a conclusion. In this section, students were not able to validate the structure of a good learning design. Therefore, the lecturer validated the model by providing the description that at least the learning design must contain three activities: preparatory, lecture, and evaluation [40]. Preparatory should consist of information related to learning objectives [41], activities to generate learning motivation [42], and exploring students' prerequisite knowledge [43], [44]. Lectures should contain problem-solving activities as a means for students to construct the mathematical concepts or formulas being studied [45]–[47]. Meanwhile, evaluation should consist of student activities in reflecting on concepts or formulas that have been learned, as well as reflecting on feelings after participating in learning [48], [49]. "Let's practice" activities that could not be done during this lecture due to insufficient lecture time. The last activity was "My Reflection". In this activity, students were able to review that at least the learning design must include preparatory, lecture, and evaluation activities. Regarding social-emotional issues, students expressed that they enjoyed attending lectures at the meeting.

Several factors made students happy during lectures, such as play activities carried out using Quizizz, the display of hypothetical lecture designs that were simple and attractive because they used QR codes, and the use of inspirational YouTube videos at the beginning of learning activities. These reasons were then in line with several previous studies [32]–[34], [36]–[38] that revealed online test applications and learning videos tended to be able to arouse student interest or motivation during lectures. The use of QR codes could also have made more information presentable. Apart from that, as previously described in the answer to the second research question, the existence of Quizizz, QR codes, and YouTube videos trained students to present ideas in various modes of representation, especially visual, verbal, and contextual.

In conclusion, the implementation of the hypothetical lecture design went according to the expected response that the researcher had anticipated. However, one obstacle was found during the implementation, namely the implementation of the lecture design, which tended to take longer so that one of the activities could not be implemented, namely “Let’s Practice”. The allocation of lecture time tended to be used quite a lot for “Let’s Watch” and “Let’s Find Out” activities because each group of students still wanted to convey ideas related to the activities given by the model lecturer. The results of this research are then in line with several previous studies [50], [51] that reveal the time allocation required for student-oriented learning tends to be more than conventional learning. This is because student-centered learning has quite a lot of mental and physical activities that must be carried out during learning. The results of this research are also in line with several studies that reveal students tend to give positive responses or responses that are in line with the researchers’ expectations when conducting learning using DDR-based learning designs [18], [31], or learning designs developed through lesson study activities [15], [16]. This is because the learning design is rich in points of view and structured based on factors that cause students to experience learning obstacles [15], [16], [52].

3.4. What is the description of the representational abilities after implementing a hypothetical lecture design?

Based on the results of the qualitative analysis of the representation ability performance sheet, future math teachers were better able to represent information after the hypothetical lecture design was used. This was because students, during lectures, had the opportunity to solve problems presented by the model lecturer. Furthermore, students were given the chance to explore, utilize, and present ideas in various forms of representation while solving these problems. The presence of trigger questions facilitated students, making it easier for them to solve problems. The integration of ICT, such as QR codes, YouTube videos, and Quizizz, proved to be quite helpful for students in creating representations during lectures. The use of smartphones and the Canva application significantly aided students in preparing mathematics learning designs during lectures. The provision of cardboard, colored markers, and various other materials also proved to be quite helpful for students when making representations. Figure 7 displays a comparison of students’ representation abilities before and after the implementation of the hypothetical lecture design. One example of a comparison of student answers when given a representation assignment before and after implementation is seen in Table 2. The table shows that before implementation, the hypothetical lecture design tended to be delivered in verbal form, whereas after implementation, it used various forms of combination representation.

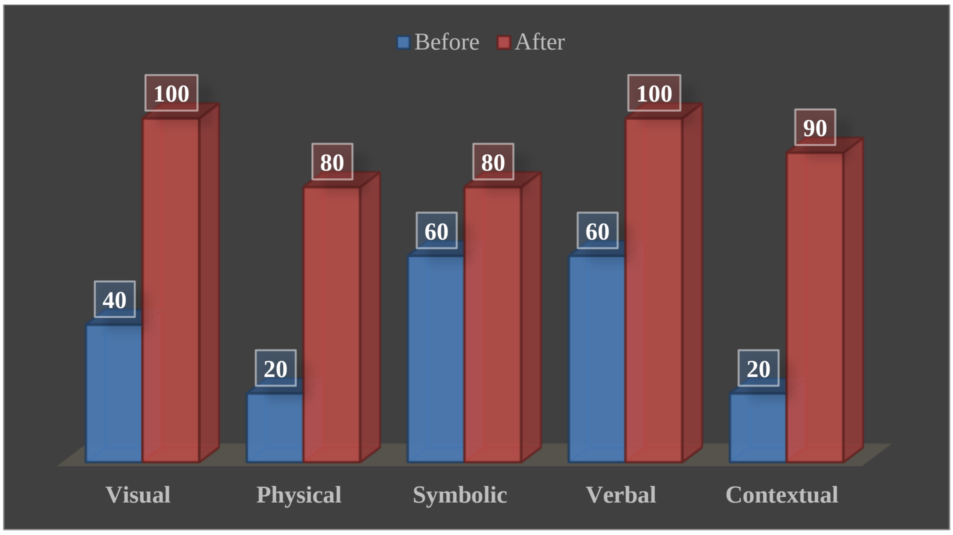
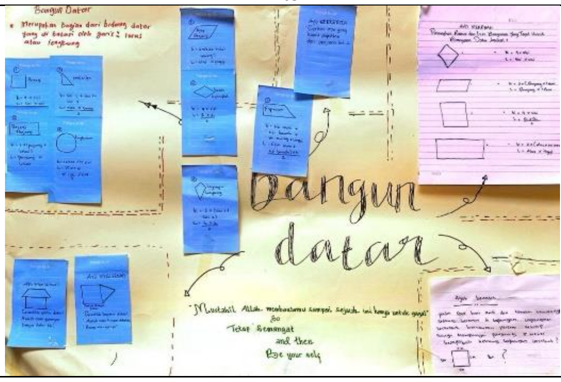


Figure 7. Comparison of students’ representation abilities before and after implementation

The results of this research are then in line with several studies [26], [27], which reveal that students’ representation abilities tend to develop optimally, especially when students are facilitated to solve problems by exploring ideas optimally. The results are also in line with previous studies [32], [39], [53], [54], which reveal that the existence of fairly complete learning support facilities, the existence of adequate learning support devices, and optimal use of ICT tend to optimize student representation abilities. The use of trigger questions also tends to make it easier for students to solve problems [55]. In summary, this research concludes that the hypothetical lecture design optimizes the representational abilities of prospective mathematics teachers.

Table 2. Comparison of student representation before and after implementation

Before	After
1. WHAT Question: What is Didactic Design Research? Didactical design research is a research that reveals learning obstacles in the learning process and aims to anticipate and eliminate learning obstacles in learning. Because it is undeniable that students have epistemological difficulties (learning obstacles) in understanding the concept of the relationship between the central angle. The length of the arc and the area of the sector of the circle. 2. WHO Question: Who discovered the DDR research? The DDR research was first discovered by a professor at the Indonesia University of Education, namely Prof. Dr. H. Didi Suryadi, M.Ed.	

3.5. How are revisions made to hypothetical lecture designs after lecture implementation?

After implementing the hypothetical lecture design, the model lecturer and several observer lecturers reflected on the lecture process that had been implemented. The reflection activity began with the model lecturer talking about feelings during the lecture. The lecturer model felt that most of the lectures had gone according to expectations. One of the observer lecturers provided suggestions for improvements to the implementation of lectures. The observer lecturer considered that there was no follow-up to the "Let's Play" activities. In fact, this activity was an activity that aimed to confirm students' prerequisite knowledge. Apart from that, as shown in Figure 5, it shows that not all students had sufficient prerequisite knowledge. Therefore, the observer lecturer recommended that the play activity have feedback from the model lecturer. This feedback could take the form of a short discussion regarding the answer that should be given to the "Let's Play" activity. This was intended so that students' prerequisite knowledge was truly ready before entering the core activities in lectures. This revision was then expected to be able to optimize students' symbolic and verbal representation abilities because students were given the opportunity to discuss verbally the answers given in the "Let's Play" activity.

Apart from that, other observer lecturers also thought that the "Let's Practice" activity could not be carried out due to time constraints. The observer lecturer considered that too much time was spent on the "Let's Tell Stories" activity. The observer lecturer recommended that students be given a time limit when making presentations. For example, each group only had five minutes to present the mathematics learning design they had created. It was hoped that this time limitation would have an impact on the implementation of "Let's Practice" activities in accordance with the appropriate time allocation. By carrying out this activity, it was hoped that students' verbal, contextual, and symbolic representation abilities would develop optimally because students were asked to carry out learning simulations that required these three representations. In summary, these revisions can be seen in Table 3.

The results of this research are in line with several previous studies [43], [44], which reveal that prerequisite knowledge is essential before students undertake learning. Additionally, feedback [56], [57] is an activity that educators must frequently carry out during learning. This is intended to ensure that interactions between educators and students run smoothly, impacting the optimal development of student competence. The results of this research are also consistent with several previous studies [58], [59], which show that providing time limits for students to express opinions has an impact on the implementation of learning activities or discussion decisions as they should be. The revised hypothetical lecture design is then referred to as the empirical lecture design. This lecture design is also included in the epistemic learning pattern because it facilitates students in constructing their own knowledge through problem-solving activities.

Table 3. Revised hypothetical lecture design

Shortcomings during implementation	Hypothetical lecture design revision
No feedback is provided on Let's Play activities.	Provide feedback on student results in Let's Play activities.
Time limitations mean that the Let's Practice activity cannot be carried out.	Limit the time allocated for students to convey opinions or ideas.

4. CONCLUSION

As described in the introduction, this research aims to develop a lecture design that optimizes the representational abilities of prospective mathematics teachers. Based on the results of research and discussion, it is concluded that the hypothetical lecture design optimizes students' representational abilities. Evidently, there is an increase in the percentage of students who can represent in various forms. This lecture design is then termed an empirical lecture design, classified as an epistemic lecture pattern. The design consists of several activities, such as "Let's Listen", "Let's Watch", "Let's Play", "Let's Find Out", "Let's Tell Stories", "Let's Summarize", "Let's Practice", and "My Reflection". The design also integrates several ICTs, such as QR codes, YouTube videos, and Quizizz.

One of the limitations of this research is that the "Let's Practice" activity cannot be carried out during lectures. However, lectures can still run well. In fact, the "Let's Practice" activity is classified as an institutionalization situation in didactic situation theory. Therefore, further research is necessary regarding whether the situation of institutionalization in learning is a mandatory thing that students must go through or not. The answer to this question could then become a finding that might be able to revise the didactical situation theory believed so far in the learning context, especially mathematics learning through didactical design research.

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