Improving trigonometric competency with GeoGebra: a quasi-experimental study in a high school

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ABSTRACT

This quasi-experimental study examines the efficacy of GeoGebra in enhancing trigonometric competence among tenth-grade students in Montería, Colombia. Comparing an experimental group that used GeoGebra with a control group receiving traditional teaching, key competencies such as reasoning and argumentation, communication, representation and modeling, and problem posing and solving were evaluated. Pre-intervention results showed that 88.19% of students in the experimental group had insufficient performance in reasoning and argumentation. After the implementation of GeoGebra, this figure decreased to 5.5%. In competencies of communication, representation, and modeling, the insufficient performance reduced from 85.7% to 5.5%, and in problem posing and solving, from 80.3% to 5.7%. These significant improvements demonstrate the positive impact of GeoGebra on the development of mathematical competencies. The study concludes that GeoGebra is an effective tool for strengthening trigonometric competence in high school students, highlighting the importance of integrating digital technologies in mathematics education. The findings suggest the need for more research on the use of technological tools in mathematics education and support investment in technological resources and teacher training.

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1. INTRODUCTION

The significance of mathematical competencies in contemporary society is undeniable [1]. Trigonometry, a fundamental component of mathematics, presents specific learning challenges for students [2]. This challenge is evident in the school in Montería (Colombia), where tenth-grade students have shown difficulties in understanding and applying trigonometry concepts and procedures [2], [3]. Despite efforts to improve academic performance, significant challenges persist in teaching and learning trigonometry.

In this context, the central hypothesis of this study is that the implementation of GeoGebra in trigonometry teaching significantly improves students' performance and mathematical skills. The research was conducted in a controlled environment with control and experimental groups, providing an effective comparison of student performance. This robust methodology offers a new perspective in mathematical pedagogy and the integration of technology in education [4].

The intervention focused on integrating GeoGebra into trigonometry teaching over six months, with specific activities and exercises designed to promote conceptual understanding. Quantitative and qualitative tests were used to assess the impact, and data were analyzed using descriptive statistics and thematic content analysis [5]. Figure 1 presents an overview of interrelated terms, highlighting "GeoGebra", "mathematical education", and "problem-solving" as central axes in the current academic discussion [6]–[8]. These nodes are closely linked to concepts such as "mathematical reasoning" and "educational technology", underlining their crucial role in mathematical pedagogy. To reflect and delve deeper into these themes, a search strategy for the literature review was developed, covering publications between 2018 and 2023, using the following search string: ("GeoGebra") and ("high school") and ("experimental study" or "case study") and ("mathematics teaching" or "educational technology"). This search was meticulously designed to capture the essence of emerging themes and direct the analysis of relevant literature.

Table 1 presents a selection of fundamental and highly relevant works in the field, identified through the systematic literature review of this study. These works, recognized as key references, can provide a solid starting point for future studies in this area [9]–[15]. This study aims to offer insights into how the integration of educational technologies such as GeoGebra can enhance the teaching of trigonometry and mathematical competencies in general [5]. This comprehensive approach seeks not only to measure the academic impact of GeoGebra but also to understand students' experiences with this tool, thereby providing a deep perspective on the integration of technology in mathematics education [16], [17]. The anticipated findings of the study are expected to offer valuable insights for educators and policy makers interested in improving both the teaching of trigonometry and general mathematical competencies.

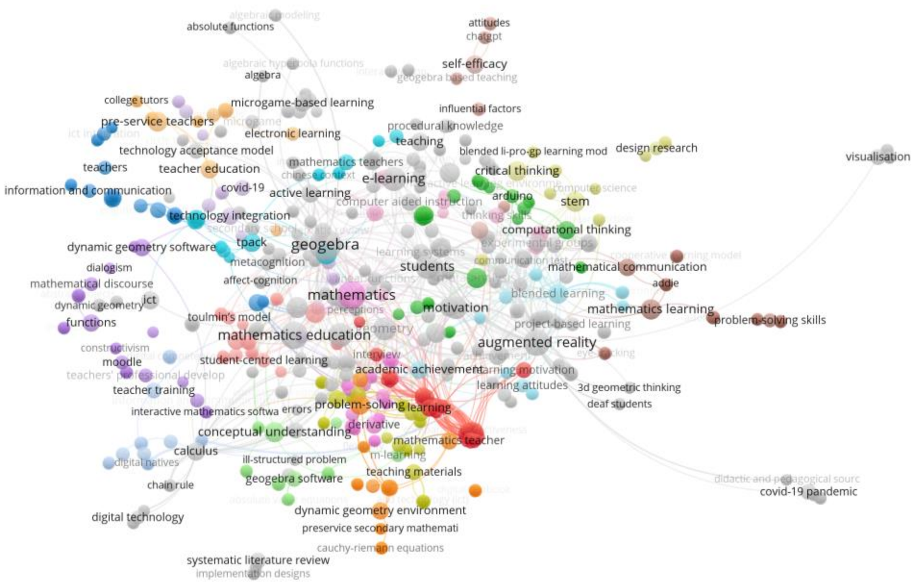

Figure 1. Key terms co-occurrence map

2. METHOD

To investigate how the GeoGebra software impacts trigonometry learning in a tenth-grade mathematics class, a case study intervention was implemented in Montería, Colombia. The study adopted a mixed-methods data collection approach, integrating both quantitative and qualitative methods. This method allowed for a more comprehensive assessment of GeoGebra's impact, capturing not only changes in academic performance but also students' perceptions and experiences.

This mixed approach was crucial to understanding GeoGebra's influence on various aspects of learning. In addition to measuring improvements in trigonometric skills, students' attitudes and responses to the use of this technological tool were explored. This section of the study provides details about the participants, the educational setting, the intervention methodology, the study's duration, the assessment tools used, and the methods employed for analyzing the collected data. By combining different research methods, the study offered a more complete and nuanced perspective on the role of GeoGebra in the mathematics classroom. The data collected through this mixed methodology not only informed about performance outcomes but also provided valuable insights into how GeoGebra can be effectively integrated into trigonometry teaching to enhance students' educational experience.

Table 1. Comparative analysis of studies on the application of GeoGebra in mathematical education

Study	Study purpose	Key discoveries	Key contributions	Study significance	Prospects
[9]	Impact of the B-Geo Module on problem-solving in rural secondary students	Improved problem-solving abilities	Effectiveness of GeoGebra in task-based teaching	Importance of digital tools in mathematical skills	Expanded use of GeoGebra in educational settings
[10]	Development of mathematical communication skills with GeoGebra	Significant improvement in communication skills	Efficacy of GeoGebra in mathematical communication	Value of interactive software in mathematical understanding	Adoption of GeoGebra in diverse educational environments
[11]	Impact of GeoGebra software on geometry performance and attitude	Experimental group outperformed control group	Empirical evidence of GeoGebra benefits in geometry	Integration of technology to enhance learning outcomes	Research on GeoGebra effects across different educational levels
[12]	GeoGebra in understanding continuity and procedural knowledge improvement	Supports procedural knowledge improvement	Importance of TPACK for effective GeoGebra use	Relevance in educational settings for visualizing mathematical concepts	Research on TPACK and social proximity factors
[13]	GeoGebra-assisted inquiry learning strategy in algebra	Improvement in mastery and interest in algebraic expressions	Effectiveness in enhancing mathematical understanding and interest	Challenge of engaging students in algebraic concepts	Broader implementation of GeoGebra in technology-assisted learning strategies
[14]	Effectiveness of instructional prompts in GeoGebra for gifted students in mathematics	Improvement in learning achievements of gifted students	Improved instructional strategies integrating prompts into GeoGebra	Contribution to education of gifted students	Enhancing gifted education through technology
[15]	Contributions, challenges, and limitations of GeoGebra in teaching mathematics	Improved interest and achievement in mathematics	Technological fluency among users and student-teacher ratio	Challenges and potential of GeoGebra in educational settings	Effective integration and future development of GeoGebra in educational contexts

2.1. Participants

To conduct this study, a careful selection of 127 tenth-grade students from a school in Montería was made. The choice of these participants was based on their academic performance in mathematics and, in particular, the challenges they faced in the area of trigonometry. This purposeful selection allowed for a more focused and relevant analysis of the impact of GeoGebra on students who showed specific needs in this discipline [18].

This targeted approach ensured that the study focused on those students for whom the intervention could be most beneficial. By concentrating on students with particular difficulties in trigonometry, the study aimed to explore not only the general efficacy of GeoGebra but also its potential to address specific learning challenges. This methodology helped ensure that the study's results were both relevant and applicable to a real educational context.

2.2. Study design

The design of this study included a quasi-experimental approach using pretest/posttest assessments. This method allowed for the observation and comparison of each student's performance in trigonometry before and after the implementation of GeoGebra. This approach provides a clear and direct evaluation of the impact that the software had on the students' learning and understanding [19].

The advantage of this quasi-experimental design lies in its ability to measure the specific effects of the intervention on the same group of students. By comparing the results of the pretest and posttest assessments, improvements can be identified not only in trigonometric skills but also in other aspects of academic performance. This approach helps to determine the effectiveness of GeoGebra in the classroom, offering valuable insights for future pedagogical strategies.

2.3. Intervention tool

In the study, the mathematical simulator GeoGebra was used as the primary intervention tool to complement traditional trigonometry teaching. This open-source educational software offers an interactive platform that enriches the learning experience in mathematics [5], [8], [20]. Its implementation allowed students to explore trigonometric concepts in a more dynamic and visual manner, facilitating a deeper and more applied understanding of the course [9].

The integration of GeoGebra into the curriculum aimed not only to improve academic performance in trigonometry but also to promote a more interactive and participatory learning approach. The intuitive and versatile nature of this software makes it a valuable tool for illustrating complex mathematical concepts, enabling students to experiment with different problems and solutions in real time. The adoption of GeoGebra in this study underscores the growing importance of combining traditional teaching methods with innovative technologies in the educational field.

2.4. Intervention procedures

The intervention in this study focused on integrating GeoGebra into trigonometry teaching sessions, spanning a period of six months. During this time, students actively used the simulator under the supervision and guidance of their teachers to delve into various concepts, theorems, and trigonometric problems. This interactive tool allowed them to explore and understand trigonometry in a more dynamic and engaging manner [21]. Furthermore, to maximize the learning potential, specific activities and exercises in GeoGebra were designed to reinforce the concepts taught in class. These activities were aimed at promoting not just the memorization of formulas and procedures, but also at fostering a deeper conceptual understanding. The inclusion of GeoGebra as a didactic tool in the trigonometry curriculum represented a significant advancement in teaching methodology, aligning with contemporary trends of incorporating educational technologies in the classroom to enhance the learning process.

2.5. Evaluation measures

To assess the impact of GeoGebra on trigonometry learning, two evaluation methods were implemented [22]. Initially, a pretest was administered to students to establish their level of competence in trigonometry before the intervention. Subsequently, a posttest was conducted after the completion of the intervention period, aiming to measure improvements in students' academic performance [23]. These assessments consisted of trigonometry problems designed to evaluate both conceptual and procedural skills.

In addition to these quantitative tests, qualitative data were collected to enrich the analysis [23], [24]. These included detailed classroom observations, learning journals kept by students, and semi-structured interviews. These methods provided a deeper understanding of how students interacted with GeoGebra and how it influenced their learning process. The combined approach of quantitative and qualitative assessments allowed for a comprehensive evaluation of GeoGebra's effect, offering a more complete view of the educational impact of the software in trigonometry teaching.

2.6. Data analysis

The quantitative data obtained from the pretest and posttest were analyzed using descriptive statistics, where the mean and standard deviation for each set of scores were calculated [25]. To identify significant differences in students' academic performance before and after the intervention, the paired samples t-test was utilized [26]. The equation for the paired samples t-test is as (1):

$$t = \frac{\bar{\bar{D}} - \mu D}{\frac{S_D}{\sqrt{n}}} \tag{1}$$

where, t is the t-test statistic, $\bar{\bar{D}}$ is the mean of the differences, μD is the population mean difference (which is generally assumed to be 0 in the null hypothesis), S_D is the standard deviation of the differences, and n is the sample size.

The qualitative data were analyzed through thematic content analysis, aiming to identify recurring themes and patterns in students' learning experiences with GeoGebra [27]. To ensure the reliability of the content analysis, Cohen's Kappa coefficient was calculated, which measures the level of agreement between two coders [28]. The equation (2) is for calculating Cohen's Kappa coefficient.

$$k = \frac{P_0 - P_e}{1 - P_e} \tag{2}$$

where, k is Cohen's Kappa coefficient, P_0 is the observed agreement proportion between the coders, and Pe is the proportion of agreement that would be expected by chance.

3. RESULTS AND DISCUSSION

The results of the study involving the use of GeoGebra as a didactic tool in a trigonometry class for an experimental group of tenth-grade students in Montería were analyzed and compared with a control group that received traditional teaching. This analysis focused on evaluating key competencies such as reasoning and argumentation, communication, representation and modeling, and problem posing and solving. Assessing these competencies was crucial to determine the effective impact of GeoGebra on student learning.

To carry out this analysis, a set of specific items detailed in Table 2, which forms part of the data collection instrument, was used. These items were carefully selected to effectively evaluate the mentioned competencies. The comparison between the experimental and control groups provided valuable insight into the effectiveness of teaching methodologies, both traditional and innovative, in improving academic performance and trigonometry skills of students.

This comparative approach allowed not only to measure the efficacy of GeoGebra in enhancing specific competencies but also to gain a broader understanding of how the integration of technology in teaching can influence the educational process. The inclusion of a traditional control group provided a significant benchmark for assessing the benefits of GeoGebra-assisted teaching compared to conventional methods. The following are the results obtained in the pretest.

Table 2. List of items for the information collection instrument

Competency	Indicators	Items from the instrument
Competency of reasoning and argumentation	– Argue, explain, and justify choices in procedures involving trigonometric ratios. – Formulate hypotheses and make conjectures, using trigonometric ratios. – Explore examples and counterexamples related to trigonometric ratios. – Generalize properties and relationships and identify patterns in trigonometric ratios.	2, 4, 9, 10, 12
Competency of communication, representation, and modeling	– Describe situations or problems using trigonometric ratios. – Interpret and distinguish the different representations of trigonometric ratios. – Interpret and translate both the formal language and symbolic language regarding trigonometric ratios.	1, 6, 8, 11, 14
Competency of problem setting and solving	– Solve hypothetical and real problems involving the use of trigonometric ratios. – Justify the choice of methods and instruments for solving problems with trigonometric ratios. – Generalize solutions and strategies in other contexts to solve new problem situations involving trigonometric ratios.	3, 5, 7, 13, 15
Total		15

3.1. Reasoning and argumentation competence

As shown in Figure 2 and Figure 3, before the intervention, the vast majority of students in both groups demonstrated insufficient performance in the competency of reasoning and argumentation. In the control group, 90.55% of the students were at the insufficient level, 3.15% at the basic level, 3.15% at the satisfactory level, and 3.15% at the advanced level. In the experimental group, 88.19% of the students reached the insufficient level, 3.15% the basic level, 5.51% the satisfactory level, and 3.15% the advanced level. This was to be expected, as the students had not previously received any direct instruction on trigonometry, particularly on trigonometric ratios.

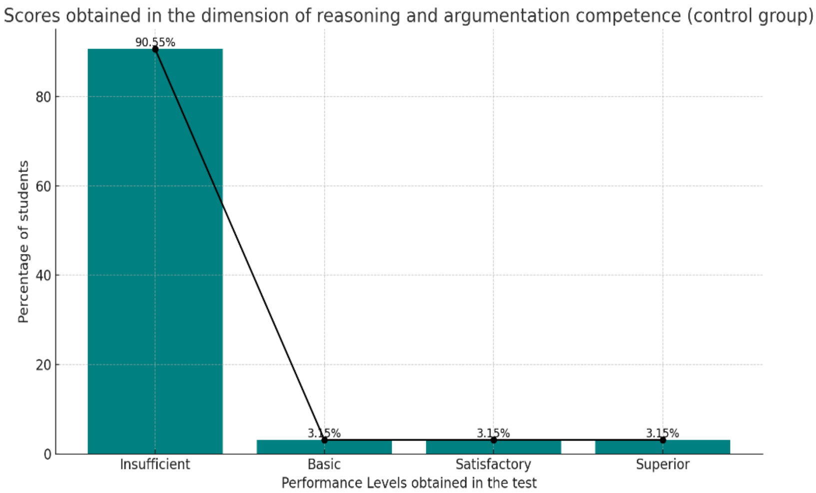

Figure 2. Reasoning and argumentation competence (control group)

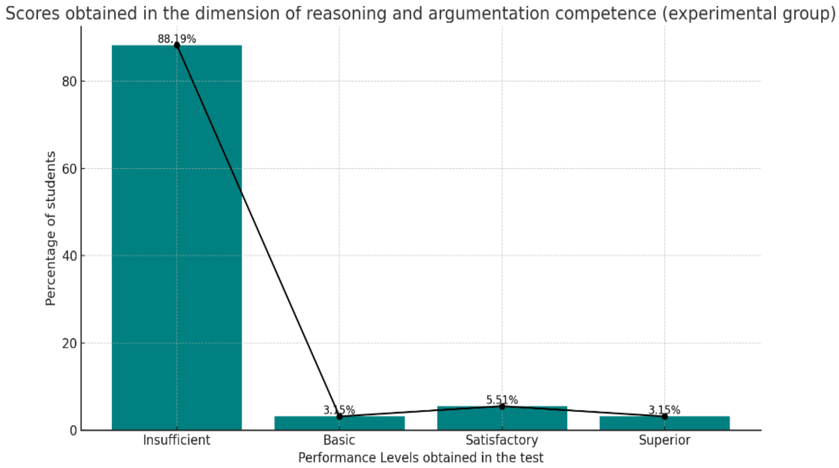

Figure 3. Reasoning and argumentation competence (experimental group)

3.2. Communication, representation, and modeling competence

Figures 4 and 5 demonstrate that, before the intervention, the vast majority of students in both groups achieved insufficient performance in the competency of communication, representation, and modeling. In the control group, 88.5% of the students showed insufficient performance, 5.7% basic performance, 2.9% satisfactory performance, and 2.9% advanced performance. Similarly, in the experimental group, 85.7% of the students achieved insufficient performance, 5.7% basic performance, 5.7% satisfactory performance, and 2.9% advanced performance. These results are predictable, as the participants had no prior knowledge regarding the measured variable.

3.3. Problem posing and problem-solving competence

As shown in Figures 6 and 7, before the intervention, the vast majority of students in both groups demonstrated insufficient performance in the competency of problem posing and solving. In the control group, 82.7% of the students were at the insufficient performance level, 5.5% at the basic level, 8.7% at the satisfactory level, and 3.1% at the advanced level. For the experimental group, 80.3% of the students achieved insufficient performance, 5.5% basic performance, 11.0% satisfactory performance, and 3.1% advanced performance. These results are predictable, albeit slightly better, as in the previous categories. The following sections present the results obtained in the posttest, breaking down the categories of reasoning and argumentation competence, communication, representation and modeling competence, and problem posing and solving competence.

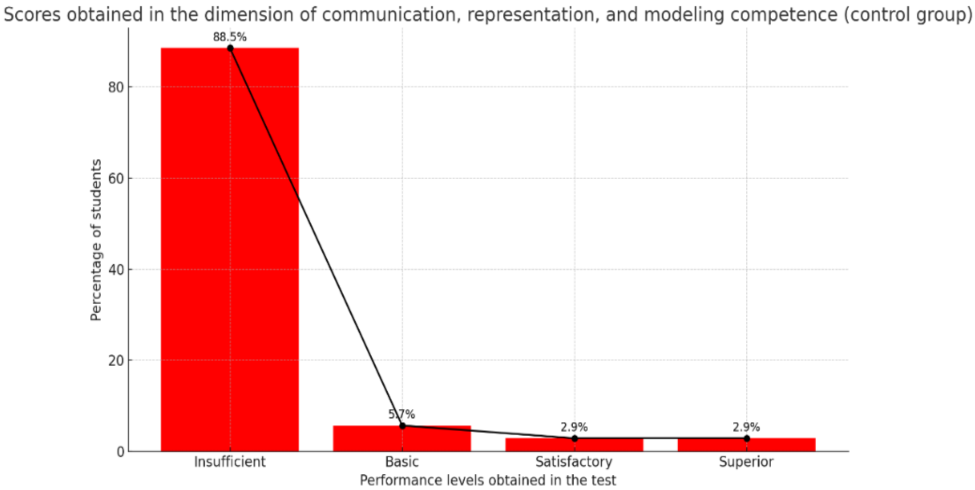

Figure 4. Communication, representation, and modeling competence (control group)

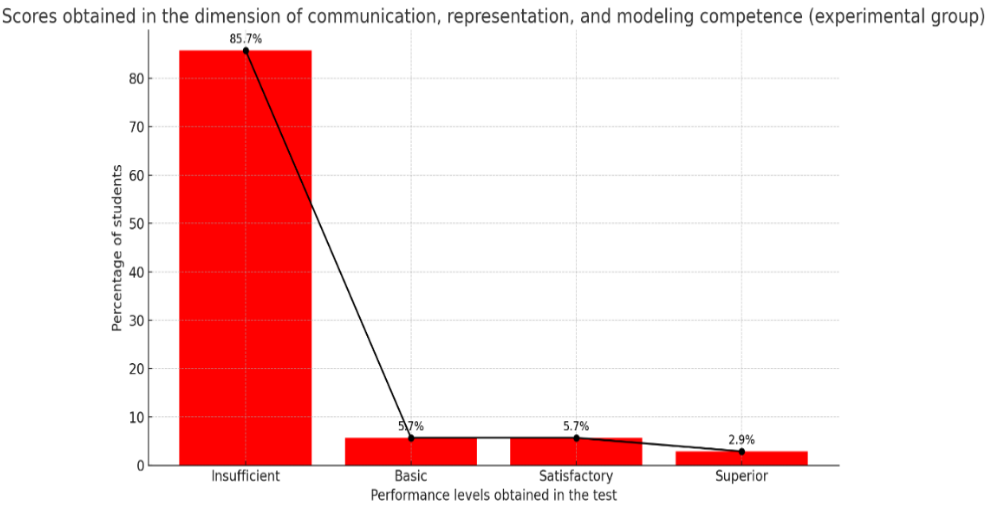

Figure 5. Communication, representation, and modeling competence (experimental group)

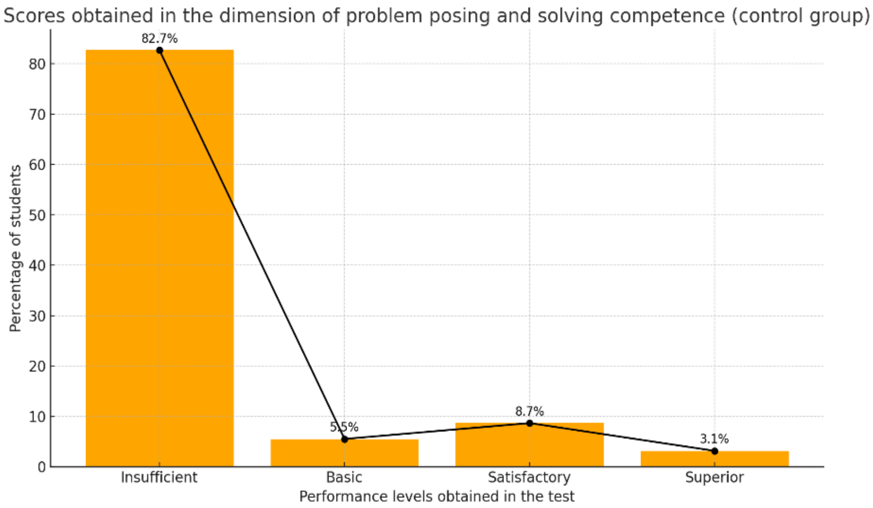

Figure 6. Problem posing and problem solving competence (control group)

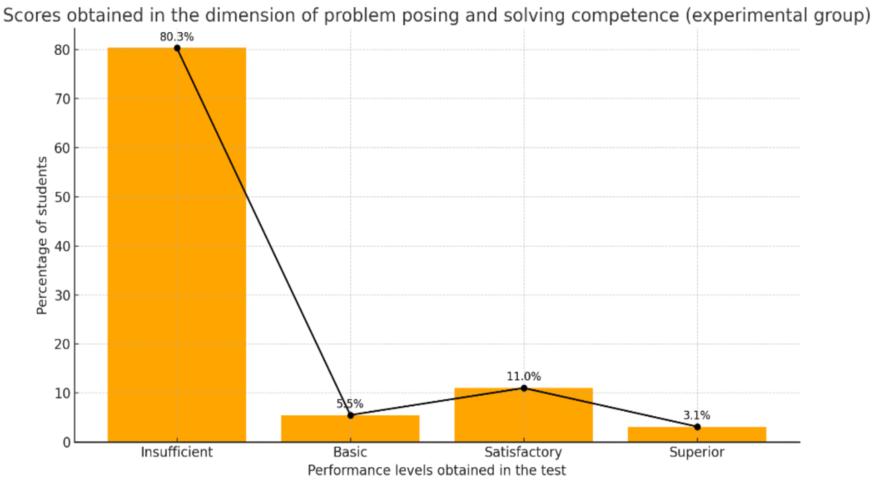

Figure 7. Problem posing and problem solving competence (experimental group)

3.4. Dimension of reasoning and argumentation competence

As illustrated in Figures 8 and 9, in the control group, 19.7% of students achieved insufficient performance, 63.0% basic performance, 11.0% satisfactory performance, and 5.5% advanced performance. However, in the experimental group, the use of GeoGebra as a teaching resource resulted in significantly better performances: 5.5% of students achieved insufficient performance, 5.5% basic performance, 63.0% satisfactory performance, and 26.0% advanced performance.

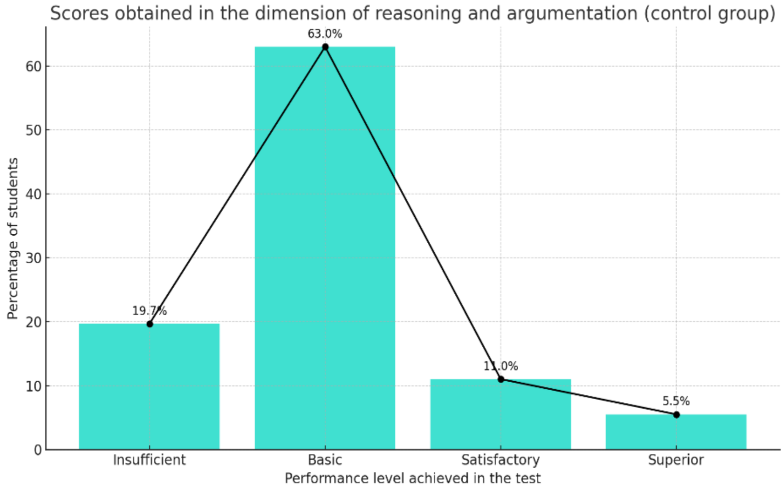

Figure 8. Reasoning and argumentation competence (control group)

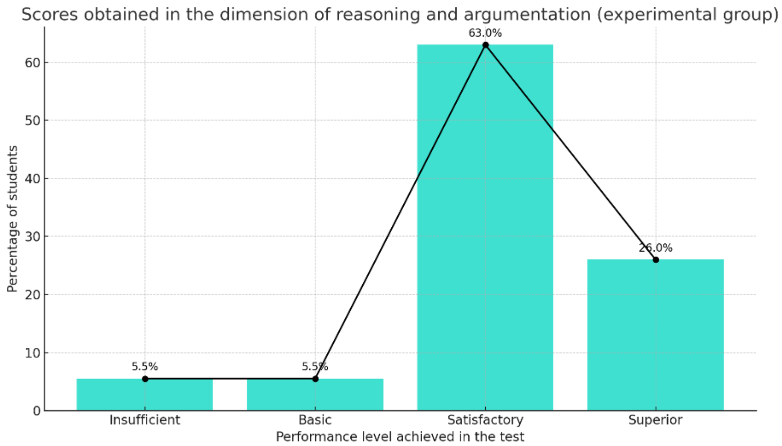

Figure 9. Reasoning and argumentation competence (experimental group)

3.5. Dimension of communication, representation, and modeling competence

As reflected in Figures 10 and 11, in the control group, 19.7% of the students were at the insufficient performance level, 59.8% at the basic level, 11.0% at the satisfactory level, and 8.7% at the advanced level. In contrast, in the experimental group, where GeoGebra was used as a teaching resource, only 5.5% achieved insufficient performance, 3.1% a basic level, 59.8% a satisfactory level, and a notable 31.5% reached an advanced level.

3.6. Problem posing and solving dimension

According to Figures 12 and 13, in the control group, 26.0% of students achieved insufficient performance, 57.5% basic performance, 11.0% satisfactory performance, and 5.5% advanced performance. In contrast, in the experimental group, where GeoGebra was implemented as a teaching resource, 5.5% of students achieved insufficient performance, 8.7% basic performance, a significant 68.5% satisfactory performance, and 17.3% advanced performance.

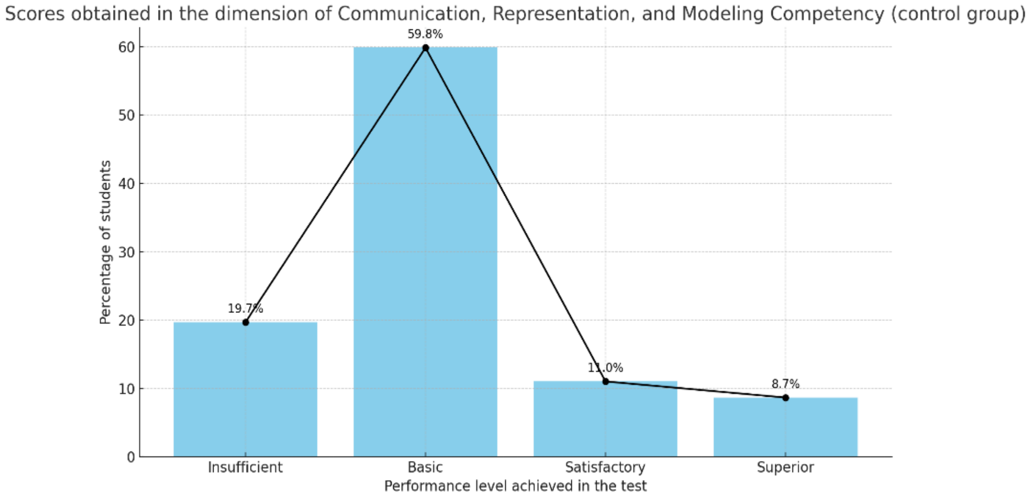

Figure 10. Communication, representation, and modeling competence (control group)

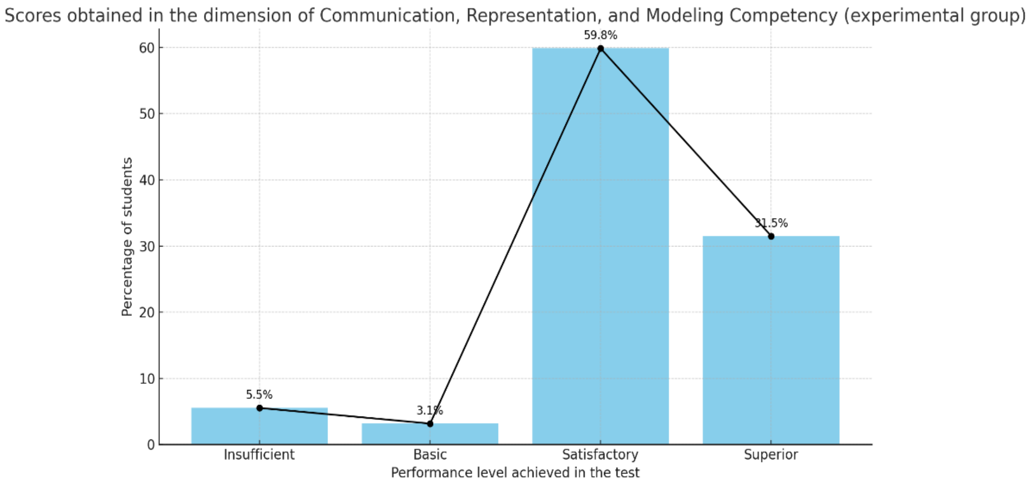

Figure 11. Communication, representation, and modeling competence (experimental group)

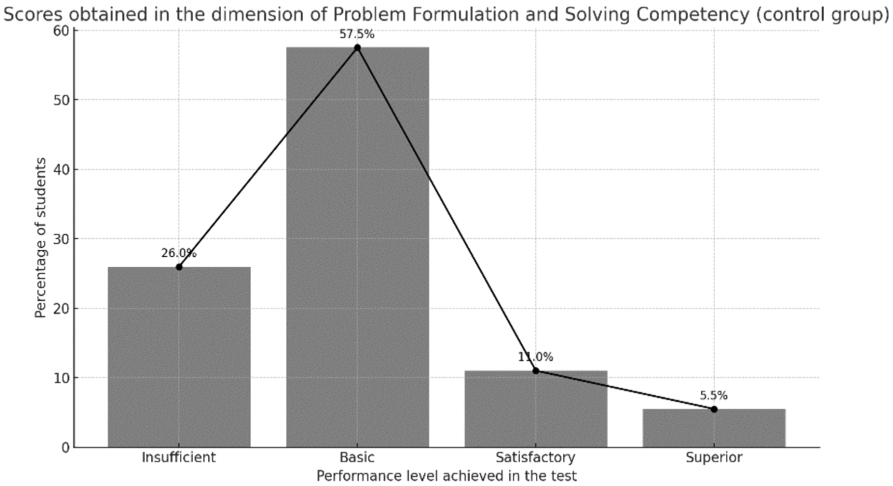

Figure 12. Problem posing and solving competence (control group)

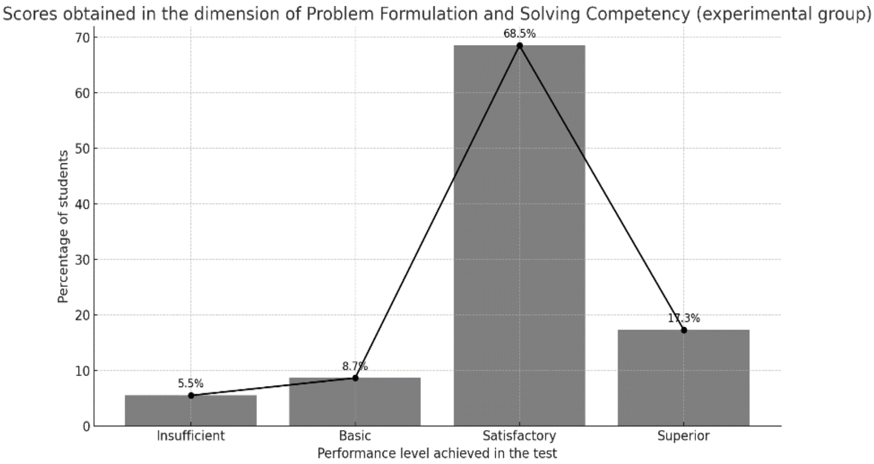

Figure 13. Problem posing and solving competence (experimental group)

3.7. Impact of the treatment on the experimental group

Comparisons were made between the means obtained before and after the intervention in the experimental group in the categories of reasoning and argumentation competence, communication, representation and modeling competence, and problem posing and solving competence using the Wilcoxon test for paired samples with a non-normal distribution as shown in Tables 3 and 4. For this comparison, the average scores obtained by students in the control and experimental groups of the study were considered, where the GeoGebra simulator software was used as a teaching resource in the latter [20], [29]–[31]. The test determined a very low bilateral asymptotic significance (p<0001) in the control group, indicating a statistically significant difference in the performances obtained by the students in this group, before and after the intervention. These results were predictable, given that the students received traditional instruction for the development of knowledge and skills in the thematic axis of trigonometric ratios. On the other hand, in the experimental group as well, a very low bilateral asymptotic significance (p<0001) was found, demonstrating a statistically significant difference between the pretest and the posttest. However, averaging the three competencies or dimensions assessed, students in the experimental group obtained, in the diagnosis or pretest, before the intervention, a performance quantified at 1.34 points (out of a maximum of 5 possible points). In contrast, after the intervention, in the posttest, students achieved a performance quantified at 4.07 points (out of a maximum of 5 possible points), which is higher than the results obtained in the control group.

Table 3. Wilcoxon test (matched samples)

Performance averages	Ranges	N	Average range	Sum of ranges
Average performances in the control group after (posttest)–before intervention (pretest)	Negative ranges	13	5.21	67.73
	Positive ranges	127	18.17	2307.17
	Ties	8	-	-
	Total	148	-	-
Average performances in the experimental group after (posttest)–before intervention (pretest)	Negative ranges	8	3.70	29.63
	Positive ranges	140	18.84	2637.37
	Ties	0	-	-
	Total	148	-	-

Table 4. Statistical summary of pre- and post-intervention performance comparisons

Test statistics	Control group	Experimental group
Z	-8.094	-7.681
Bilateral asymptotic significance	$5.75e^{-16}$	$1.58e^{-14}$

Notes: statistics used are Wilcoxon signed-rank test and results based on negative ranges

Based on the Table 4, the research hypothesis is confirmed, and the null hypothesis is rejected. It is concluded from the empirical findings presented here that the use of the GeoGebra simulator as a didactic resource has significant effects on the level of competence in learning trigonometry (geometric-metric mathematical thinking) in tenth-grade students of a school in the city of Montería. Furthermore, the corroboration of the hypothesis in this research confirms previous research and theories in the field.

In particular, it confirms that the use of digital simulations, such as those produced in GeoGebra, promotes the acquisition of skills in reasoning and argumentation, communication, representation and modeling, and problem posing and solving. This study conducted in a school in the city of Montería with tenth-grade high school students provides a valuable advancement in mathematical pedagogy and educational technology [32]. The research, focused on examining the effect of the GeoGebra didactic tool on the learning of trigonometry in tenth-grade students, has generated significant and promising empirical evidence, contributing notably to the existing scientific literature in this field [33].

The study initially identified insufficient student performance in various critical competencies, including reasoning and argumentation, communication, representation and modeling, and problem posing and solving in the field of trigonometry [15]. After the intervention with GeoGebra, a drastic decrease in insufficient performance in all these competencies was observed. Specifically in reasoning and argumentation, the decrease was recorded from 90.55% and 88.19% in the control and experimental groups, respectively, to 19.7% and 5.5% after the intervention. This result reflects a significant improvement in both groups, with particularly more pronounced changes in the experimental group that incorporated GeoGebra into its methodology. This improvement was quantified in an increase in performance in the competencies evaluated in the experimental group, going from 1.34 points in the preliminary test to 4.07 points in the final test, an increase that far exceeded that achieved by the control group. These quantitative findings support the hypothesis that the implementation of GeoGebra has a positive and significant impact on trigonometry learning and the development of mathematical skills, aligning with sustainable development goals 4 and 9, which promote quality education and innovation in infrastructure, respectively [34], [35].

Regarding the competency of communication, representation, and modeling, the rates of insufficient performance decreased from 88.6% and 85.7% to 20% and 5.7% in the control and experimental groups, respectively. Again, the improvement was more notable in the experimental group, with an increase of 80% compared to 68.6% in the control group. In a similar trend, in the competency of problem posing and solving, the percentages of insufficient performance reduced from 82.9% and 80% to 25.7% and 5.7% in the control and experimental groups, respectively. Once more, the improvement was more significant in the experimental group, experiencing an increase of 74.3%, compared to 57.2% in the control group. The advanced performance after the intervention was also significantly higher in the experimental group in all evaluated competencies. The average in the evaluated competencies in the experimental group significantly increased, going from 1.34 points in the preliminary test to 4.07 points in the final test, an increase of 203.7%, an improvement that far exceeded that achieved by the control group [36]. These quantitative findings support the hypothesis that the implementation of GeoGebra has a positive and significant impact on trigonometry learning and the development of mathematical skills. It is also crucial to recognize that, although the traditional teaching method produced improvements, these were notably less than those observed in the experimental group.

These results not only provide solid empirical evidence that GeoGebra can significantly improve mathematical skills in trigonometry learning, but also expand our understanding of how digital technologies can be effectively implemented in mathematics education. Therefore, this research significantly contributes to the body of scientific knowledge in the fields of mathematical pedagogy, educational technology, and the application of teaching and learning strategies [37], [38]. The educational community is urged to incorporate tools like GeoGebra in mathematics teaching, and the research community is encouraged to continue exploring the potential of digital tools to enrich mathematics teaching and learning.

The study on the integration of GeoGebra in trigonometry teaching yields significant results with multifaceted implications. From a practical perspective, the inclusion of technological tools in mathematics teaching suggests a positive impact on the development of students' mathematical competencies [17]. These findings have important implications for educational practice, suggesting that educators consider incorporating digital technologies in their teaching methodologies to promote deeper and more effective learning [39]. From an educational policy, these results support investment in technological resources and teacher training in their use, which is fundamental for the improvement of mathematics education [40].

Theoretically, this study contributes to existing knowledge in mathematics teaching and learning, providing empirical evidence of the value of digital tools in the development of mathematical competencies. This approach supports and expands existing theories on mathematical learning and technology integration in education [41]. Methodologically, the use of an experimental design with control and experimental groups provides a robust model for future research in this field, highlighting the importance of comparative methods in teaching [17], [39]. This study demonstrates the effectiveness of GeoGebra as a didactic tool in trigonometry teaching and provides valuable guidance for educators and educational policymakers in improving mathematics teaching through the integration of digital technologies. Future research is recommended to continue exploring the impact of various technological tools in different areas of mathematics and in various educational contexts.

4. CONCLUSION

The current study on the implementation of GeoGebra in trigonometry teaching offers revealing insights with profound implications in the field of mathematical education. The results highlight a substantial improvement in students' mathematical competencies, particularly in reasoning and argumentation, where a decrease in insufficient performance from 88.19% to 5.5% in the experimental group was observed following the adoption of GeoGebra. This advancement underscores the effectiveness of integrating digital tools in teaching, highlighting the need for a more interactive and technologically advanced mathematical education.

This study reinforces the idea that investment in technological resources and teacher training are essential to achieve the United Nations sustainable development goals (SDGs) 4 and 9, advocating for quality education and innovation in infrastructure. Theoretically, the research expands knowledge in the field of mathematics teaching and learning, emphasizing the significant role of digital tools in the development of fundamental mathematical skills. The integration of GeoGebra in trigonometry teaching has not only improved students' mathematical skills but has also proven to be an effective model for modernizing mathematics teaching in the 21st century. This approach contributes not only to a deeper understanding of how technology can enrich the teaching and learning process in mathematics but also opens pathways for future research. The educational community is encouraged to explore the impact of various technological tools in different areas of mathematics and in varied educational contexts, which could reveal innovative strategies to address contemporary educational challenges and promote a more comprehensive education, tailored to the needs of today's world.

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