International Journal of Evaluation and Research in Education (IJERE)
Vol. 14, No. 1, February 2025, pp. 229~239
ISSN: 2252-8822, DOI: 10.11591/ijere.v14i1.28342

Students' metacognitive activities in contextual mathematical problem solving

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Article Info

Article history:

Received Aug 15, 2023
Revised Aug 11, 2024
Accepted Sep 26, 2024

Keywords:

Contextual mathematical
Metacognitive
Metacognitive awareness
Metacognitive evaluation
Metacognitive regulation

ABSTRACT

Metacognition is essential in supporting students' academic success, problem-solving abilities, and overall achievement. Therefore, this study aimed to provide a detailed description of metacognitive activities of students when engaging in contextual mathematical problem-solving. The participants comprised 11th-grade students from SMA Negeri 3 Jayapura, Papua Province, Indonesia. Descriptive and qualitative methods were adopted, and the data were collected using various test instruments and interview guidelines. Subsequently, the data were analyzed through the phases of reduction, presentation, as well as conclusions, and triangulation methods were used to ensure robustness and reliability. The results showed that metacognitive awareness occurred when students considered previous knowledge and experiences in solving contextual mathematical problem. Metacognitive evaluation activities occurred when relevant methods and steps were assessed, while metacognitive regulation manifested when contemplating the actions required to solve problem. This showed that students used various metacognitive activities, namely awareness, evaluation, and regulation, to address contextual mathematical problem.

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1. INTRODUCTION

Metacognition is essential in supporting students' academic success, problem-solving abilities, and overall achievement. Studies have consistently shown that students with excellent metacognition tended to use effective problem-solving strategies [1]–[3]. The awareness of this skill significantly enhanced information processing within long-term memory, and also facilitated information, perception, and memory [4]–[6]. A meta-analysis on metacognition indicated that when awareness occurred in an individual, it could remain relatively unchanged throughout developmental stages and into elderly years [7]. Therefore, this cognitive construction was strongly correlated to individual intelligence [6].

Several studies have confirmed the crucial role of metacognition in problem-solving skills and general intelligence [7]. For instance, Berardi-Coletta *et al.* [8] found that people with higher metacognition exhibited significantly better problem-solving strategies in training and task transfer compared to those with lower abilities. Rickey and Stacey [9] also suggested that the enhancement of metacognition improved problem-solving abilities. Cikrici and Odaci [10] described metacognition as a crucial component for advanced mental performance and effective learning process.

Metacognition can be assessed in students during problem-solving processes using specific indicators. Schraw and Dennison [11] developed metacognitive awareness inventory (MAI), which

comprised metacognitive components, namely declarative knowledge, procedural knowledge, conditional knowledge, planning, information management strategies, monitoring understanding, debugging methods, and learning evaluation. Wilson and Clarke [12] also formulated three metacognitive activities, namely awareness, evaluation, and regulation, in solving mathematical problem.

Several improvements in mathematical education mainly focus on mathematical reasoning, problem-solving skills, and their applicability in real-life situations. The use of real-life cases or contexts in mathematical learning also aids students in comprehending abstract concepts by constructing their thoughts from familiar and realistic situations [13]. Context is defined as a recognizable situation that captures students attention [14]. This situation can either be imaginary or authentic, aiding students to recall the existing knowledge acquired through experience, such as informal problem-solving procedures. It can also foster a meaningful learning process [13], as appropriate learning context guides toward active thinking processes [15].

The use of contextual math problem is essential for enabling students to develop more complex thinking patterns, as they engage both formal and informal mathematical knowledge. Furthermore, by solving contextual mathematical problem, students are encouraged to unleash their full psychological potential, specifically regarding cognitive processes [1]. The objective is to motivate students to understand the subject matter [16], [17], as this enhances their metacognition in solving contextual problem [18], [19]. Problem-solving is a complex process that includes several cognitive operations, such as gathering and selecting information, heuristic, and metacognitive strategies [20]. Metacognition is an important strategy associated with academic achievement and problem-solving ability since it comprises self-examination, cognitive strategies, awareness, and thoughtful planning [21]. Metacognitive-based contextual learning effectively enhances problem-solving and mathematical communication skills, making it a recommended method for teaching mathematical [18].

An empirical study conducted at State Senior High School 3 Jayapura, Papua, Indonesia, suggested that students had poor mathematical problem-solving skills. Furthermore, interviews with teachers showed that students still struggled with daily exercises due to the complexity of understanding the narratives of questions. This result was consistent with Tjalla and Putriyani [22], stating that students generally had poor metacognition and frequently struggled with activities requiring metacognitive awareness, specifically, solving of mathematical problem. Therefore, the current study aimed to develop an alternative method to help students solve mathematical problem, using contextual math questions. The use of contextual questions could enhance the motivation to solve problem and comprehend learning materials. Another objective was to improve students' metacognition and environmental awareness. This was in line with Mumu and Tanujaya [23], showing that the use of environmental math problem relevant to the context of students could improve metacognition and awareness of environmental problem.

Several studies have described the importance of metacognitive awareness in learning mathematical, specifically in mathematical proving [24], informal deductive thinking in geometric exercises [25], [26], literacy [18], [27], [28], problem-solving [29]–[32], positioning and emotion in mathematical activities [10], [33], and enhancing mathematical skills [34], [35]. While these investigations solely described metacognitive processes of students and their relationships with mathematical problem-solving and thinking skills, the current study focused on metacognitive activities in solving contextual mathematical problem. This area of study was relevant as it represented the capacity of students to regulate information processing in their long-term memory. In addition, this skill exhibits relative consistency throughout the developmental trajectory of an individual into advanced age. This study analyzed and described metacognitive activities of high school students in solving contextual mathematical problem. In addition, the indicator proposed by Wilson and Clarke [12], which integrated the Polya problem-solving stages was adopted to evaluate students' metacognition.

2. METHOD

Descriptive and qualitative methods were specifically used in this study to effectively describe students' metacognitive activities in solving contextual mathematical problem. These methods were also used due to the nature of the collected data, which primarily consisted of textual information in the form of sentences [36]. A similar method had been used in some studies on metacognition [14], [37], [38]. The participants comprised eleventh graders from State Senior High School 3 Jayapura, Indonesia. Contextual mathematical problem-solving test was administered to 25 students in the first stage of participant selection, while those who successfully completed the test and provided think-aloud responses were chosen in the second stage. The three students who fulfilled these criteria were identified as PK, VP, and EDM. Other studies with similar objective and sample size were [39]–[43].

Data collection was carried out using test instruments and think-aloud methods, as well as interview guidelines, which were validated by mathematical expert at the State University of Malang. The validation results showed that the instrument was suitable for achieving the study objectives. Interviews were conducted

to clarify the results of the written test and capture metacognitive thoughts or processes of students. The interviews were also carried out to confirm the results of the think-aloud process, which had been video recorded. This method was adopted because not all metacognitive indicators were evident in the written responses, and some thinking processes could not be adequately transcribed into an answer sheet. The context of mathematical problem is shown in Figure 1.

Beto and Boas went shopping at the Jayapura Mall, and were interested in the clothes brands A and B. Beto bought two brand A clothes and three brand B clothes for IDR 900,000. Boas bought three brand A clothes and two brand B clothes for 850,000.

a. Without calculating the actual price, which brand is more expensive? Describe your reasons.
b. How much does one brand A shirt cost? How do you calculate it?

Figure 1. Contextual mathematical exercise

The data analysis adhered to the model outlined by Miles *et al.* [44], and was conducted interactively and continuously until data saturation was achieved. The analysis was completed through three stages, namely data reduction, show, and verification or conclusion drawing. A triangulation method was used to ensure data credibility and validity assessment. Source triangulation was specifically adopted, wherein the reliability of information obtained from observations and interviews was compared and verified.

The analysis commenced with the distribution of tests to the participants, who were instructed to engage in think-aloud process. The video recordings of participant metacognition in solving contextual math problem, were subsequently analyzed. The think-aloud results and interviews from each participant were analyzed based on the modified metacognitive indicators proposed by Wilson and Clarke [12]. Table 1 presents the description code for metacognitive activities in solving contextual mathematical problem.

Table 1. Description code for students' metacognitive activities in solving contextual mathematical problem

Metacognitive activities	Wilson and Clarke [12]	Indicators	Code
Metacognitive awareness (AM)	Activities related to individual awareness during problem-solving process using specific content knowledge and the knowledge about personal learning or problem-solving strategies. This includes what needs to be done, what has been done, and what should be done in solving problem.	1. Thinking about knowledge related to the provided problem	AM1
		2. Thinking about past experiences similar to the provided problem	AM2
		3. Thinking about the strategies to solve problem	AM3
		4. Thinking about the possible alternative to solve problem	AM4
Metacognitive evaluation (EM)	Activities related to individual judgments about the processes, capacities, and limitations of thinking when solving problem. For example, judgments about the effectiveness of thinking or strategy can be formulated.	1. Thinking about the possible steps to solve problem	EM1
		2. Thinking and examining the steps of solving mathematical problem	EM2
		3. Thinking of the appropriate problem-solving strategy	EM3
		4. Thinking of the successful possibility to solve problem	EM4
Metacognitive regulation (RM)	Occurs when metacognition is used in directing knowledge and thoughts.	1. Formulating the plan for solving mathematical problem	RM1
		2. Thinking of adopting a different mathematical problem-solving strategy	RM2
		3. Thinking about the next strategy to solve mathematical problem	RM3
		4. Thinking of adopting different strategies to solve mathematical problem	RM4

3. RESULTS AND DISCUSSION

3.1. Results

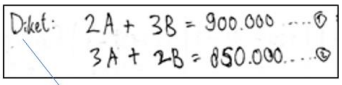
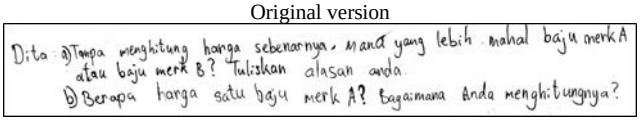
The description of students' metacognitive process in solving contextual mathematical problem is presented in the following section. Three students identified as PK, VP, and EDM were extracted to describe the students' metacognitive activity process. The data on metacognitive activities were obtained using think-aloud method, in conjunction with written answers and interview test results.

3.1.1. Metacognitive activities of the first participant (PK)

The results of the think-aloud session and written tests on contextual mathematical problem suggested that PK exhibited metacognitive awareness in solving problem. Metacognitive process included

the identification of information from the provided problem (AM1), reflecting on past experiences (AM2), and formulating a strategy to solve problem (AM3). In AM1, PK recorded essential information on worksheet, showed an awareness of relevant information for solving contextual problem, and was capable of communicating using mathematical symbols. During AM2, PK recalled prior experiences in solving similar situations, and also formulated a strategy, particularly substitution and elimination methods for AM3. Excerpts from the think-aloud process are presented in Table 2.

Table 2. Utterance/behavior of PK in the aspect of metacognitive awareness

Utterance/behavior	Written answer
(Examine the question paper, read the question and contemplate) Ohhh… yes, I remember how to solve this problem (AM2). This should be exemplified first and transformed into an equation (AM3). (Examine the question paper and provide an answer) The first equation is for Beto who bought two clothes, brand A, and three clothes, brand B with a price IDR 900.000, should be $2A + 3B = 900000$. Meanwhile the second equation is for Boas who bought three clothes brand A and two clothes brand B at IDR 850.000, so $3A + 2B = 850000$ (AM1). (Solve and write answer on the worksheet) The questions are: (a) Which is more expensive without calculating the actual price between brand A and B clothes? Write down your reasons, and (b) how much does one shirt from brand A cost? How do you calculate it? (AM1)	 Known Original version  Translated version Asked: a) Without calculating the actual price, which brand is more expensive? Describe your reasons. b) How much does one brand A shirt cost? How do you calculate it?

Similar data were obtained during the interview, where PK emphasized the importance of relevant information in solving contextual problem and transcribing into worksheet. PK further showed an ability to represent information using mathematical symbols. Excerpts from the interview, focusing on metacognitive awareness, are presented:

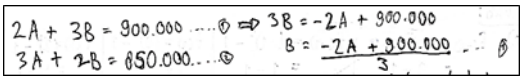
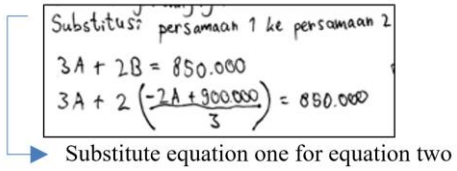
P : *Why do you search for your existing knowledge relevant to problem?*
PK : *Before solving problem, I have to understand the information provided in the question item (AM1). For example, the ways to transform the clothing brands purchased by Beto and Boas into an equation.*
P : *What have you done to understand the question item?*
PK : *I rewrite the information provided within the question item on my worksheet.*
P : *What is mathematical model for problem?*
PK : *I equate clothes from brand A and clothes from brand B, and transform them into an equation (AM3)*

Based on the written answers and think-aloud process, metacognitive regulation of PK in solving contextual mathematical problem included the construction of problem-solving plan (RM1) and the formulation of additional strategy (RM3). During RM1, PK used the substitution method to resolve problem, modified the original equation, and integrated it with the second equation, in RM3. Excerpts from the think-aloud process are presented in Table 3.

A similar conclusion was drawn from the interview results with PK, who explicitly mentioned that the question could be resolved using the substitution method as part of metacognitive regulation. PK altered the initial equation, transforming it into a new mathematical expression, which was subsequently incorporated into the second equation. The interview excerpt is presented:

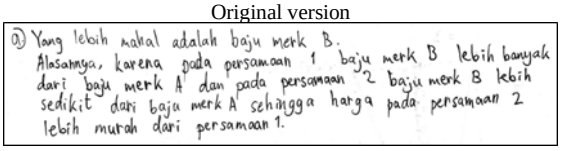
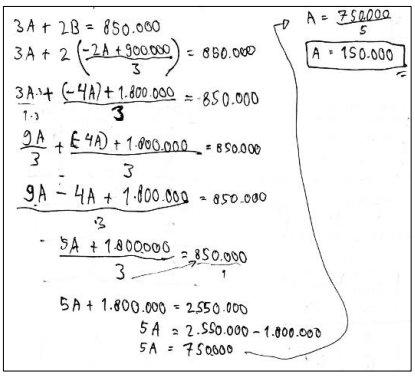
P : *How did you plan to solve this problem?*
PK : *I decided to use substitution method after comprehending problem.*
P : *How did you solve this problem?*
PK : *I integrated equation one with equation two to obtain A grade.*

Table 3. Utterance/behavior of PK in metacognitive regulation aspect

Utterance/behavior	Written answer
(Examine the worksheet) From the known equation, this can be solved using the substitution method (RM1) (Look at the worksheets and write) Therefore, equation one is changed to $3B = -2A + 900.000$ $B = \frac{-2A+900.000}{3}$ and substituted into equation two (RM2). (Write on the worksheet) Substitute equation one with equation two (RM2). (write)	-   Substitute equation one for equation two

The results of think-aloud sessions and the written answer for contextual mathematical problem suggested that metacognitive evaluation by PK was characterized by deliberation on the steps to solve problem (EM1), contemplation and verification (EM2), and reflection on the accuracy of problem solution (EM3). During EM1, PK speculated the substitution method to solve question (b), and elaborated question (a) by explaining why clothing brand B was more expensive, as well as the rationale behind the answer. During both EM2 and EM3, PK operated and integrated the newly formulated equation into equation two. Subsequently, PK performed estimations until value A was obtained. The examples of the utterances or behaviors during the think-aloud sessions are summarized in Table 4.

Table 4. Statement/behavior of PK in metacognitive evaluation aspect

Statement/behavior	Written answer
(Examine and write the answer) Question (a) Which clothes are more expensive…Hmhmhm. The ones that are more expensive are clothes from brand B. The reason is that in equation one, brand B clothes are purchased more than brand A clothes. Meanwhile, in equation two, fewer brand B clothes are purchased than brand A clothes. Therefore, the price obtained in equation two is cheaper than in equation one (EM1).	Original version  a) Yang lebih mahal adalah baju merk B. Alasannya, karena pada persamaan 1 baju merk B lebih banyak dari baju merk A dan pada persamaan 2 baju merk B lebih sedikit dari baju merk A sehingga harga pada persamaan 2 lebih murah dari persamaan 1. Translate version a) The ones that are more expensive are clothes from brand B. The reason is that in equation one, brand B clothes are purchased more than brand A clothes. Meanwhile, in equation two, fewer brand B clothes are purchased than brand A clothes, so the price obtained in equation two is cheaper than in equation one
(Writing on the worksheet) $3A + 2B = 850.000$ $3A + 2\left(\frac{-2A + 900.000}{3}\right) = 850.000$ $3A + \frac{(-4A) + 1.800.000}{3} = 850.000$ $\frac{9A}{3} + \frac{(-4A) + 1.800.000}{3} = 850.000$ $\frac{9A - 4A + 1.800.000}{3} = 850.000$ $\frac{5A + 1.800.000}{3} = 850.000$ $5A + 1.800.000 = 2.550.000$ $5A = 2.550.000 - 1.800.000$ $5A = 750.000$ $A = \frac{750.000}{5}$ $A = 150.000$ (EM2 and EM3)	

Similar data were obtained from the interview results. In metacognitive evaluation, PK mentioned re-examining problem-solving steps and the final outcome, while also verifying the answers for questions (a) and (b). The interview excerpt is presented:

P : *How did you check your work?*
PK : *I re-examined my problem-solving steps and the results. I related my answer with question (a) and decided that shirt B was more expensive (EM1).*
P : *Were you sure about your answer and the stages of problem-solving?*
PK : *Yes, I'm sure about my answers and steps for solving problem (EM2, EM3)*

3.1.2. Metacognitive activities of second participants (VP)

Based on the results of the think-aloud process and the written answers for contextual mathematical problem, metacognitive awareness by VP in solving contextual mathematical problem was characterized by reflecting on past experiences in solving similar problem (AM2) and contemplating the necessary steps to address the current issue (AM3). During AM2, VP recalled encountering a similar experience when solving analogous problem, while deliberating on the current course of action and deciding to use the elimination method, during AM3. The interview excerpt is presented:

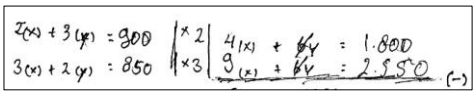
VP : *(Examine question paper, reads the question, and thinks). Hmh (thinking)(AM2), this problem could be solved using substitution or elimination method (AM3)*

Similar data were obtained from the interview results. In terms of metacognitive awareness, VP recalled past experiences in solving similar problem and attempted to apply the acquired knowledge. The interview excerpt is presented:

P : *Why did you ruminate on problem you are currently facing?*
VP : *To solve problem easily*
P : *What did you do to understand problem given?*
VP : *I remember working on a similar problem using elimination or substitution (AM2).*
P : *What is mathematical model for problem?*
VP : *I take, for example, brand A and brand B clothes, and turn them into an equation (AM3).*

In solving contextual mathematical problem, metacognitive regulation of VP included formulating a plan for solving mathematical problem (RM1) and contemplating the subsequent steps in mathematical problem-solving process (RM3). During RM1, VP deliberated on using the elimination method, and multiplied both equations, specifically equation one by two, and equation two by three, in RM3. This resulted in $4x + 6y = 1800$ (equation one) and $9x + 6y = 2550$ (equation two). Subsequently, the equations were manipulated to identify the value of x. Instances of subject utterances or behaviors through the think-aloud method are presented in Table 5.

Table 5. Statement/behavior of VP in metacognitive regulation

Statement/Behavior	Written answer
(Examine the worksheet)	-
With the provided equations, this problem can be solved using elimination method (RM1)	
(Examine the worksheet and write) $2x + 3y = 900 \mid \times 2 \mid 4x + 6y = 1.800$ $3x + 2y = 850 \mid \times 3 \mid 9x + 6y = 2.550$ (RM3)	

Similar data were obtained from the interview results. In terms of metacognitive regulation, VP mentioned using elimination method and manipulating the two equations provided. The interview excerpt is presented:

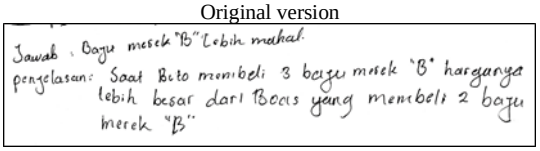
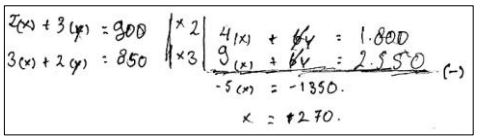
P : *How did you plan to solve this problem?*
VP : *I used elimination method after remembering how to solve problem (RM1).*
P : *How did you solve this problem?*
VP : *I switched the two equations in order to eliminate one variable (RM3).*

The think-aloud and written results for contextual mathematical problem showed that metacognitive evaluation of VP was characterized by contemplating the steps for solving problem (EM1), considering and reviewing problem-solving phases (EM2), and speculating the feasibility of completing those phases (EM4).

During EM1, VP considered using elimination method to solve question (b), and expounded on question (a) by explaining why clothing brand B was more expensive, along with the rationale behind the answer. Excerpts of utterances or behaviors during the think-aloud session are presented in Table 6. VP acknowledged experiencing hesitation in problem-solving process and verifying answers for both question (a) and question (b). The excerpt from the interview in the dimension of metacognitive evaluation is:

P : *How did you check your work?*
VP : *I had doubts about my answer, but I'm sure with the steps (EM1).*
P : *Were you sure about your answer and problem-solving procedures?*
VP : *No, I just realized that my answer was wrong.*

Table 6. Utterance/behavior of VP in metacognitive evaluation aspect

Statement/behavior	Written answer
(Examine the question item and write) for (a) the clothes from brand B is more expensive. Description, when buying 3 clothes from brand B, Beto paid higher than Boas, who bought two clothes from the same brand (EM1).	Original version  Translate version Answer: Brand B clothes are more expensive Explanation: when Beto buys 3 shirts of brand B, the price is greater than Boas who buys 2 shirts of brand B.
(Write on the worksheet) $2x + 3y = 900 \mid\times 2 \mid 4x + 6y = 1.800$ $3x + 2y = 850 \mid\times 3 \mid 9x + 6y = 2.550$ $-5x = -1350$ $x = 270$	
(EM2) (Seems to contemplate) Is this the right answer?	-

3.1.3. Metacognitive activities of the third subject (EDM)

The think-aloud and the written answer for solving contextual mathematical problem suggested that metacognitive awareness of EDM included recalling past experiences in solving similar problem (AM2) and contemplating the necessary steps to address the current problem (AM3). During AM2, EDM recollected encountering a similar experience when solving analogues problem, and also considered resolving using elimination method during AM3. The interview excerpt is presented:

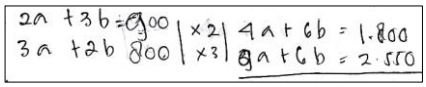
EDM: *(Look at the question item, read the questions, and seems thinking). It seems that I can use elimination (AM3). I've been doing this before… (AM2)*

Similar data were obtained from the interview results. In terms of metacognitive awareness, EDM mentioned recalling experiences in addressing analogous problem and attempting to apply the acquired knowledge. The interview excerpt is presented:

P : *Why did you ruminate on problem you are currently facing?*
EDM: *I had worked on similar questions before. While reading the questions, I immediately remembered I could use elimination or substitution (AM2, AM3).*
P : *What had you done to understand problem given?*
EDM: *I immediately wrote down the equation from problem.*
P : *What was mathematical model of problem?*
EDM: *I take, for example, brand A and brand B clothes and turn them into an equation (AM3).*

The think-aloud and written results for contextual mathematical problem showed that metacognitive regulation of EDM was characterized by formulating a plan (RM1) and contemplating subsequent steps in problem-solving process (RM3). During RM1, EDM considered using elimination method, while multiplying both equations, specifically equation one by two, and equation two by three, during RM3. This resulted in $4a + 6b = 1800$ (equation one) and $9a + 6b = 2550$ (equation two), and the equations were subsequently manipulated to derive answer. The utterances or behaviors of EDM during the think-aloud sessions are presented in Table 7.

Table 7. Utterance/ behavior of EDM in metacognitive regulation aspect

Utterance/behavior	Written answer
(See worksheet) The two provided equations allow the use of elimination method, making it an easy task (RM1)	-
(Examine the worksheet and write) $2a + 3b = 900 \mid \times 2 \mid 4a + 6b = 1.800$ $3a + 2b = 850 \mid \times 3 \mid 9a + 6b = 2.550$ (RM3)	

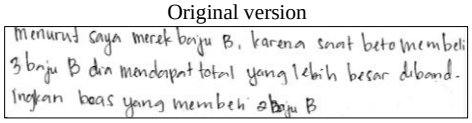
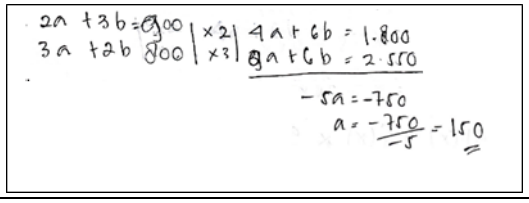
Concordant data were obtained from the interview results, wherein EDM mentioned the potential use of elimination method to solve problem, while manipulating both equations formed. The interview excerpt is presented:

P : *How did you plan to solve this problem?*
EDM: *I used the elimination method (RM1).*
P : *How did you complete the answer?*
EDM: *I substituted the two equations to eliminate one variable (RM3).*

The think-aloud results and the written answers for contextual mathematical problem showed that metacognitive evaluation by EDM was characterized by contemplating the steps to solve problem (EM1), considering and reviewing the steps (EM2), and evaluating the validity of the solution (EM3). During EM1 stage, EDM deliberated on using elimination method to answer question (b), elaborated on question (a), and explained that clothing brand B was more expensive. The interview excerpt is presented in Table 8. Similar data were obtained from the interview results, wherein EDM expressed instances of uncertainty regarding the provided answers and problem-solving stages, while verifying the answers for questions (a) and (b).

P : *How did you check your work?*
VP : *I was confident in my work and had checked it (EM1).*
P : *Were you sure about your answer and the actions you took in solving problem?*
VP : *Yes (EM3)*

Table 8. Statement/behavior of EDM in metacognitive evaluation aspect

Statement/behavior	Written answer
(Examine and write) I think it is brand B clothes because when Beto bought 3 B clothes, he paid higher than Boas, who bought 2 clothes from the same brand (EM1).	Original version  Translate version I think it's shirt brand B, because when Beto buys 3 B shirts, he gets a bigger total than Boas who buys 2 B shirts.
(Write on the worksheet) $2a + 3b = 900\ (\times 2) \rightarrow 4a + 6b = 1800$ $3a + 2b = 800\ (\times 3) \rightarrow 9a + 6b = 2550$ $-5a = -750$ $a = \frac{-750}{-5}$ $= 150$ Therefore, cloth from brand A=IDR 150.000 (EM2)	

3.2. Discussion

The analysis showed that all participants engaged in metacognitive activities, namely awareness, evaluation, and regulation. metacognitive awareness occurred when students recognized that their prior knowledge could assist in solving problem. This was in line with Wilson [12], stating that knowledge related to problem constituted metacognitive awareness. Rofii *et al.* [25] also described the role of metacognitive awareness during planning process to enhance knowledge and the effect on decision-making when selecting or identifying alternative strategies. Furthermore, Arum *et al.* [29] identified components of metacognitive awareness, including knowledge about cognition and cognitive regulation that impact the resolution of mathematical problem among students.

The current study showed that metacognitive regulation occurred when the participants decided to reconsider the steps repeatedly used before attempting different methods. Purnomo *et al.* [45] asserted that the repeated checking of answers before reaching a conclusion was a characteristic of metacognitive regulation activities. Metacognitive regulation refers to the mental activities used in managing cognitive strategies in problem-solving [46]. This is specifically noted when participants alter problem-solving method, while recording this alteration on the worksheet includes cognitive activities.

The results showed that metacognitive evaluation occurred when students judge their answers to be correct after repeated checking. This was consistent with Magiera and Zawojewski [47], suggesting that repetitive checks before making judgments reflected metacognitive evaluation. It can also occur when judgements are made about the effectiveness of cognition or strategy used [5], [12].

Based on the explanation above, students engaged in metacognitive activities in the forms of awareness, evaluation and regulation when solving contextual math problem. Metacognition plays an important role in influencing the ability to solve problem, hence students who are aware of their metacognition tend to use effective strategies [1], [9], [10], [12]. It is also related to the ability of students to organize or evaluate thoughts, since they can control information processing [4], [5], [7]. The incorporation of metacognitive activities into contextual mathematical problem solving can aid in identifying and rectifying problem [29], [30], [33]. In addition, the integration of mathematical problem-solving with the environment can enhance metacognition as well as the awareness of environmental issues [23]. Metacognitive-based contextual learning typically improves problem-solving and mathematical communication skills, making it a recommended method for teaching mathematical [18].

4. CONCLUSION

In conclusion, metacognitive activities experienced by participants in solving contextual mathematical problem included metacognitive awareness, evaluation, and regulation. These three activities occurred alternately with varying frequencies. The results showed that metacognitive process encouraged students to engage more frequently in evaluating problem-solving processes. This indicated the importance of training students to use metacognition when solving mathematical problem.

Following the conclusions, several recommendations were proposed to benefit educators and educational practitioners. Firstly, the incorporation of metacognitive activities in problem-solving was crucial as it instilled a habit of effectively engaging and managing knowledge. Secondly, students who were adept at metacognition developed thinking patterns that were more critical, active, creative, and well-controlled when encountering problem. Thirdly, the awareness of metacognition enhanced problem-solving skills. Fourthly, the development or adaptation of mathematical problem into contextual forms was necessary to elevate both mathematical problem-solving abilities and metacognition.

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