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The influence of student's mathematical beliefs on metacognitive skills in solving mathematical problem

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ABSTRACT

The current research aimed to understand the effect of mathematical beliefs of middle school students on their metacognitive skills in solving mathematical problems. In examining the matter, the study utilized a mixed method. In the first step, a linear regression test was utilized to determine the effect of belief on students' metacognitive skills in solving geometry problems. Furthermore, a qualitative approach was used to compare the metacognitive skills of high and low-belief students. This study involved 72 middle school students sitting in the 8th grade at Tarakan 1 State Junior High School. Based on the linear regression results, it is known that students' beliefs positively influenced their metacognitive skills in solving geometric problems. Furthermore, it was found that when both selected subjects with high and low beliefs started solving the problems, they started by planning. Then, they monitored what they had done, but there were differences in evaluating the solutions. Additionally, students who believe strongly in problem solving will be more aware of what they are thinking and thus have an impact on improving their learning outcomes.

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1. INTRODUCTION

One of the main goals of the mathematics learning process is for students to be able to solve problems. However, there are still many students who experience difficulties and feel anxious when learning mathematical or solving mathematical problems [1]. Many things can affect students in solving problems, for example working memory capacity [1], [2], cognitive awareness [3], belief in solving mathematical problems [4], fear of mathematics [5], and strategic learning [6].

Cognitive awareness and understanding of the problems encountered are required by students when solving mathematical problems. Through awareness, students can improve their skills [7]. However, the results of a study by Suryaningtyas and Setyaningrum [8] indicate that students did not maximize their awareness when solving mathematical problems. The principles of metacognition are the importance of being aware of knowledge and knowing what to do to solve a problem. Metacognition also positively influences the problem-solving process [9] and development of geometric thinking [10]. However, most of the difficulties in solving problems are generally caused by failure to organize processes or mathematical problems, choose the most effective strategies, analyze, understand the essence of the problem, and monitor and control the processes carried out [11]. Therefore, it is imperative to discuss in this research the emergence of problem solving in relation to metacognition.

The concept of metacognition was coined to assert how a person can control their thinking while learning and solving a problem, particularly when experiencing cognitive failure and difficulties in dealing with problem-solving and information processing [12]. Metacognition is also one of three aspects that influence the mathematical problem-solving process [13], [14]. Metacognitive aspects are influential to students' ability to organize their thinking. Lioe *et al.* [15] also found that metacognition is one of the key components in solving mathematical problems, emphasizing the ability of students to monitor their own thinking. This finding is also consistent with the concept of metacognition developed by Flavell [16], which refers to the awareness of students on their own cognitive processes and the regulation of cognitive processes in achieving specific goals. Therefore, students who solve problems well always monitor their thinking and evaluate the results. The students know when to use an appropriate strategy and when to switch strategy in order to be consistent with a specific goal. There are three components of metacognition, which are metacognitive knowledge, metacognitive skills, and metacognitive experience [16], [17].

Metacognitive knowledge is a person's awareness of factors related to cognitive activity [18]. It includes one's knowledge of strategies, tasks, and oneself that influence the direction and outcome of cognitive effort (including declarative, procedural, and conditional knowledge). Metacognitive skills are a person’s awareness of employing strategies with the goal of controlling their cognition. The skills include planning, monitoring, and evaluation. Lastly, metacognitive experience is an awareness of one's feelings when finding a solution to a particular problem and the feeling of receiving information related to the solution received. Metacognitive experiences include feelings experienced in connection with the task at hand.

In addition, metacognitive knowledge influences awareness of thought processes in learning. When awareness is realized, metacognitive skills emerge, where students can control their minds by designing, monitoring, and evaluating what needs to be learned. Suliani *et al.* [19], [20] have found that while differences can be found in the representation of students’ thoughts in problem-solving when viewed from their genders, mathematical ability, cognitive activities are still involved, particularly in the form of planning, monitoring, and evaluation. Based on that, the focus of the present study is on metacognitive skills.

Efforts to increase students’ learning activities, which have an impact on improving learning outcomes, cannot be separated from metacognitive skills which are an important role in learning. In addition, there are other efforts such as students’ confidence in solving problems. The research findings of Ozturk and Guven [21] concluded that beliefs not only influence the process of problem solving, but also influence personal factors such as life experiences. In addition, Ishida [22] reported that belief in solving a problem has an impact on the ability to solve problems correctly.

Students’ beliefs in mathematical problem solving relate to what is believed to be correct, which in this case is related to mathematical problem-solving. The mathematical problem-solving is viewed under six aspects by Kloosterman and Stage [4] including belief in the time needed to solve mathematical problems, the steps needed to solve the problems, the understanding of the found solutions, the understanding that there are multiple ways of solving the problems, the efforts in improving mathematical skills, and the usefulness of mathematics in everyday life. The students’ beliefs have a profound impact on their working memory. Deep beliefs have an impact on what and how students remember the events they experienced. Pajares [23] noted that beliefs are thought to affect perception, and in turn, it affects behavior that is consistent with and reinforces original beliefs. If students like mathematical, they will be enthusiastic as they participate in a mathematical class and when faced with problem situations. They always solve problems in different ways based on their knowledge and learning experience. Students who are unenthusiastic about mathematics, however, seem to be reluctant to take mathematics lessons. When confronted with mathematical problem situations, they hesitate to solve them, thinking that the problems posed are difficult to solve.

Students with different mathematical beliefs may influence their metacognitive knowledge. Characteristics of students with high confidence in solving geometric problems, namely time management, are needed when solving mathematical problems. Meanwhile, students who have low confidence in solving geometric problems do not consider the time required. This means that students with these characteristics when solving a problem tend to give up easily to survive solving the problems they face.

Geometry is itself a challenge for learners, particularly when it is being learned remotely. In addition to space and form, geometry includes the distance, scale, and relative position of figures. Numerous occupations, such as architects, mechanical engineers, technicians, draughtsman, utilize geometry. Geometry is a very essential branch of mathematics. The majority of learners find geometry difficult to study and have no desire to do so. This is due to the fact that learners frequently feel unsure of themselves about what they have learned, experience anxiety when studying it, and are unable to use geometric theory to solve their problems [24].

Middle school students are typically within the age range of 11-15 years. Based on Piaget’s developmental stages, it is at this age that students’ thinking skills are incorporated into the formal action phase. This allows them to have more flexible thinking when it comes to thinking about more possible solutions, especially mathematical problems. In general, research regarding the effects of beliefs on middle school students' metacognitive skills in solving mathematical problems is still very limited [25], [26], although it is important to be explored as it involves students' beliefs about solving mathematical problems to identify their metacognitive skills. For example, when a student is faced with a problem, there are two possible states: he/she is aware and believes in solving the problem, or he/she is aware but not sure if he/she can solve the problem. Furthermore, a phenomenon is produced by a study of mathematics beliefs that examines student beliefs about how long it takes to solve a problem. Students who have a high level of confidence in mathematics are more likely to abandon the process of problem-solving when they experience failure [27].

In relation to the previous studies, this study tries to find the influence of middle school students' mathematical beliefs on their metacognitive skill in mathematical problem-solving and describe how belief mathematical effect on metacognitive skills of middle school students with high and low beliefs. This study is hoped to be useful to determine the appropriate learning methods to increase middle school students' belief in solving mathematical problems. Additionally, students who believe strongly in problem solving will be more aware of what they are thinking and thus have an impact on improving their learning outcomes.

2. RESEARCH METHOD

2.1. Research design

A mixed-method design is utilized to answer the research objectives. The design refers to the collection, analysis, and integration of both qualitative and quantitative data at multiple stages of a research [28]. The sequential explanatory design is one of the approaches in mixed methods. This design goes through two consecutive phases. At the first phase, the researchers collect and analyze quantitative data. Then, at the second phase, qualitative data are collected and analyzed to help explain the previously mentioned quantitative results [29].

In the present study, quantitative research method was used to scrutinize the effect of mathematical belief on middle school students' metacognitive ability to solve mathematical problems. In this case, the task of solving geometry problems is a task situation that measures awareness of students' metacognitive abilities. The score for students' confidence in solving mathematical problems is the total score of the Indiana mathematics belief (IMB) scale questionnaire, and the students' metacognitive ability score is the total score of the questionnaire on metacognitive perception in solving mathematical problems. The linear regression test was performed twice, where the dependent variable was the students' metacognitive abilities, and the independent variable was the students' mathematical beliefs. Aside from seeing the effect of students' beliefs on their metacognitive skill in solving mathematical problems, a qualitative research method was also used in an attempt to analyze the matter deeper. The qualitative research was carried out by selecting six participants. There were three subjects with high mathematical belief and three subjects with low mathematical belief. The subjects were given a task to solve geometry problems.

2.2. Sample and data collection

Stratified random sampling was employed to select the subjects. Sampling is the process of taking samples. The sampling conducted by researchers aims to obtain a representative sample of the population. The sample consisted of 11 groups, in which every group is homogeneous, so from each group, six to seven students were randomly selected. There are 36 students for each group, all within the age of 13-15 years. The total sample in this study is 72 eighth grade students studying at Tarakan 1 State Junior High School. This is in line with Roscoe [30], who stated that an appropriate sample size in research is between 30 and 500. The data collection was carried out in two consecutive phases. The first phase was the quantitative approach, namely using the mathematical belief scale and the metacognition scale of middle school students. After that, the second stage was the qualitative approach, namely observation, documentation, and interviews.

2.3. Instruments and data analysis

The Indiana mathematics belief (IMB) scale was used to measure students' mathematical belief in solving mathematical problems [4]. The IMB questionnaire consists of 36 items, including: six statements regarding the time needed for problem-solving, six statements about steps in problem-solving, six statements about understanding of the found solutions, six statements about different ways of solving problems, six statements about effort to improve mathematical skills, and six statements about the usefulness of mathematics in daily life. This instrument uses a Likert scale with a range of 1 to 5, where a value of 1 is for "strongly disagree", a value of 2 for "disagree", a value of 3 for "unsure", a value of 4 for "agree", and a value of 5 for "strongly agree". The IMB score ranges from 36 to 180. Subjects are placed in the high-belief group if the score is less than 131 and in the low-belief group if the score is equal to or more than 132.

The selected subjects were analyzed to be explored on their metacognition in solving mathematical problems based on considerations that the students had confidence in solving mathematical problems and were willing and able to verbally express their ideas orally and in writing. These considerations made it easier when conducting in-depth interviews to recognize the metacognitive abilities of students with high and low mathematical beliefs using geometry material. The research was conducted in the academic year 2021/2022, specifically in the even semester.

The questionnaire on students' metacognition awareness in solving mathematical problems was used to measure students' metacognition in solving mathematical problems. This questionnaire was from Schraw and Dennison [17] and adapted to fit the characteristics of middle school students for metacognitive skills consisting of planning, monitoring, and evaluation. Based on the results of the validity of each statement item calculated using the correlation formula, out of 18 statements related to middle school students' metacognitive abilities in solving mathematical problems, they were found to be valid and reliable (Cronbach's alpha=0.934). This means that the questionnaire was reliable to be used as a data collection tool.

The mathematical test being used in this study comprised five questions including numbers, algebra, geometry, statistics, and probability. Consisting of various materials considered representative of mathematics, this test was designed for grade 8th students to complete. The mathematical test scores were used to measure the subject's equivalent mathematical ability, with the difference in the outcomes being less than 5 on a scale of 0 to 100. This difference is quite small, so it might be assumed that their mathematical skills were on par. With subjects of equal ability, researchers could avoid the possibility of not receiving metacognitive data as a result of one of the subjects that could not complete the task. The chosen subjects were given problem-solving tasks in the second phase of the study, namely simple geometric problems, but there are no routine procedures for solving them. To accomplish this task, the problem was as: "Firman creates a square photo frame with an outside diagonal of the frame being $80\sqrt{2}$ cm. The subjects were asked to calculate the total length of the timber that Firman would use, and the minimum cost Firman would incur if the price per meter of the timber is Rp40,000." The effect of mathematical belief on metacognition in solving mathematical problems was analyzed from the significance value of the linear regression test, with the dependent variable being the total score on the student's metacognition questionnaire in solving geometric problems. The geometric problem-solving task was analyzed by comparing the results of both groups, namely the high mathematical belief group versus the low mathematical belief group.

3. RESULTS AND DISCUSSION

3.1. The effect of students' beliefs on their metacognitive skills in solving mathematical problems

A total of 72 eighth grade students took part in this study. All subjects completed a series of tests, namely a basic mathematic test, then completed the student confidence in geometric problem-solving questionnaire. After that, they completed the geometric problem-solving task, followed by completing the student geometric problem-solving metacognition questionnaires. The students who took part in this study were 36 males and 36 females. Furthermore, 15 students were classified as having high mathematical belief, consisting of 6 males and 9 females. On the other hand, 57 students were classified as having low mathematical belief (30 males and 27 females). The test results description is presented in Table 1.

Table 1 demonstrates that the average level of beliefs in solving mathematical problems of participants was 125.19 and the mean level of metacognitive skill was 66.86. This exerts that students lack confidence in solving mathematical problems so that it has an impact on their metacognitive skills. The researchers used the SPSS application to test the normality of data on students' metacognitive skills and beliefs when solving mathematical problems. The data used were scores of students' confidence in solving mathematical problems and scores of students' metacognitive ability. The residuals of the regression should be in line with a normal distribution. Furthermore, this condition is met by utilizing a normal predicted probability (P-P) plot. Figure 1 gives information on the normal predicted probability (P-P) plot of the data of belief and metacognitive skill.

Table 1. High/low-belief and metacognitive skill

	Mean	Standard deviation	Minimum	Maximum
High/low belief	125.19	9.015	108	147
Metacognitive skill	66.86	12.065	39	90

It is learned from Figure 1 that the plot of metacognitive skills score points coincide with the diagonal line of normality: this indicates that the normality conditions were met. Furthermore, homoscedasticity, to know whether or not the residuals are equally distributed, is to be found and this condition is able to be known with a scatter plot of the residual. Using scatter plots, Figure 2 presents the residuals for metacognitive skill score.

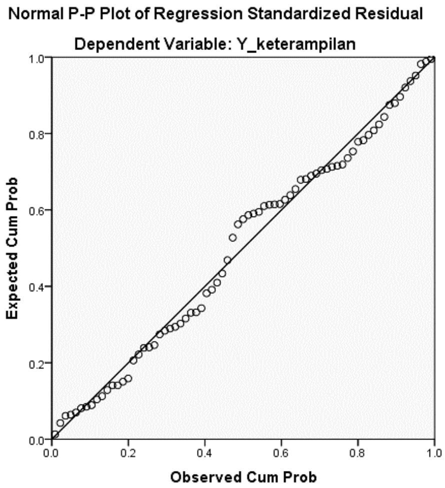


Figure 1. Normal predicted probability plot

Figure 2 shows that special pattern is nowhere to be found. It can be seen that the points are distributed equally both on the X-axis and the Y-axis. Thus, the homoscedasticity requirement was achieved. It is confirmed that the residuals were normally distributed and homoscedastic. Thus, the predictor variables in the regression have a linear relationship with the outcome variable. The linear regression test was used to determine whether there was a significant effect between students' beliefs and metacognitive ability when solving mathematical problems in a linear population.

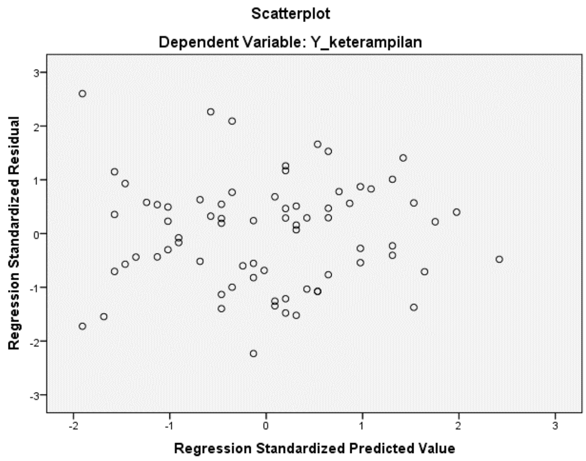


Figure 2. The scatter plots of the residuals

In Table 2, output summary results using SPSS from regression analysis on students' beliefs and metacognition in solving mathematical problems yielded an R-squared value of 0.131, indicating that the effect of mathematical beliefs as the independent variable on the students' metacognition is 13.1%, while the other variables explain the rest. After that, the following step was to determine the regression model, examine the suitability of model, and investigate the variables that affect metacognitive skill. The coefficient and the significant values of the linear regression analysis are shown in Table 3 and Table 4.

Table 2. Regression analysis of students' metacognition and belief in solving mathematical problems

Model	R	R square	Adjusted R square	Std. error of estimation
1	0.362[a]	0.131	0.119	11.325

a. predictors: (constant), X_{belief}

Table 3. ANOVA from the regression analysis of students' metacognition and belief in solving mathematical problems

Model	Sum of squares	df	Root mean square	F	Sig p-value
Relapse	1374.465	1	1374.465	10.717	0.002
Residual value	9106.165	71	128.256		
In total	10480.630	72			

Table 4. Output coefficient of the regression analysis out students' metacognition and belief in solving mathematical problems

Model	Non-standard coefficients		Standardized coefficients	t	Sig p-value
	B	Std error	Beta		
Constanta	6.187	18.582		0.333	0.740
High/low belief	0.485	0.148	0.362	3.274	0.002

Dependent variable: metacognitive skill

Demonstrated by the results of the ANOVA table test as shown in Table 3, it was found that $F = 10.717$ with Sig. $p - value = 0.002 < 0.05$. Therefore, the overall regression model fits the data. The regression model (Table 4) was: $y = 6.187 + 0.485x$, with variable y was defined as the dependent variable, namely students' metacognitive skills in solving mathematical problems, while the variable x was defined as a variable independent, namely the students' belief in solving mathematical problems.

Thus, students' belief in solving mathematical problems affected their metacognitive abilities in solving mathematical problems. That is, if there are students who have high belief in solving mathematical problems, it is predicted that those students will have high metacognitive ability in solving mathematical problems. This means that if students have high confidence in solving mathematical problems, it is predicted that their metacognitive skills will be better at solving mathematical problems, affecting their learning outcomes in mathematics. This is consistent with the research by Setyawati and Indrasari [31] showing that students who are more confident in their mathematical skills use better metacognitive strategies. It can be said that the determinants of students' success in solving mathematical problems depend not only on perceptions of their thought processes, but also on their beliefs in solving mathematical problems. When students have good beliefs, they can better improve their cognitive skills [21], [32]. Consistent with the results of the research by Guven and Belet [25], it was found that belief in learning as an effort rather than a skill raises more metacognitive awareness, and monitors the learning process. The scatterplot in Figure 3 shows that the higher the belief score, the higher the metacognitive ability of middle school students in solving mathematical problems. The two variables have a positive correlation.

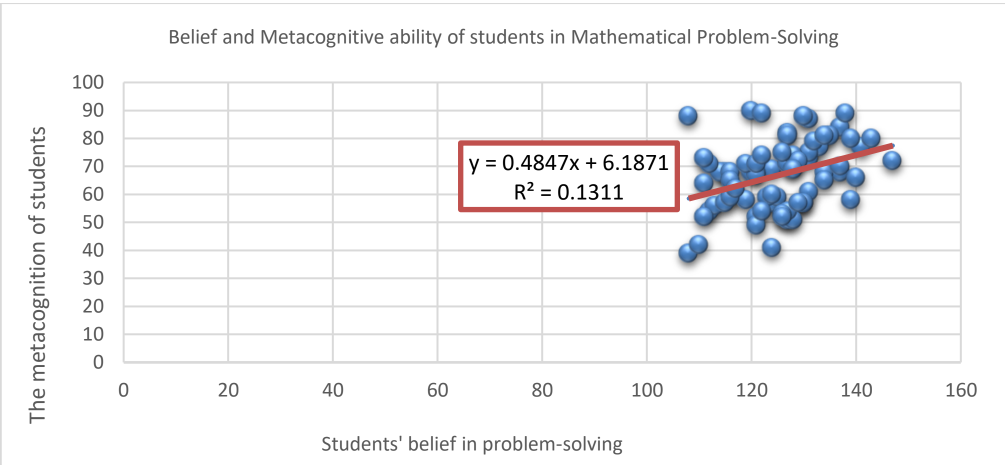


Figure 3. Scatterplot of students' belief and metacognition when solving mathematical problems

To know the effect of these factors on metacognitive abilities, a qualitative study was conducted by assigning six participants mathematical problems. The participants were selected to consist of three subjects with high belief in solving mathematical problems and three subjects with low belief in solving mathematical problems.

From the answers given by the participants, two students were selected, consisting of one high belief individual and one low belief individual, who were identified through targeted sampling as research subjects who have a unique problem-solving strategy and are able to communicate their ideas when solving mathematical problems.

Based on the results of regression analysis, students' beliefs positively and significantly affect their metacognitive abilities, hence it continued by examining and identifying the subject's metacognitive abilities in solving mathematical problems. The subject with high belief category had a score of 138, meaning the subject score in the "high-belief in solving mathematical problems" category ($132 \leq x \leq 180$), while the subject with low-belief category had a score of 108, meaning the subject scored in the "low belief in solving mathematical problems" category ($36 \leq x < 132$). The subject with high belief in problem-solving is referred to as skills, knowledge, and tools (SKT), while the subject with low belief in problem solving is referred to as safe keeping receipt (SKR).

In general, based on the results of analyzing the responses of the two subjects, namely SKT and SKR, they could correctly solve the problem. However, they followed different strategies when implementing the problem-solving. The strategy used by SKT was more systematic in solving problems than SKR. The time required for the two subjects was also different. Furthermore, SKT in problem solving was more systematic in the way it was done compared to SKR. SKT was more aware of his knowledge when faced with a problem, while SKR needed support when faced with a problem.

When solving mathematical problems, SKT and SKR both started by understanding the purpose of the problems at hand, then planned a solution method based on their learning experience, and then apply the prepared plans to find solutions that meet the goals of the given problems after. When they got the expected results, they re-evaluated the steps taken to resolve the issue. This was useful for identifying things that did not align with the goals of the given problem, so they can immediately correct the mistakes they made. However, SKR did not evaluate what had been identified as a solution to the problems. On the other hand, SKT thought about the plan, then monitored, and evaluated what he thought. In contrast, SKR did not evaluate what he had done when solving problems.

3.2. The metacognitive skills of SKT and SKR in solving mathematical problems

The first step SKT took in solving mathematical problems was to understand the problem to find out the purpose of the problem at hand. SKT tried to understand the intent and purpose of the problems by reading in a low voice while pointing to the questions. SKT tried to identify important information and organize it well. SKT also rewrote it to state the problem at hand. In addition, SKT also thought about a plan that would be used to understand the problem by reading it repeatedly to be able to find important information in the questions.

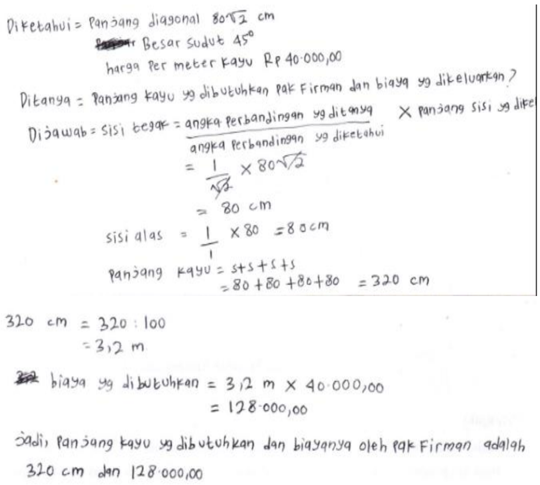
SKT used its knowledge to know what to do to understand the problem and knowledge of the purpose of a particular strategy when the strategy was appropriate and effective to solve the problem at hand. This showed that SKT was aware of the need to set goals before understanding the problem and think about multiple ways to solve the problem as well as choosing the best strategy. SKT retold the question and gave symbols in the form of picture frames to the questions, aimed at helping SKT understood the questions better. Furthermore, in an effort to understand the problems, SKT also considered the material related to the given problem, namely geometry. SKT used certain strategies to be more effective. Based on the results of the SKT interview, SKT could provide reasons for procedures and strategies to understand the problem. SKT began reading, writing, and drawing square shapes, which affected SKT's understanding, in which SKT claimed that these methods were better than the others.

To understand the problem, SKT made a plan by realizing certain formulas that would be used to solve the problem. SKT used the formula for the ratio of the sides of an isosceles triangle. SKT also understood the importance of reading the questions repeatedly to check again if there was any information that was not clear, so that later it would not be difficult to resolve the problems at hand. Additionally, to understand the problem, SKT monitored his understanding by incorporating prior knowledge and experience. SKT also predicted a good time to understand the problem. Before taking the next step in solving the problem, SKT re-evaluated the results.

The next step in solving geometry problems was to plan a solution method based on the subject's learning experience. SKT, in planning for the problem-solving, involved prior knowledge and experience, by linking the known information and the asked problem to the question and choosing the most effective strategy to be able to solve the given problem. In addition, STK made plans by creating a problem-solving plan, then SKT monitored what it had planned to solve the problems. At the end, SKT also re-evaluated the created plan by re-reading the questions and looking back at what he had written in accordance with the given problems. SKT also estimated the time it took to develop a problem resolution plan.

SKT, in solving the problem, first identified the size of the angle on each side of the flat rectangle. Since the length of the outside diagonal of the square was known, the angles formed by the diagonal had a measurement of 45°. Then, two isosceles triangles were formed from a square shape that had one diagonal.

In this case, SKT focused on an isosceles triangle. Therefore, SKT first calculated the perpendicular side of the isosceles triangle. SKT calculated the vertical side from the ratio of the sides in an isosceles right triangle, namely $1 : \sqrt{2} : 1$. SKT explained that the angles formed from the triangle are $90°, 45°$, and $45°$. Therefore, to determine the vertical side by calculating the reference number in question multiplied by the known side length, the result was divided by the known reference number. SKT was therefore aware of using declarative knowledge, namely linking factual information with developed strategies in order to arrive at the right solution. This can be seen in Figure 4, which shows a section of the results of the solution.



Translated.
Is know: diagonal length $80\sqrt{2}$ cm
Big angle $45°$
Price per meter of wood Rp40,000.00
Asked: Length of wood does Mr Firman need and costs incurred?
Answer:

$$upright\ side = \frac{the\ comparison\ figures\ asked\ for}{know\ comparison\ figures} \times know\ side\ lengths$$

$$= \frac{1}{\sqrt{2}} \times 80\sqrt{2} = 80 \text{ cm}$$

$$base\ side = \frac{1}{1} \times 80 = 80 \text{ cm}$$

$$Length\ of\ wood = s + s + s + s$$

$$= 80 + 80 + 80 + 80 = 320 \text{ cm}$$

320 cm = 320 ÷ 100 = 3.2 m
Costs required= 3.2 m × 40,000.00 = 128,000.00
So, the length of wood needed and the cost according to Mr. Firman is 320 cm and Rp128,000.00

Figure 4. The troubleshooting stages provided by SKT

The last stage conducted was to re-examine the results of the obtained solutions. In this phase, SKT re-examined the results by recalculating the answers that were given by looking back at the problem and matching the results of the answers received. This characterized SKT who was aware of using declarative knowledge when re-verifying the obtained solutions. SKT also checked the arithmetic operations used and the units used. SKT believed that the results corresponded to the purposes and goals of the given problems. This shows that SKT was aware of including procedural knowledge and conditional knowledge in the verification of the solutions obtained. This is in line with the results of research by Sutama *et al.* [33], which states that students with high confidence are able to carry out planning steps, make important decisions for themselves, and solve problems well. Therefore, a student with a high belief in solving mathematical problems would have increased awareness related to their metacognitive activities, ranging from planning, monitoring, and evaluating the results achieved.

SKR, in an effort to understand the issue, first identified the important information contained in the problem. To understand the problem, SKR read it over and over again. However, SKR did not rewrite the information on the provided answer sheet. SKR used the knowledge that he had previously associated with the given problem, so that he knew the goals and objectives of the questions asked correctly and could determine the appropriate and effective strategies to solve the problem. Even without rewriting the factual information contained in the question, SKR was still able to complete it well. What SKR identified included the knowledge and learning experience the subject already had regarding the problem.

SKR planned to use the ratio of the sides of an isosceles right triangle. After obtaining the side of the isosceles right triangle, SKR calculated the total length of the timber needed and then calculated the money to spend. Therefore, SKR could incorporate previous knowledge and experience related to geometry material in the subsection of the Pythagorean theorem with a comparison of the sides of an equilateral right triangle. SKR, however, could not determine the time required to develop a problem resolution plan. This identified SKR in an effort to develop a problem-solving plan that involved procedural knowledge, namely realizing the application of knowledge related to the steps in the problem-solving process.

SKR directly wrote the aspect ratio of the isosceles right triangle. SKR had previously created two isosceles right triangles, the first of which was the length of the hypotenuse $80\sqrt{2}$cm, while isosceles right triangle times the length of the hypotenuse was $\sqrt{2}$. SKR compared the sides of the isosceles right triangle the way the upright side was compared to the hypotenuse. Figure 5 shows the results of the troubleshooting provided by SKR. SKR also could not predict how long it would take to solve the mathematical problem using such a solution. This characterized SKR who consciously used procedural and conditional knowledge when performing geometric problem solving.

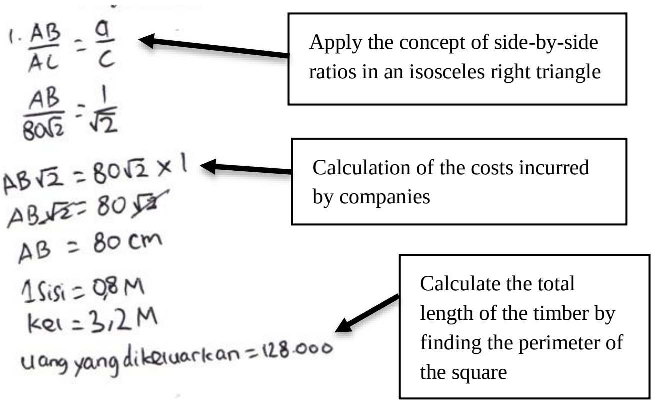


Figure 5. Stages of problem resolution specified by SKR

SKR did not carry out the phase of re-examination of the given solutions. SKR was satisfied that the solution given was correct and therefore considered it unnecessary to re-check the results obtained. This indicated that SKR was not aware of the need of evaluation of the results obtained. This also yielded that a student with low belief in solving mathematical problems might influence awareness related to their metacognitive activities, beginning with planning and monitoring, but not engaging in any activities to re-evaluate the results.

It can be seen that there are differences and similarities in both of the subjects' metacognitive abilities to understand geometric problems, and to determine the solutions of the problem at hand, namely about the planning, the appropriate strategies, and the incorporation of their existing knowledge. The plan utilized by the subjects to understand the problem was by reading the questions carefully. If they had not found the purpose of the questions asked, both subjects decided to read the questions slowly and repeatedly, so that they could find the important information contained in the questions, which in this case related to the information known and the information requested. This is consistent with Tobias and Everson's idea [34] which says that the ability to differentiate the known information from the unknown is prominent for academic success. This is also in line with Woolfolk *et al.* [35] who state that choosing which strategy to use, how to begin, and which one to apply first are included in the planning of metacognitive problem-solving skills.

In addition, the two subjects also observed what they were thinking in order to understand the problem. Both of them recognized important information and determined the formulas for finding the solution. If they understood the problem, they could also ask questions to themselves to reflect on whether the plans laid down were consistent with the results achieved. This finding demonstrates their direct awareness on how to perform cognitive activities. This result is consistent with several studies [35]–[37]. According to the studies, monitoring activity is a direct awareness of a person's cognitive activities which also manifests the dimension of checking progress towards the goals of a problem.

The SKT subject continued to evaluate the found solution. This is different to the SKR subject, who immediately determined the solution from his answer without re-evaluating the found solution. According to Pulmones [37], evaluation activities might be in the form of a renewed review of goal achievement and reflect which problem-solving strategies are more efficient. Furthermore, evaluation is an activity in which a person remembers and reflects on their experiences [38]. A person who has the ability to reflect their thoughts not only understands what he/she knows well, but he/she also has the ability in making conscious decisions and correcting their mistakes.

4. CONCLUSION

In conclusion, students' belief in solving mathematical problems affects their metacognitive skill in solving mathematical problems ($r = 0.36$ and Sig. $p - value = 0.002 < 0.05$). It means that, if there are students with high mathematical belief, it is predicted that those students would have high scores in metacognitive ability in solving mathematical problems. High-belief students use metacognitive problem-solving skills to find the right solution to the problem at hand. Students with low-belief in solving problems do not always involve their metacognitive skills at every stage of solving mathematical problems. However, with every activity student would think about the plans they will carry out and then monitor what they think. Although the two subjects can use different problem-solving strategies to find the right solution, they have the same meaning. Middle school students with high belief in problem solving are aware of factors related to their cognitive activities. Furthermore, these are cognitive and affective experiences that arise consciously in any type of processing task.

In contrast, students with low mathematical belief tend to not evaluate the solutions they find, although they are aware of using the knowledge to implement strategies to solve mathematical problems. Students with low belief also tend to avoid seriousness when solving problems, thereby reducing awareness of their thought processes. This can affect student achievement. The findings and primary outcome could potentially serve as a resource for schoolteachers. Teachers can determine the appropriate learning method to increase junior high school students' belief in solving math problems. In addition, by understanding the effectiveness of metacognitive skills in learning mathematics, teachers can modify instructional strategies so that students can also develop their metacognitive skills for other subjects at school.

REFERENCES

[1] D. Juniati and I. K. Budayasa, "Working memory capacity and mathematics anxiety of mathematics undergraduate students and its effect on mathematics achievement," *Journal for the Education of Gifted Young Scientists*, vol. 8, no. 1, pp. 279–291, 2020, doi: 10.17478/jegys.653518.

[2] D. Anjariyah, D. Juniati, and T. Y. E. Siswono, "How does working memory capacity affect students' mathematical problem solving?" *European Journal of Educational Research*, vol. 11, no. 3, pp. 1427–1439, 2022, doi: 10.12973/eu-jer.11.3.1427.

[3] N. X. Fincham and G. Li, "Metacognitive knowledge and language learning in a web-based distance learning context," in *Research Anthology on Remote Teaching and Learning and the Future of Online Education*, IGI Global, 2022, pp. 449–472. doi: 10.4018/978-1-6684-7540-9.ch023.

[4] P. Kloosterman and F. K. Stage, "Measuring beliefs about mathematical problem solving," *School Science and Mathematics*, vol. 92, no. 3, pp. 109–115, 1992, doi: 10.1111/j.1949-8594.1992.tb12154.x.

[5] D. Juniati and I. K. Budayasa, "The mathematics anxiety: Do prospective math teachers also experience it?" *Journal of Physics: Conference Series*, vol. 1663, no. 1, 2020, doi: 10.1088/1742-6596/1663/1/012032.

[6] D. Juniati and I. K. Budayasa, "Field-based tasks with technology to reduce mathematics anxiety and improve performance," *World Transactions on Engineering and Technology Education*, vol. 19, no. 1, pp. 58–64, 2021.

[7] A. In'am, *Demonstrate the solution to math problems*. Indonesia: Aditya Media Publishing (in Indonesian), 2015.

[8] S. Suryaningtyas and W. Setyaningrum, "Analyzing the metacognitive skills of high school students in the grade xi science program to solve math problems," *Journal of Mathematics Education Research*, vol. 7, no. 1, pp. 74–87, 2020, doi: 10.21831/jrpm.v7i1.16049.

[9] A. E. Kesici, D. Güvercin, and H. Küçükakça, "Metacognition researches in Turkey, Japan and Singapore," *International Journal of Evaluation and Research in Education (IJERE)*, vol. 10, no. 2, pp. 535–544, 2021, doi: 10.11591/ijere.v10i2.20790.

[10] M. A. Naufal, A. H. Abdullah, S. Osman, M. S. Abu, H. Ihsan, and Rondiyah, "Reviewing the Van Hiele model and the application of metacognition on geometric thinking," *International Journal of Evaluation and Research in Education (IJERE)*, vol. 10, no. 2, pp. 597–605, 2021, doi: 10.11591/ijere.v10i2.21185.

[11] A. M. Victor, "The effects of metacognitive instruction on the planning and academic achievement of first and second grade children," Doctoral Thesis, Graduate College of the Illinois Institute of Technology, 2004.

[12] A. Efklides and G. D. Sideridis, "Assessing cognitive failures," *European Journal of Psychological Assessment*, vol. 25, no. 2, pp. 69–72, 2009, doi: 10.1027/1015-5759.25.2.69.

[13] R. I. Charles, F. K. Lester, and P. G. O'Daffer, *How to evaluate progress in problem solving*. National Council of Teachers of Mathematics, Reston, Va., 1987.

[14] A. Baroody and R. T. Coslick, *Problem solving, reasoning, and communicating (K-8): helping kids think mathematically*. New York: Macmillan Publishing, 1993.

[15] L. T. Lioe, H. K. Fai, and J. G. Hedberg, "Students' metacognitive problem-solving strategies in solving open-ended problems in pairs," *Redesigning Pedagogy*, Sense Publishers, 2019, pp. 243–259, doi: 10.1163/9789087900977_018.

[16] J. H. Flavell, "Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry," *American Psychologist*, vol. 34, no. 10, pp. 906–911, 1979, doi: 10.1037/0003-066X.34.10.906.

[17] G. Schraw and R. S. Dennison, "Assessing metacognitive awareness," *Contemporary Educational Psychology*, vol. 19, no. 4, pp. 460–475, 1994, doi: 10.1006/ceps.1994.1033.

[18] A. Efklides, "Metacognitive experiences in problem solving: Metacognition, motivation, and self-regulation," in *Trends and Prospects in Motivation Research*, Dordrecht, The Netherlands: Kluwer, 2001, pp. 297–323, doi: 10.1007/0-306-47676-2.

[19] M. Suliani, D. Juniati, and A. Lukito, "Analysis of students' metacognition in solving mathematics problem," *AIP Conference Proceedings*, vol. 2577, 2022, doi: 10.1063/5.0096032.

[20] M. Suliani, D. Juniati, and A. Lukito, "Students' metacognitive processes to solve arithmetic sequence problems in terms of mathematical ability," *Proceedings of the 3Rd Ahmad Dahlan International Conference on Mathematics and Mathematics Education 2021*, vol. 2733, p. 030013, 2023, doi: 10.1063/5.0140157.

[21] T. Ozturk and B. Guven, "Evaluating students' beliefs in problem solving process: A case study," *Eurasia Journal of Mathematics, Science and Technology Education*, vol. 12, no. 3, pp. 411–429, 2016, doi: 10.12973/eurasia.2016.1208a.

[22] J. Ishida, "Students' evaluation of their strategies when they find several solution methods," *Journal of Mathematical Behavior*, vol. 21, no. 1, pp. 49–56, 2002, doi: 10.1016/S0732-3123(02)00102-5.

[23] M. F. Pajares, "Teachers' beliefs and educational research: cleaning up a messy construct," *Review of Educational Research*, vol. 62, no. 3, pp. 307–332, 1992, doi: 10.3102/00346543062003307.

[24] D. Juniati and I. K. Budayasa, "The influence of cognitive and affective factors on the performance of prospective mathematics teachers," *European Journal of Educational Research*, vol. 11, no. 3, pp. 1379–1391, 2022, doi: 10.12973/eu-jer.11.3.1379.

[25] M. Guven and D. Belet, "Primary school teacher trainees' opinions on epistemological beliefs and metacognition," *İlköğretim Online*, vol. 9, no. 1, pp. 361–378, 2010.

[26] R. A. Tarmizi and M. A. A. Tarmizi, "Analysis of mathematical beliefs of Malaysian secondary school students," *Procedia - Social and Behavioral Sciences*, vol. 2, no. 2, pp. 4702–4706, 2010, doi: 10.1016/j.sbspro.2010.03.753.
[27] A. H. Schoenfeld, "Learning to think mathematically: problem solving, metacognition, and sense making in mathematics," in *Handbook for Research on Mathematics Teaching and Learning*, New York: Macmillan, 1992.
[28] J. W. Creswell, *Research design: Qualitative, quantitative, and mixed methods approaches*, 3rd ed. Thousand Oaks, CA: Sage Publications, Inc, 2009.
[29] N. V. Ivankova, J. W. Creswell, and S. L. Stick, "Using mixed-methods sequential explanatory design: from theory to practice," *Field Methods*, vol. 18, no. 1, pp. 3–20, 2006, doi: 10.1177/1525822X05282260.
[30] Roscoe, *Research Methods for Business*. New York: Mc Graw Hill, 1982.
[31] J. I. Setyawati and S. Y. Indrasari, "Mathematics belief and the use of metacognitive strategy in arithmetics word problem completion among 3rd elementary school students," *Proceedings of the Universitas Indonesia International Psychology Symposium for Undergraduate Research (UIPSUR 2017)*, vol. 139, 2018, doi: 10.2991/uipsur-17.2018.29.
[32] A. H. Schoenfeld, "Explorations of students' mathematical beliefs and behavior," *Journal for Research in Mathematics Education*, vol. 20, no. 4, p. 338, 1989, doi: 10.2307/749440.
[33] S. Sutama *et al.*, "Metacognition of junior high school students in mathematics problem solving based on cognitive style," *Asian Journal of University Education*, vol. 17, no. 1, pp. 134–144, 2021, doi: 10.24191/ajue.v17i1.12604.
[34] S. Tobias and H. T. Everson, "Knowing What You Know and What You Don't: Further Research on Metacognitive Knowledge Monitoring," Research Report No. 2002-3, College Entrance Examination Board, 2002.
[35] A. W. Woolfolk, M. Hughes, and V. Walkup, *Psychology in education*. London: Pearson, Longman, 2008.
[36] J. Wilson and D. Clarke, "Towards the modelling of mathematical metacognition," *Mathematics Education Research Journal*, vol. 16, no. 2, pp. 25–48, 2004, doi: 10.1007/BF03217394.
[37] R. Pulmones, "Learning chemistry in a metacognitive environment," *The Asia-Pacific Education Researcher*, vol. 16, no. 2, pp. 165–183, 2008, doi: 10.3860/taper.v16i2.258.
[38] D. Boud, R. Keogh, and D. Walker, *Reflection: turning experience into learning*. London: Kogan Page, 1985.

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