International Journal of Evaluation and Research in Education (IJERE)
Vol. 12, No. 4, December 2023, pp. 2230~2245
ISSN: 2252-8822, DOI: 10.11591/ijere.v12i4.25577

Mathematical competencies with applications of mirror classes in regular basic education

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Article Info

Article history:

Received Sep 27, 2022
Revised Sep 19, 2023
Accepted Oct 10, 2023

Keywords:

Mathematical competence
Mirror class
Pedagogical strategy
Virtual education

ABSTRACT

The problem about learning in mathematics lies in the lack of application of teaching strategies for the resolution of the calculation in mathematical problems, problems of movement, mathematical regularity, equivalence, and uncertainty; in populations with difficulties to learn collaboratively. Here the application of mirror classes in populations with interactive problems is demonstrated, thus contributing to strengthening their knowledge from cooperative information management, the use of analysis skills based on co-evaluation. In this sense, the purpose of the study was to evaluate the mirror class, as a pedagogical strategy, in order to optimize the competence of solution of quantity problems in basic school students of two educational institutions in Lima, Peru. The study was conducted under the action research approach where the diagnostic and exit test, question guide and field diary were the instruments used for data collection. It was concluded that the mirror class as a pedagogical strategy favors the optimization of competition and solves quantity problems in basic education students. So, they also developed concrete knowledge, to assign them in cognition more enduringly as a form of social learning.

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1. INTRODUCTION

Mathematical competencies promote the development of mathematical processes such as thinking, reasoning, argumentation, interpretation, among others, in students [1]. However, national and international standardized evaluations show worrying results regarding mathematics learning in Peru. Thus, the regional comparative and explanatory study (ERCE 2019) conducted in Latin American and Caribbean countries showed that 61% of Peruvian students in sixth grade of primary school are below the minimum level of the competencies established in the area of mathematics [2]. Likewise, the program for international student assessment (PISA) in 2018 reported that the academic performance of Peruvian students, in general, is low, ranking 64th out of 79 countries [3].

Mathematical competence is defined as the ability to understand the relevance of mathematics in the world and thus use it to exercise the role of constructive, committed and reflective citizens [4]. In this regard, D'Amore, Godino, and Pinilla [5] pointed out that achieving mathematical competencies is a challenge and requires considering four didactic requirements: i) Epistemological requirement, which is the theoretical reference that guides the teacher in the teaching and learning process; ii) Cognitive requirement, which is the theoretical construction of a learning object by the student; iii) Communicative requirement, which consists

of discursive interaction in a learning situation; and iv) Socio-cultural requirement, which refers to the relationships that develop among all the elements that are part of the classroom. In Peru, the national curriculum proposes in the area of mathematics, the approach focused on problem solving. From this approach, four competencies are considered for the development of mathematical thinking: i) Solution of quantity problems; ii) Solution of form, movement and location problems; iii) Solution of regularity, equivalence and change problems; and iv) Solution of data management and uncertainty problems.

The results of the research contribute to demonstrate the effects of developing mirror classes in the quantity problem solving competence in more than half of the participants, developing significant levels of achievement. At the same time, it is shown that, in motivation, teacher accompaniment, as well as teaching strategies are elements of the session that improve the beliefs of the students, as well as their own parents, and the apprehension that resources applied strategies generate greater collaboration and intrinsic motivation to learn. Finally, teachers begin to accept that the reflection of knowledge in others as a strategy of the zone of proximal development, allows progressively staggering increasingly positive performances in learning mathematics.

Given this, the hypothesis is raised that the effects of the mirror class can cause improvements in the way students learn, associating information to solve problems collaboratively, with stronger motivational interrelationships for the development of mathematics, both in learning, as well as in the perception of self-efficacy of teachers. This will also influence the parental expectation towards the children about the solution they arrive at, changing the traditional classes for classes with potentially cognitive interrelationships. It is consistent with the review of the literature that, the position that the interrelationships can be more cooperative if models of proximal learning are provided through classes that reflect the development of the skills of other more expert subjects over those who are apprentices or have poor school performance.

The mathematical competency: solution of quantity problems demands greater use of time in class with students because it addresses several thematic fields. This competency encourages students to solve and formulate problems that allow them to comprehensively construct the notions of number, number systems, their operations and properties. It also requires students to use strategies and develop logical reasoning, based on the mobilization of the following skills: i) Translation of quantities into numerical expressions; ii) Communication of understanding of numbers and operations; iii) Use of estimation and calculation strategies and procedures; and iv) Argument of statements about numerical relationships and operations [6]. In that sense, competencies differ from abilities in that they are developed in the long term while skills are acquired in a specific learning experience [7].

The translation of a verbal statement into a symbolic representation requires understanding the variables and relationships established within the verbal statement, in addition to the syntactic characteristics of the symbolic representation [8], being the verbal-to-graphic representation one of the most difficult for students [9]. On the other hand, Font, Godino, and D'Amore [10] refer that comprehension is an ability that the student evidences in practice. This allows students to develop a more flexible understanding by making several representations [11]. Likewise, they contribute to organize students' thinking and make mathematical ideas more concrete [12]. Hence, other research [13] points out that representations should be considered as instruments that allow mathematical competence to be generated. In this sense, going through several equivalent representations improves the learning of mathematics [14].

On the other hand, Alsina *et al*. [15] explained that the strategies used in problem solving acquire an important role because they favor the learning of mathematical concepts, consolidating comprehension, expression and their application. Reciprocally, the problems done in class should encourage students to express their ideas or explain how they have solved them [16]. In this regard, communication allows students to formulate questions and argue in and with mathematics, thus contributing to the development of mathematical competences [1]. Hence, the current teaching of mathematics requires the implementation of new pedagogical strategies that favor the understanding the mathematical ideas of students. Therefore, they contribute to the development of competencies in the context of virtual classes as a result of the COVID-19 pandemic [17].

In this sense, mirror classes promote interaction between students and teachers where knowledge is shared through a virtual platform between two national or international institutions [18]. In this regard, the research literature has so far revealed that the cognitive mechanisms of students increase as much as their knowledge when they use the expertise of their peers to improve their cognitive or procedural skills. This means that this conception is based on socio-cultural and imitative perspectives of learning, since the mirror class method has revealed certain empirical glimpses in research. It shows approaches towards its realization for the improvement of the abilities described. Mirror class is a strategy of pedagogical internationalization that allows strengthening the competence of students by establishing a connection between collaborative learning, the theory of research competences, the application of information and communication technologies (ICT) and identifying the communicative tools in virtual learning environments [19].

Nowadays, with the dominance of technologies in learning at the secondary or university level [19] have found that mirrored learning (oriented to mirror classes), generates positive effects on the participants of these types of classes by introducing telephony and artificial media in learning. In essence, they found that learning among Swedes and Finns was much higher in the face-to-face and screen-based interaction strategies, which allowed significant improvement of this ability in the classroom. A study aimed at understanding the method steps of the teacher [20] found that teachers are more predisposed to generate command strategies in classrooms where learning exchanges are carried out by the same teacher but in an experiential way, as opposed to those who only work under information and communication systems in the classrooms in which they participate, since there is greater co-occurrence of knowledge generation in classrooms where they work in a participatory (experiential) way.

On the other hand, this strategy seems to have appeared in other researches, which with similar structure have oriented their research work towards mirror activities [21] who with an addition of inverted classroom, found improvements in students of an internship on surgery, and which, under constructivist strategies of collaboration allowed finding more evidence of self-learning in students who carry out this experience. In the school field, Shimpi, Akhtar, and Moore [22] reported that imitation as a learning replication strategy in young children, who imitated the learning acts of others who were unknown to them. Although an attempt was made to control the interaction variable between them, first sketches were generated about autonomous learning and imitation of strategies among totally unknown infants. Although the age of the participants should be taken into account, since in the initial stage the cognitive capacities influence the quality of the denial of the interaction as to make a mirror learning.

In the pedagogical context, we can find that the imitative effects in learning also depend on the quality of the learning itself, from the ecologist theory, this quality by which neurons perform imitation is enduring with respect to the goals of the recipients of such learning. Less quality in the strategy (durability, exemplarism, balance, and motivation), can affect the reception of learning in subjects with less expertise, denoting that learning by mirror method depends both on the relationship: subject-expert/subject-inexpert, and on the mediation of the teacher to regulate the factors of the quality of the imitated learning [23]. Current evidence has determined that in addition to the acquisition of learning and the development of skills, mirror learning may include the acquisition of goals, attitudes, and other aspects such as confidence, self-belief, about the visionary achievement of learning in their educational mirroring interactions [24], [25].

In view of that, we could focus on the Vygostkian perspective, alluding to the use of the proximal potential with the acquired potential among the participants of an experience, although experimental evidence differs in the inclusion of the interaction variable in mirror learning [22], [26]. Although the situationally of this learning can also occur through sociocultural processes of teaching from the more expert to the more inexperienced, the inclusion of the interrelation variable in the method with refractory or mirror classrooms is necessary. This model also belongs to a certain extent to Chevallard's approach in 1985, regarding the didactic transposition among learners [27], since there is an origin that must be transferred to other participants so that they can take advantage of the substantial aspects of knowledge, in order to use the borrowed sources as didactic elements in the teaching of knowledge to others. In this sense, we also focus these contributions on the co-constructive method analyzed [28] who argue that learning is both negotiated and bounded, in interactive segments, which we predict as convenient among the peers performing the learning. For this reason, the perspectives complement each other, from their socio-biological aspect to the socio-cultural aspect described during the history of mankind.

From the Vygotskian perspective, the study contributes to the development of mathematics, based on proximal and potential development, it contributes to the achievement of skills to develop problems, exemplifying mathematical classes with students with greater knowledge and skills, with others with low potential. to make it. When carrying out student exchanges, the development of low-level skills is promoted, supporting them in the experience of those who, if they have the potential to achieve it, therefore, mirror classes based on student exchange are applied. For all the explanation, this study has the aim to evaluate the mirror class as a pedagogical strategy, in order to optimize the competence of solution of quantity problems in basic school students of two educational institutions of Lima, Peru.

2. RESEARCH METHOD

2.1. Participants

The aim of the study was to evaluate the mirror class as a pedagogical strategy to optimize the problem-solving competence in quantity in basic school students in two educational institutions in Lima, Peru. In this sense, this research was developed under the qualitative approach, with an action research design. Under this design, four phases were assumed: planning of an action plan to optimize the quantity problem-solving competence, action to execute the action plan, observation to collect evidence from the

implementation of the plan, and reflection on the actions recorded. The sample consisted of 40 students of both sexes whose ages ranged between 8 and 10 years old, chosen from a total of 64 corresponding to the third grade of basic education from two public educational institutions from different districts of Lima, Peru, selected by means of purposive sampling.

2.2. Instruments and procedure

For data collection, two semi-structured interview guides were used as instruments for both the parents and the teachers who taught the mirror classes; a diagnostic test and an exit test were also used, in addition to the field diary. The first instrument used was the diagnostic test, which was developed by the students individually and online. It consisted of five multiplicative problems in order to collect information on the current situation of the level of development of the competency solve quantity problems. The problems were oriented to the skills that make up the aforementioned competency: translation of quantities into numerical expressions, communication of understanding of numbers and operations, use of estimation and calculation strategies and procedures, and argument of statements about numerical relationships and operations. Scores were classified into the following achievement levels: beginning (0-10), process (11-14), and satisfactory (15-20). Subsequently, 12 mirror classes were developed, focused on multiplicative problem solving (multiplication and division problems), conducted virtually due to the context of the COVID-19 pandemic. The duration of each class was 150 minutes. In the development of the classes, the use of unstructured concrete material and teamwork were prioritized as strategies. At the end of the 12 mirror classes, the students were evaluated with an exit test consisting of five multiplicative problems, to corroborate the progress in the development of the solution of quantity problems competency. Similar to the pedagogical test, the questions were oriented to the skills of the competency Solve quantity problems. The achievement levels considered were beginning (0-10), process (11-14), and satisfactory (15-20).

On the other hand, the two semi-structured interview guides were applied, consisting of 8 questions each, addressed to the 2 teachers and 8 parents of both classrooms, on their perception of the mirror class as a pedagogical strategy in the achievement of the solution of quantity problems competence in the students. The field diary was used to record everything observed in each mirror class. The research followed the four phases [29], namely planning, execution, observation, and reflection.

2.2.1. Planning phase

Coordination meetings were held among the responsible teachers, in these meetings an action plan was established to optimize the solution of quantity problems competence, specifically in the resolution of multiplicative problems. For this purpose, first, a review of the curricular planning of each classroom was considered in order to determine the possible subtopics that could be included in the resolution of multiplicative problems. Secondly, an agreement matrix was organized to establish the pertinent guidelines for the development of the mirror classes (topic and subtopics, date and time of the mirror classes, virtual learning tool to be used in the meetings, materials and evaluation instrument). Subsequently, the mirror classes referred to multiplicative problems and the evaluation instruments (rubric) were designed. Finally, the diagnostic evaluation was applied to third grade students of the selected educational institutions, both located in different districts of Lima-Peru, in order to identify the level of achievement of the Solution of quantity problems competence.

2.2.2. Action plan execution phase

The 12 mirror classes referred to multiplicative problems were carried out and the evaluation instruments (rubric) were applied. In the development of the mirror classes, the stipulations of the agreement matrix had to be considered, in addition to complying with the following recommendations: all meetings are synchronous, teachers must be connected virtually in all sessions, innovative and relevant activities must be generated, students will work in teams to solve, create and argue the resolution of multiplicative problems, the means of communication will be kept open between students and; between teachers and students, all materials will be shared with students before the mirror classes (videos, worksheets, and practices) in order to address queries or doubts. At the end of the 12 mirror classes, an exit evaluation was applied to the students to identify the level of achievement reached in the competency solve quantity problems.

2.2.3. Evidence observation phase

The actions carried out in the plan implementation phase were observed and evidence was collected using diagnostic and exit tests, interview guides and the field diary. The preliminary tests were developed with performance tests, instead, the interviews were carried out personally with those parents who agreed to the evaluation. The teacher's diary was a clear tool for the teacher's auxiliary record and diary according to the development of their classes.

2.2.4. Reflection on actions phase

Weekly meetings were developed between the teachers of both educational institutions to evaluate the results obtained in the previous phase, so that improvements and aspects to be strengthened in the research were identified. The meetings consisted of questioning and proposal stages. In the first stage, it was allowed to focus on the problems that arose, as well as the solutions that did not work in the development in class. On the other hand, the proposals made it possible to complete cyclical learning processes in the classroom, progressively improving those that were deficient.

3. RESULTS AND DISCUSSION

3.1. Interpretation and analysis

The analysis of the results was based on the data obtained from the diagnostic test and the exit test, in addition to the development of interviews with parents, teachers, and the analysis of the field diary applied in 12 activities to third grade students in two educational institutions in Lima, Peru. To this end, we began with the transcription and organization of the information and the theoretical and data triangulation. It allowed us to argue the study from different expert authors and the definition of the Peruvian Ministry of National Education (Ministerio de Educación Nacional del Perú) in relation to the solution of quantity problems competence and its capabilities in the area of mathematics, in addition to the responses of the participants that allowed us to know their perception in relation to each of the categories of the study.

The information given by the participants of instrument 1, teacher interview, instrument 2 and the observation record card of the field diary were processed in the Atlas.ti software version 8.4.24, in which each one was enlisted according to the categories of the study; solution of quantity problems competence and mirror classes. According to the data, memos were created, associated with the theory that underlies the study, and finally semantic networks were constructed that demonstrate the relationships between each of the categories, subcategories and memos, in order to understand and interpret what is the perception of teachers, students and parents about the implementation of mirror classes as a pedagogical strategy for the development of skills, such as: translating quantities into numerical expressions; communicating understanding about numbers and operations; using estimation and calculation strategies and procedures; arguing statements about numerical relationships and operations.

3.2. Data coding

In the coding process, the selective coding method [30] was considered, which allows starting the processing from the definition of a list of codes previously established from the method design of the research and the categories and subcategories of the study. In this case, four indicators were reported on the evaluated competence. Thus, the semantic network of codes of the research is proposed, which are made up of the categories, solution of quantity problems competence and mirror classes as shown in Figure 1.

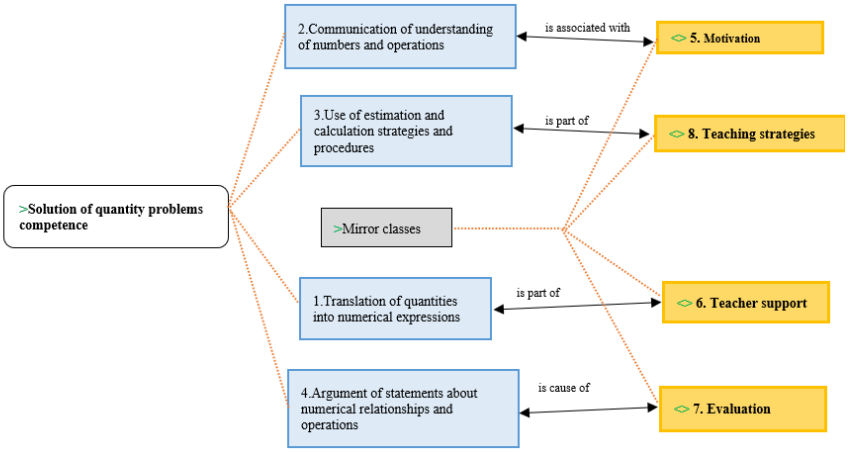

Figure 1. Research codes and code groups

Thus, between the category solution of quantity problems competence and mirror classes; an association relationship was evidenced with the subcategory motivation, two relationships of belonging with the subcategories teaching strategies and teaching support and a causality relationship with the subcategory

evaluation. Although the evaluation has been more relational with the arguments, the teaching accompaniment with the translation of the quantities, as well as the teaching strategies seem to come together better with the development of numerical estimation. Finally, motivation has been associated in a better predisposition with the forms of communication of students when learning mathematics.

3.3. Category analysis

3.3.1. Solution of quantity problems competence

The purpose of the competency solves quantity problems is for the student to solve and formulate problems that make it possible to understand numerical systems, their operations and properties through the implementation of different resources [31]. Thus, for this category, four subcategories have been established for the analysis, which have been established according to the skills that the child must develop to achieve this competence: translating quantities into numerical expressions, communicating understanding about numbers and operations, using estimation and calculation strategies, and arguing statements about numerical relationships and operations. The analysis associated with each of the subcategories is presented, based on the instruments applied to teachers, parents and elementary school students from educational institutions in Lima, Peru.

a. Translation of quantities into numerical expressions

In the analysis of this subcategory, the contribution of the mirror classes in the development of this ability in the students was evidenced. Their application allowed the creation of problems based on a given situation or requirement. Likewise, it allowed the resolution of multiplicative problems from the translation of the verbal statement of the problem to a concrete, graphic and later symbolic representation, using unstructured didactic materials, the guidance of the teacher and the accompaniment of the parent. In this regard, several researchers [32]–[34] stated that the creation and resolution of problems are essential tasks for the development of competencies and therefore of mathematical thinking. On the other hand, the opportunity to provide them with autonomy in the development of their activities strengthened the teaching process, teamwork and the development of student learning through the virtual sessions as shown in Figure 2.

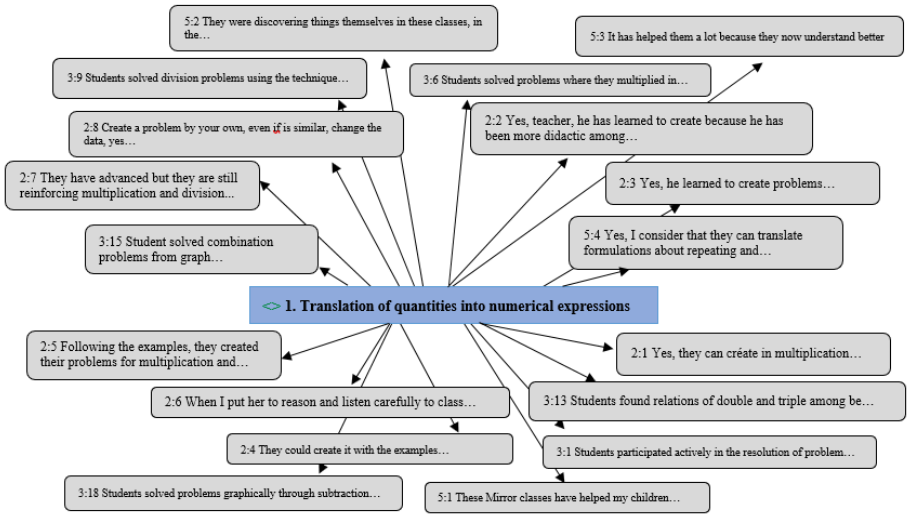

Figure 2. Network of codes and quotations of the translation of quantities into numerical expressions ability

It can be seen from the teachers' answers to the question: Do you consider that your students can translate actions of repeating and distributing quantities, to multiplication and division expressions with natural numbers, when solving problems from the mirror classes? Why?

(348:418)-D1: *These mirror classes have particularly helped my children a lot ...*
(820:878)-D1: *It has helped them a lot because now they understand better*.
(592:687)-D2: *They themselves were discovering things in these classes, being in the virtual classes*.
(1459:1576)-D2: *Yes, I consider that they can translate statements about repeating and distributing into multiplication and division expressions.*

Now, with respect to the question addressed to the parents, do you consider that your child can create problems on multiplication and division and solve them from the mirror classes? Why? It was evidenced:

(215:266)-P1: *Yes, in multiplication and division they can also create ...*
(416:495)-P2: *Yes, they have learned to create because it has been more didactic among classmates*
(575:630)-P3: *Yes, he learned to create problems by himself, he does it and solves it...*
(691:742)-P4: *They could create it with the same examples they were given ...*
(873:960)-P5: *They created their multiplication and division problems following the examples and solved them ...*
(1098:1283)-P6: *When she reasons and listens carefully to the class is where she herself sometimes tells me mom this is how it is, we add, then we subtract, then we multiply by the amount ...*
(1337:1418)-P7: *They have advanced but they are still reinforcing multiplication and division ...*
(2401:2476)-P8: *Yes, a little more because they helped each other among classmates to do the problems.*

On the other hand, in the analysis of the field diary applied to the students it was found:

(1644:1903)-DC1: *The students actively participated in the resolution of the multiplication problems by representing them with concrete material such as the Mac kinder box, the rulers or base ten, then graphically and finally through multiplication ...*
(2581:2740)-DC2: *One of the problems was worked in teams and then one member of each team explained how they solved it. The teacher gave pertinent feedback ...*
(4201:4286)-DC3: *They represented the problems graphically with drawings, diagrams or number lines ...*
(9188:9377)-DC4: *They used the drawings, the table of points, the table of direct proportionality, the number line, the decomposition, as they found easier to solve the problems*.

b. Translation of quantities into numerical expressions

In the analysis of this subcategory, the contribution of mirror classes in the development of the ability communication of understanding of numbers and operations, through representations in a concrete, graphic and symbolic way, from the use of didactic and ludic strategies such as the use of unstructured didactic material and the application of virtual games, was evidenced. In this perspective, [35] point out that representations favor the understanding of mathematical concepts. They contribute to organize students' thinking and make mathematical ideas more concrete [12]. On the other hand, ICT implemented in the teaching and learning process enhance the understanding of mathematical content [36]. For their part, parents recognize the added value of mirror classes in their children's learning, in addition to the development of other skills such as teamwork, as shown in Figure 3.

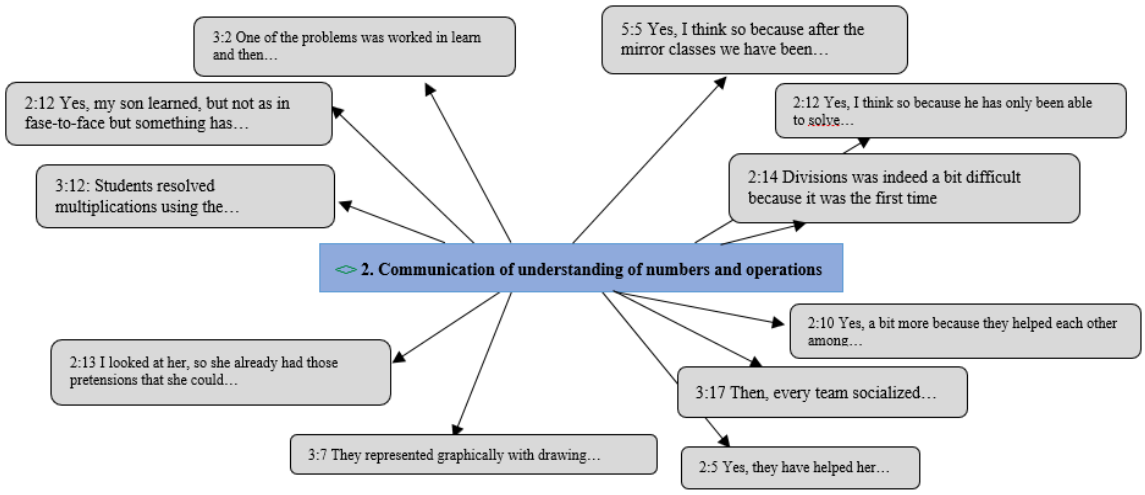

Figure 3. Network of codes and quotations of the communication of understanding of numbers and operations ability

It can be seen from the teachers' answers to the question: Do you think that your students acquired greater understanding of the meaning of multiplication and division from the mirror classes? Why?

> (2666:2842)-D1: *Yes, because after the mirror classes we have been analyzing some problems and they already realize when it is multiplication, when it is necessary to divide and they have even realized the importance of knowing multiplication ...*
> (3340:3458)-D2: *Yes, because in the development of the mirror classes the children participated and answered the questions, besides that in their practices they solved and created multiplicative problems."*

Now, with respect to the question addressed to the parents: Do you think that your child acquired greater understanding of the meaning of multiplication and division from the mirror classes? Why? It was evidenced:

> (2480:2568)-P3: *Yes, my son learned, now he solves by himself ...*
> (2572:2762)-P4: *Yes, I think so because he has been able to solve his own problems. When they said multiplication, he said double, triple, he did it by himself or also with his small caps and with the beans ...*
> (2907:2975)-P5: *I was looking at her so she already had those hints that she could do ....*
> (3035:3157)-P6: *The divisions were a little difficult for her because it was her first time, but she had already made progress in multiplication ...*
> (3366:3410)-P7: *They have helped her, she did not understand almost anything ...*
> (1337:1418)-P7: *They have made progress but they are still in reinforcement with multiplication and division ...*
> (2401:2476)-P8: *My child made progress with multiplication and division.*

On the other hand, in the analysis of the field diary applied to the students it was found:

> (2743:2860)-DC 1: *Most students were very attentive and participative since it was a topic that generated much interest in them ...*
> (4288:4382)-DC 2: *Then they explained the process of the resolution and received feedback in a timely manner ...*
> (5314:5482)-DC 3: *Students found double and triple relationships between the factors of the multiplications from graphs and problems posed. In addition, they completed them ...*"

Here the motivation or attention to the task converge, the relationship seems to have achieved a better understanding of the problems with semiotic representation of quantities. Likewise, it has been found that shared evaluation can support the generation of new responses in cognitively less favored students. In this sense, assessment can promote a sense of learning and mathematical learning because of students' own motivations.

c. Use of estimation and calculation strategies and procedures

It was found that estimation and calculation procedures were worked on through problem solving. In this context, the strategies used by the students were mostly oriented to the use of didactic resources such as unstructured materials, among which the Mackinder box and homemade objects from their environment such as small caps, chip-taps, marbles and seeds stand out in the development of mathematical activities. In this regard, Torrecilla, Carrasco, and Cerezo [37] pointed out the importance of the use of educational material in the teaching and learning of mathematics, since it contributes to the development of logical thinking due to the manipulation that allows for greater understanding of concepts. In this perspective, strategies play an important role because they favor the learning of mathematical concepts by consolidating understanding [15]. On the other hand, it allowed parents to identify the motivation involved in student learning, the implementation of playful, participatory strategies that encourage teamwork as shown in Figure 4.

It can be seen from the teachers' answers to the question: "What strategies used in the teaching and learning process of multiplication and division in the mirror classes allowed their students to acquire greater understanding?"

> (4075:4236)-D1: *With the materials and graphics, the children who did not understand multiplication and division very well were able to understand ...*
> (4506:4527)-D2: *Use of the Mackinder box, the seeds or objects that the children used to solve the multiplicative problems. In addition, the graphic representation ...*

Now, with respect to the question addressed to parents, "What strategies or activities used in the teaching and learning process of multiplication and division in the mirror classes allowed their child to acquire greater understanding?" It was evidenced:

(5801:5815)-P3: *Mackinder box …*
(2572:2762)-P3: *We had a bottle with small plates and with that he used to solve the multiplication problems, for division it was through pure multiplication ...*
(6395:6403)-P4: *Mackinder ...*
(6423:6587)-P4: *Small plates, my child used to do it with beans and also those little division and multiplication games for them to answer, with roulette they also did that a lot ...*
(6964:6980)-P5: *Little Mackinder box ...*
(7275:7336)-P6: *They used the Mackinder box for multiplication and division ...*
(7409:7436)-P7: *I have not used any purchased material, but I have used the small caps and seeds we have at home ...*
(7963:8006)-P8: *Used seeds, soda caps*.

On the other hand, in the analysis of the field diary applied to the students it was found:

(3261:3325)-DC1: *They justified the reason for the representations they used ...*
(3148:3260)-DC2: *The students represented multiplications in different ways, using the Mackinder box, in different situations*.

d. Argument of statements about numerical relationships and operations

In the analysis of this subcategory, the ability of most students to argue and explain the strategies and procedures applied in the development of multiplicative problems, that is, multiplication and division, was evidenced, which allows highlighting the contribution of the mirror classes. In this perspective, the problems solved in class should encourage students to express their ideas or explain how they have solved them, otherwise it makes no sense to consider the competence to communicate as an objective to be achieved [16]. In this regard, Solar *et al.* [1] points out that oral or written communication contributes to give meaning to ideas and to share them with others. Hence, classroom is the appropriate space for students to develop communicative and argumentative practices [38]. However, some parents pointed out that in the development of the classes their children, due to their shyness and insecurity to express themselves in front of others, presented limitations to share their knowledge, as shown in Figure 5. This could also be demonstrated in internationalization classes as in other evidence found [39].

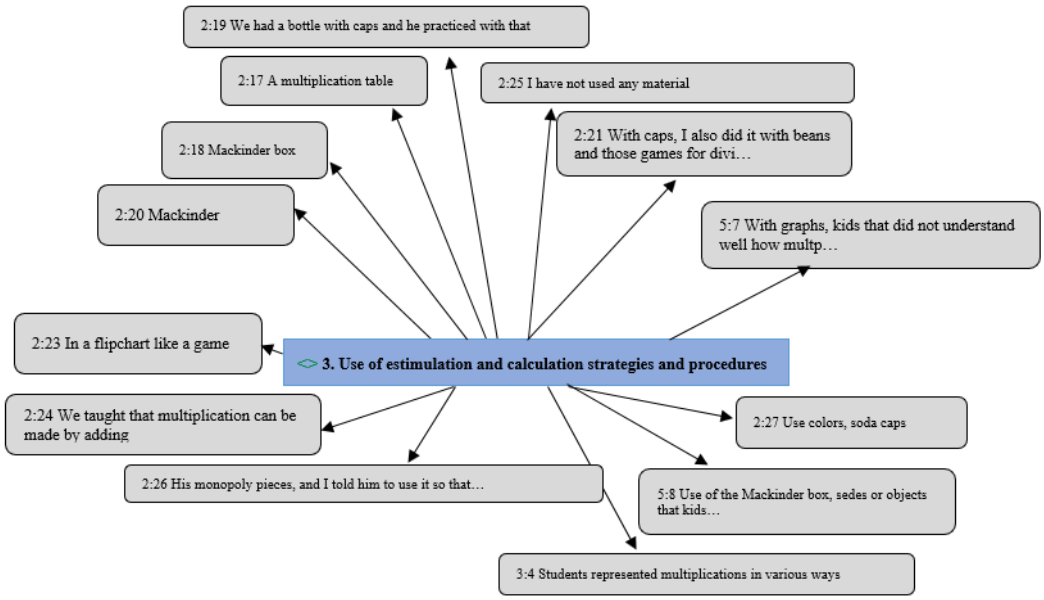

Figure 4. Network of codes and quotations of the use of estimation and calculation strategies and procedure's ability

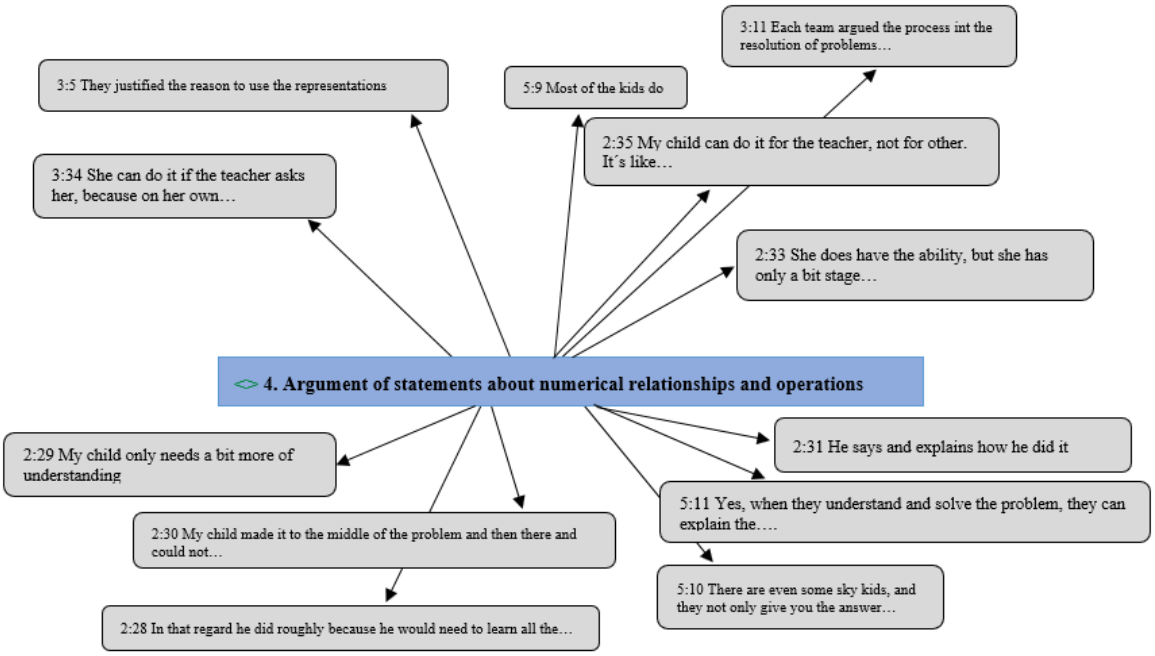

Figure 5. Network of codes and quotations of the argument of statements about numerical relationships and operations ability

It can be seen from the teachers' answers to the question: "Do you consider that your students can make arguments or explanations about multiplication and division problems? Why?"

> (4506:4527: 5432:5660)-D1: *Most of the children do ... some are shy and do not give you the answer, they just tell you that they have already done it, but when you take them by teaspoonful, they explain how they have been doing it, but others are more confident.*
> (5945:6053: 6470:6487)-D2: *Yes, when they understand and solve the problem, they can explain the process they have followed to solve it ... definitely yes.*

Now, with respect to the question addressed to the parents, "Do you consider that your child can make arguments or explanations about multiplication and division problems? Why?" It was evidenced:

> (8343:8433)-P1: *In that regard he did roughly because they are shy to speak in public ...*
> (8820:8860)-P2: *He would need a little more to get around ...*
> (9161:9238)-P3: *Mi child explains to you how they managed to solve the problem ...*
> (9270:9307)-P4: *He says and explains how he did it ...*
> (9506:9591)-P5: *She is a little bit more shy to express herself in public, but at home she does it very well ...*
> (9774:9844)-P6: *Yes, she does, the only thing is that she has a little bit of stage fright ...*
> (9894: 9998)-P7: *Yes, I can do it but if the teacher asks her because she doesn't do it on her own, she is very shy ...*
> (10054:10152)-P8: *Yes, they can do it for their teacher, not for others. They are a little bit suspicious, a little bit shy.*

On the other hand, in the analysis of the field diary applied to the students it was found:

> (3918:4198)-DC1: *The students solved problems where they multiplied in a rectangular way, that is, rows by columns and vice versa to find the product, thus discovering that the order of the factors does not alter the product, relating it to the commutative property in multiplication ...*
> (4668:4827)-DC2: *The students solved multiplications using the board of dots. This also allowed them to strengthen their understanding of the commutative property.*

3.3.2. Mirror class

The category, mirror class is defined as a pedagogical strategy that promotes interaction between students and teachers where knowledge is shared through a virtual platform between two national or international institutions [18]. For this category, learning sessions have been indicated as a subcategory. For its analysis, the following domains of analysis were established: motivation, teacher support, evaluation and teaching strategies.

a. Learning sessions

The analysis of this subcategory allowed identifying, from the perception of teachers, parents and students, those factors that are related to the development of each of the 12 activities proposed in the mirror classes for the development of learning and skills of the competency solve quantity problems. The aspects, such as motivation, teacher support, evaluation, and teaching strategies were identified as shown in Figure 6 to Figure 9, respectively. Motivational components were analyzed under convergent restructuring, and the citation code network for the analyzed database.

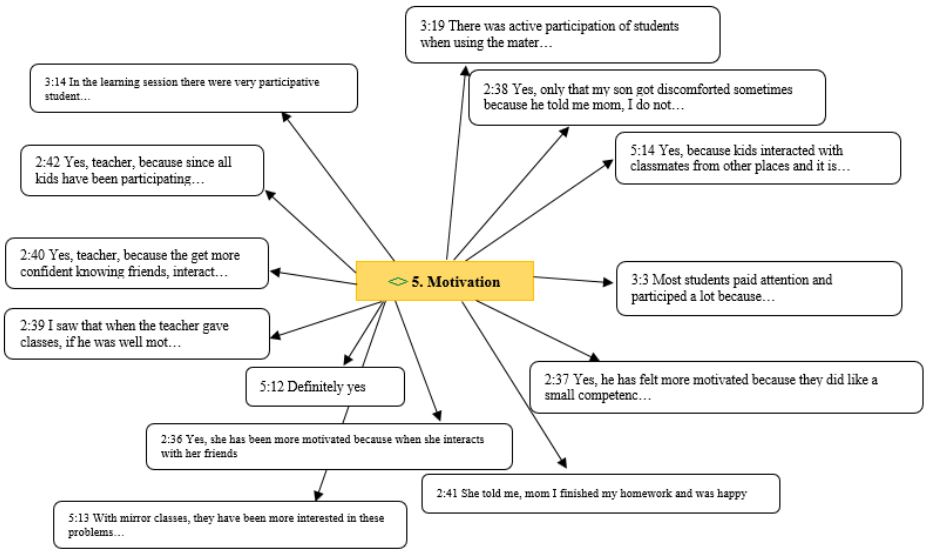

Figure 6. Network of codes and citations of the motivation domain

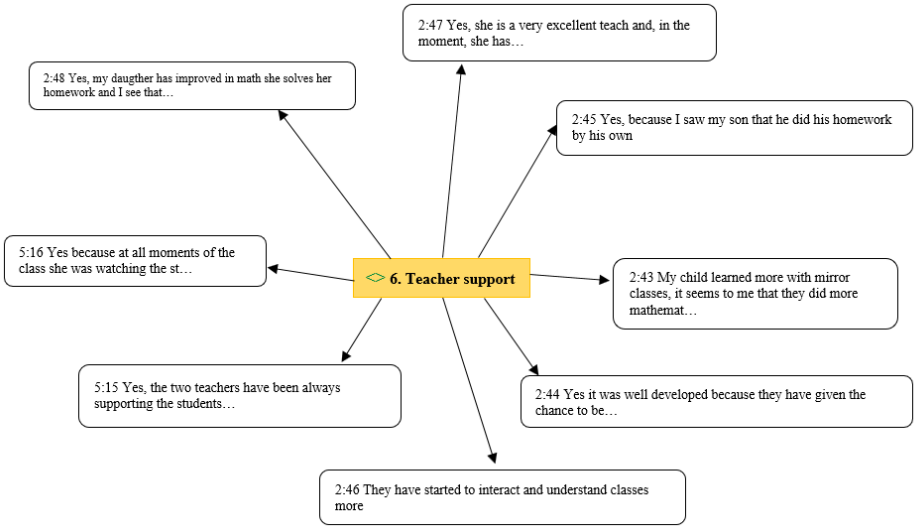

Figure 7. Network of codes and citations of the teaching accompaniment domain

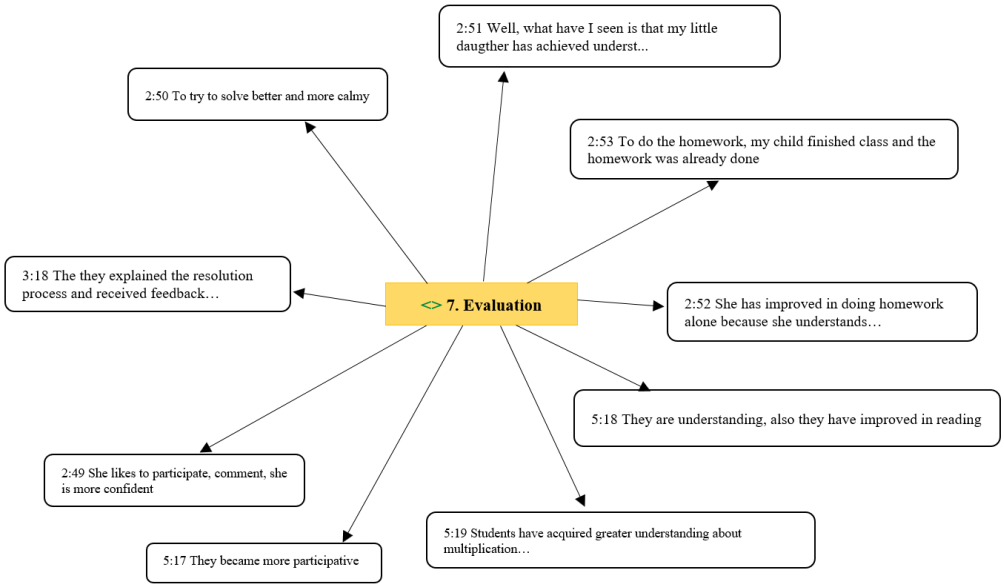

Figure 8. Network of codes and citations of the evaluation domain

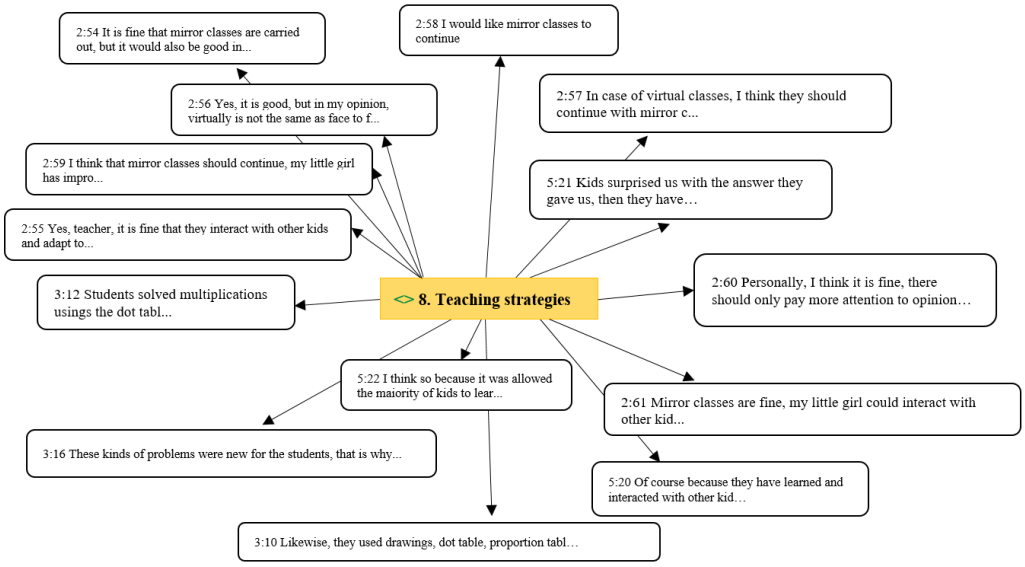

Figure 9. Network of codes and citations of the teaching strategies domain

It can be seen from the following questions and answers to the teachers: "Do you consider that the mirror classes contributed to make your students feel more motivated to learn about multiplication and division operations? Why?"

> (6600:6796)-D1: *With the mirror classes they have become more interested in these multiplication and division problems because we saw that they were two classroom groups which interacted even in the work groups ...*
> (7460:7645)-D2: *Yes, because the children interacted with classmates from another place and that was new for them; also, working with different activities and strategies such as the use of didactic material ...*

In relation to the question: "Do you consider that the teacher support in the mirror classes has contributed to the development of your students' mathematical skills?" The teachers answered:

(8399:8574)-D1: *Yes, because the students were being observed at all times during the classes and feedback was given as needed, not only in the classes themselves but also outside of them ...*
(8871:8902)-D2: *Yes, we have always been supporting the students to give them feedback when necessary, so that they can continue advancing in their learning ...*

On the other hand, in response to the question: "What improvements have you observed in your students' learning as a result of the mirror classes?" It was found:

(9218:9281)-D1: *Students have acquired greater understanding of multiplication and division, they have implemented the use of concrete material and its graphic representation in problem solving; in addition, most of them are now more participative ...*
(10006:10076)-D2: *The children did not participate much now with the mirror classes I see that they have become more participatory, they have not been so afraid to participate, they have realized that when they participated if for A or B they made a mistake, they were given feedback and they understood ...*

Finally, in response to the question: "Do you consider that mirror classes are a good strategy for your students' learning?" We obtained:

(9218:9281)-D1: *I think so, because it has allowed most of the children to learn comprehensively about multiplication and division through various strategies, in addition to interacting and learning from other children …*
(10006:10076)-D2: *Of course, it is, because they have learned and interacted with other children*.

Now, with respect to the questions addressed to the parents: "Do you consider that the mirror classes helped your child feel more motivated to learn about multiplication and division operations? Why?" It was found:

(10386:10487)-P1: *Yes, she has been more motivated because when she interacts with her friends, she wanted to find out one thing, another thing ...*
(10769:10864)-P2: *Yes, he has been more motivated because they had a little competition among his classmates ...*
(10868:10951)-P3: *Yes, he always wanted to answer when the teachers asked ....*
(11116:11195)-P4: *I could see that when there were mirror classes, he was well motivated there*.
(11199:11270)-P5: *Yes, teacher, because he begins to let go and meet friends, he interacts ...*
(11199:11270)-P6: *Yes, he is more motivated ...*
(12008:12067)-P7: *She told me mom, I finished my homework and she was happy ...*
(12071:12190)-P8: *Yes, teacher, because as all the children have been participating in both classrooms, she also wanted to be there*.

On the other hand, in relation to the question: "Do you consider that the teacher accompaniment in the mirror classes has contributed to the development of mathematical skills in your child? Why?" It was evidenced:

(12388:12462)-P1: *They learned more with the mirror classes, it seems to me that they have given more mathematics ...*
(12702:12798)-P2: *Yes, they have developed very well because they have given the opportunity to participate several times ...*
(13194:13258)-P3: *Yes, my son has been able to understand ...*
(13194:13258)-P4: *Yes, because I watched my son solve his homework by himself ...*
(13415:13472)-P5: *My child has started to interact and understand the classes more ...*
(13633:13755)-P6: *When she has needed her teacher, she has always been there ...*
(2572:2762)-P7: *I think so ...*
(13980:14062)-P8: *Yes, my little girl has improved in mathematics, she solves her homework and I see that she understands*.

On the other hand, in relation to the question: "What improvements have you observed in your child's learning as a result of the mirror classes?" It was identified:

(14172:14220)-P1: *She likes to participate, to give her opinion, she is more confident ...*
(14374:14413)-P2: *In trying to solve better more calmly ...*
(15779:15922)-P3: *Well in what I have seen that my little daughter has achieved is that she can understand better, she can also let go, express herself and can solve problems ...*
(16089:16204)-P4: *She has improved in solving alone the tasks that are left for her because she understands and I see that she likes math more ...*
(16208:16407)-P5: *In doing her homework, she finished her class and already had her homework ready. The other thing they have learned is to express themselves, not 100%, I could say 70% about emotions and to give opinions ...*
(17177:17256)-P6: *It is good that mirror classes are held, it would be good in other areas as well".*

Finally, in relation to the question: "What is your opinion about mirror classes?" We obtained:

(17177:17256)-P1: *It is good that mirror classes are held, it would be good in other areas as well.*
(17729:17810)-P2: *Yes, it is good, teacher, that they interact with other children and adapt to changes ...*
(17814: 17887)-P3: *Yes, it is good, but in my opinion virtually is not the same as face-to-face ...*
(18137:18245)-P4: *In case of virtual classes, I think they should continue with mirror classes, this way as they have done ...*
(18249: 18297)-P5: *I would like them to continue with mirror classes ...*
(18413:18500)-P6: *I think they should continue with mirror classes, my little daughter has improved in mathematics ...*
(18630:18726)-P7: *Personally, I have seen it well ...*
(18730:18830)-P8: *The mirror classes are good, my little daughter could interact with other children and solved her problems on her own …*

On the other hand, in the analysis of the field diary applied to the students it was found:

(3918:4198)-DC1: *The students represented multiplications in different ways, using rulers, in various situations ...*
(3148:3260)-DC2: *Students solved division problems using the technique of division between one and two digits in the quotient ...*
(9063:9185)-DC3: *Students solved combination problems from graphs, double-entry tables and explained how they solved the combination in each case*.

Otherwise, the results obtained in the diagnostic and exit test applied to the participating students are presented in Table 1. As shown in the table, the results obtained in the exit test corroborate that the mirror classes as a pedagogical strategy favored the optimization of the solution of quantity problems competence in basic education students in Lima, Peru. The mirror classes have allowed students to reflect on ways of solving mathematical problems expressed collaboratively.

Table 1. Achievement level of the solution of quantity problems competence

Achievement level	Diagnostic test	Exit test
A (achieved)	13 (33%)	38 (95%)
B (process)	24 (60%)	2 (5%)
C (beginning)	3 (7%)	0 (0%)
Total	40 (100%)	

4. CONCLUSION

The application of the mirror class as a pedagogical strategy optimizes the development of the solution of quantity problems competence in basic education students because it provides spaces for interaction between students and teachers that generate an exchange of knowledge and reflection on the pedagogical work among peers. Going through different ways of representing a problem allows for a better understanding of mathematical concepts in students because these concepts become more concrete. In this sense, the use of didactic materials favors the understanding of mathematical concepts by allowing the student to manipulate them in a concrete way. Teachers should give students the opportunity to express their ideas or explain how they have solved the problem, thus contributing to the development of the argument of statements about numerical relationships and operations ability.

ACKNOWLEDGEMENTS

The research was financed by the Competitive Funds of the Norbert Wiener Private University (UPNW/N°. 859-2021).

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