Identifying common errors in polynomials of eighth grade students

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ABSTRACT

This research aims to study and classify errors in polynomials made by secondary school students. The data for error identification was collected from exercise books of 72 eighth grade students. Three types of errors were examined: careless, computational, and conceptual errors. The errors were considered according to four topics in polynomials: similar terms of monomials; addition of polynomials; subtraction of polynomials; and multiplication of polynomials. It is found that students made the highest computational errors in identifying monomials' similarity, which accounts for 17.86%. They have the highest percentage of making computational errors in the addition and subtraction of polynomials, which account for 10.88% and 12.04%, respectively. Lastly, they have the highest percentage of making careless errors in the multiplication of polynomials, which accounts for 14.44%. Furthermore, it can be seen that the source of errors is learners' carelessness when writing the question and its answer. In addition, the basic knowledge of computing addition, subtraction, and multiplication of integers is the most crucial factor that leads to incorrect answers. Nevertheless, most students understand the principle of polynomials, but frequently make errors on other issues.

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1. INTRODUCTION

Mathematics plays an essential role in constructing 21st-century skills [1], and it is intimately related to the details of everyday human life and activities [2]. Nowadays, mathematics is a necessary subject and a crucial requirement in every field [3], developing human thinking to be more creative, logical, and able to analyze problems or conditions extensively. Furthermore, mathematics can assist people in anticipating, planning, deciding, and appropriately solving any problems encountered in daily life [4], [5]. Because mathematics is both abstract and concrete, most students find it challenging to comprehend when solving mathematics problems since it contains several rules, formulas, and definitions that are unrelated to life. These are causes that make students lack understanding and may generate difficulties that might give rise to errors [6]. Therefore, it is crucial for mathematics teachers to assist learners to learn from their errors and mistakes [7] to improve their understanding of the higher levels of mathematics.

Learning mathematics can be defined as a learning process in which students actively engage in constructing mathematical knowledge [8] and combine various abilities to master a variety of mathematical concepts to apply them to solve problems in everyday life [9]. Every mathematical knowledge is essential for students' understanding of the contents of higher concepts in mathematics [10] or other subjects. Many factors can influence students' mathematical learning success [11]. Capuno *et al.* [12] discovered that students have a positive attitude toward mathematics, which is one of the elements that indicate the students' learning achievement in mathematics, and that students have confidence in focusing more attention on one subject than on others [13]. In contrast, there are factors that cause students to struggle with learning, which is an impediment to mathematical achievement. Both internal factors such as problem-solving, intelligence, learning motivation, subject mindset, and personality, and external factors like a teacher's performance, family support, campus environment, and learning methods all have an impact [14].

For all levels of mathematics education, algebra is necessary for achieving success in all branches of mathematics [15]. It is one of the most essential topics in mathematics because its application is very fundamental and integrated into other mathematical concepts [16]. In learning abstract concepts, students' acquisition of abstract thinking ability is essential [17], and they would like to make those concepts more concrete to be simple to understand. Furthermore, algebra concepts help students to form the relationship between numbers and real-life applications [18]. On the other hand, algebra is one of the primary topics in mathematics that students commonly make errors [19] and is difficult to understand which has an impact on their ability to apply concepts in other topics and subjects [20].

Polynomials are also one of the topics in algebra, they are material that requires reasoning and is not enough to just memorize it. Students must understand the concepts of the formulas presented so that they can solve the problem well and, in the end, can be seen as a picture of student understanding [21]. If students do not understand the fundamental concept, it will have an impact on their ability to understand higher-level content, such as factoring polynomials, which will require them to apply addition and multiplication of polynomials to solve problems. Consequently, polynomials are essential for pupils to comprehend at a fundamental level to learn at a higher level with ease.

When students want to solve problems in mathematics, they need to have two types of knowledge, namely, procedural and conceptual knowledge. Procedural knowledge involves the ability to carry out memory of definitions, rules, principles and procedures in mathematics, and to utilize them when solving problems without substantially understanding of them, while conceptual knowledge refers to mathematical concepts and interconnected components of mathematical knowledge which contribute to an understanding of mathematical concepts, rules, and propositions [22]. It is important for students to be able to relate conceptual understanding to procedural skills. In particular, students are expected to explain the concept and choose steps that will be applied for solving mathematical problems [23].

Many researchers recognize that these two types of mathematical knowledge are useful in learning and aid student comprehension. On the other hand, school teachers tend to be more concerned with acquiring accurate answers using rules or procedures than with the concept about why and how procedures work [24]. In other words, they emphasize procedural rather than conceptual knowledge when teaching mathematics. For instance, some teachers prepare several exercises as a repetitive process to demonstrate whether students understand the content. Unfortunately, students with only procedural knowledge cannot solve real-world situations as a result of a lack of conceptual knowledge. They cannot make the connections between the concepts and the problem-solving situations [25], which corresponds to the findings in previous studies [26], [27] that the process of describing and justifying solutions for accurate and inaccurate examples is more valuable for attaining learning outcomes than describing and justifying solutions for the accurate solutions only. Teaching with a focus only on procedural skills will diminish learning in the classroom. It is not adequate to provide students with mathematical skills for the future [24]. Therefore, they can easily forget and may not be aware when they make a mistake.

If there is a problem with some knowledge, problem-solving errors may occur. According to Pomalato *et al.* [28], mistakes are a word used in science and mathematics to describe systematic, consistent, or unintentional deviations from an accurate value, which corresponds to Riantini *et al.* [29], in which an error is described as an alteration from the real solution of a problem. As defined in mathematics, an error is a deviation from the correct solution to a problem. Errors can also be observed in incorrectly answered problems from students' learning styles when solving a mathematical problem [30]. It impedes mathematics learning as it prevents students from achieving their learning goals. These errors can occur for some students. According to Luneta and Makonye [31], errors in mathematics could be caused by various factors, including carelessness, a lack of concentration, or a pattern of mistakes. Errors do not occur regularly, but they can occur through the existing basic knowledge. The answer may be wrong, but students can improve, or correctness can be achieved more readily.

Many researchers have divided types of errors from different perspectives. Baidoo [32] studied types of errors in algebraic fractions and identified students' errors in four types. These four types are: i) Mathematical language errors which result from a learner's lack of comprehension of mathematical technical jargon, a misunderstanding of how to use operation symbols, and a misunderstanding of how to use letters; ii) Procedural errors which occur when rules and formulas are mixed up when solving mathematical problems; iii) Concept errors which occur when learners connect their understanding to a topic that they do not comprehend its principles or qualities; and iv) Application errors which occur when learners understand the concept but are unable to utilize it correctly to solve problems. According to Agustyaningrum *et al.* [33], there are three sorts of errors: i) Careless errors which occur when students are not paying attention or working too quickly in mathematics; ii) Computational errors which arise when students are unable to identify a sign, digit, or place value, or when they utilize the wrong formula; and iii) Conceptual errors which occur from misconceptions about the fundamental principles and concepts associated with the mathematical problem.

In another study [34], errors in mathematics made by students are categorized into three categories, including: i) Factual mistakes which are errors that occur when students are not mastering in a fact required to solve the problem; ii) Procedural mistakes which are errors occurs when students inaccurately apply mathematical operations; iii) Conceptual mistakes which are errors caused by misunderstandings or misconceptions about the theories and concepts related to the problem. Oktaviani [35] identified four types of errors as: i) Conceptual errors which are made when students have a limited understanding of mathematical concepts and misconceptions; ii) Procedural errors which occur when students work in the wrong order; iii) Factual errors, also called, computational errors, which happen when students are unable to identify a sign, a digit, and a place value, or when they employ the wrong formula; and iv) Careless errors occurring when students are not paying concentration or working too quickly in mathematics. In study by Makhubele, Nkhoma, and Luneta [36], three types of errors are identified which are: i) Slips, little blunders made by students who are in a hurry; ii) Conceptual errors which occur when learners lack conceptual understanding as a result of a lack of comprehension of basic concepts, facts, and skills; and iii) Procedural errors occurring when student understand a concept but are unable to utilize it to solve a problem. They carry out the calculation without fully comprehending what they are doing.

According to Herholdt and Sapire [37], error analysis is performed to find interpretations for the reasoning of errors and mistakes formed in learners' work. Besides, it can assist teachers in identifying students' weaknesses, allowing them to identify problems with any topic and solve them to get on point, allowing students to learn mathematics in a simple and more effective way. Moreover, error analysis is the process of reviewing errors in order to provide feedback and remedial instruction to enhance learning and performance [38]. According to Lee [39], the analysis of students' work from worksheets and exercise books would help teachers to understand the students' process of understanding and problems with conceptual understanding in mathematics. It is critical to provide opportunities for students to practice, review, or reinforce the material already covered in the class, and determine whether they have comprehended the materials and have achieved the expected learning outcomes [40]. Furthermore, it allows students to direct their own learning and select how and where to apply assigned tasks [41]. These errors will occur when students complete the task, and teachers should check virtually all of them for causes of errors.

As polynomials is a basic concept for consequent topics in mathematics and is one of the topics that students frequently make mistakes, it is crucial to identify common errors made by students. This research aims to answer the following questions: i) What are common errors made by students in polynomials according to similar terms of monomials, addition of polynomials, subtraction of polynomials, and multiplication of polynomials?; ii) Which type of errors among careless error, computational error, and conceptual error is the most common errors made by students in polynomials? The main contribution in this paper is that the errors identification and classification were considered in detail for each topic, which is in sequential, including similar terms of monomials, addition of polynomials, subtraction of polynomials, and multiplication of polynomials. Therefore, it could help instructors to be aware of difficulties that would affect students' understanding of polynomials. For that reason, the researchers are interested in analyzing and classifying errors on polynomial topics with a focus on mistakes made by students while working in their exercise books.

2. RESEARCH METHOD

2.1. Research design

This research employs quantitative approaches to examine student errors in polynomials. Students' comprehension provides the required conditions for learning a higher level of mathematics. Therefore, an error assessment is required to effectively enhance the teaching at the next chance and help students to be able to apply the knowledge at a higher level of education. The concept of quantitative approaches used in this research can be summarized in Figure 1.

2.2. Sample

A total of 97 eighth grade students from a school in Bangkok, Thailand was used to create a sample. A purposive sampling strategy was used to select the sample, which included 72 students in eighth grade of the English and Mini-English programs. The sample consisted of students who had already studied polynomials and had an understanding of the mathematical terms used in the test.

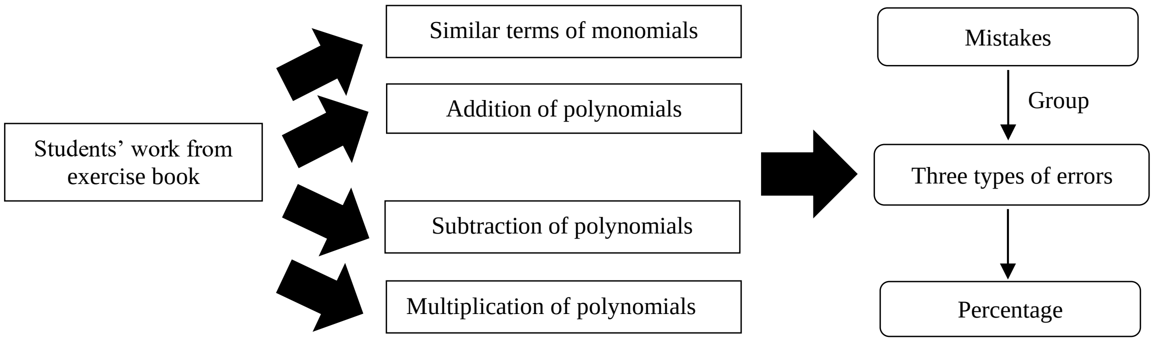

Figure 1. Quantitative approach for identifying errors

2.3. Data collection

Students' work from an exercise book was collected for error analysis on polynomials. There were several patterns in each exercise, such as true or false questions for similar terms, and open-ended question requiring solution details for addition, subtraction, and multiplication of polynomials as indicated in Table 1. Students' mistakes for each topic were collected and categorized into three types of errors, after which the percentage of students with an incorrect answer was calculated. For students whose mistakes differed from others, an interview was used to acquire the reason for mistakes found in each topic, which is also used for the discussion to support the data analysis.

To analyze mistakes found in students' exercise books, the researchers categorized mistakes into three types of errors as careless error, computational error, and conceptual error [34]. Their definitions and examples are given in Table 2. This process is carried out for each topic in polynomials.

Table 1. The topic for error analysis and type of questions in polynomials

No	Topic	Type of question
1	Similar terms of monomials	True-false
2	Addition of polynomials	Open-ended question
3	Subtraction of polynomials	Open-ended question
4	Multiplication of polynomials	Open-ended question

Table 2. Type of errors

Types of errors	Definition	Example
Careless error	Students lack concentration and are careless from working too fast when doing mathematics.	Find the sum of $4a^3 – 8a^2 + 5a + 3$ and $6a^3 + 7a^2 – 5$. $4a^3 – 8a^2 + 5a + 3$ $\underline{6a^3 + 7a^2 \quad – 5}$ + $\underline{10 – a^2 + 5a – 2}$ It shows that students do not write the term a^3 because they are careless in writing the answer.
Computational error	Students know the concept, but they make mistakes when adding, subtracting, multiplying, or dividing.	Find the difference of $9 + 3x – 7x^2$ and $5x^2 – 13x$. $9 + 3x – 7x^2$ $\underline{– 13x + 5x^2}$ – $\underline{9 + 16x – 2x^2}$ The example shows that students make mistakes in the calculation of the coefficient of x^2.
Conceptual error	Students have a poor understanding of the principles and ideas connected to mathematical concepts or cannot apply their knowledge correctly to mathematical concepts.	$(6x + 2)(3x^2 – 5)$ $= 18x^3 – 30x + 6x^2 – 10$ $= 18x^3 + 6x^2 – 40$ The example shows that students add -30x and -10 together to get the result is -40 but these two terms are not similar terms so they cannot be added together which is not consistent with the principle of adding two monomials.

2.4. Data analysis

In this research, a quantitative approach is used to analyze and classify mistakes and errors. The mistakes made by students are collected for each topic in polynomials. For the topic of similar terms, there are seven common mistakes, called ST1-ST7, which are classified as careless error, computational error or conceptual error shown in Table 3. There were two careless errors, two computational errors and three conceptual errors. For each item, the percentage of students with an incorrect answer is subsequently calculated.

As shown in Table 3, the mistake ST1 shows that students thought that two monomials xy and yx have different variables, so they answered that the two monomials are not similar. Secondly, the mistake ST2 indicates that students thought that when two monomials have different coefficients, it means the monomials are not similar terms. Thirdly, the mistake ST3 shows that students thought that two monomials are not similar because they are in different forms. Fourthly, the mistake ST4 found that students thought that two monomials have the same variables: x and y, and they concluded that two monomials are similar terms.

Next, the mistake ST5 shows that students thought that two constants which are different types of numbers, such as 15 is a positive number and –10 is a negative number, are not equal numbers, so they answered that these two constants are not similar terms of monomials. Sixthly, the mistake ST6 shows that students answered that these two monomials are similar terms because they have the exponent of the variables more than 1 even though the exponent of the same variable is not equal. It makes students confused as to whether the two monomials are similar or not. Lastly, for the mistake ST7, it is found from interviews that students looked at the exponent of variables, so they concluded that two monomials are similar terms.

Table 3. Type of mistakes and errors in similar terms

Topic in polynomials	Type of errors	Mistakes	Code	Example of mistakes
Similar terms	Careless	Two monomials with the same variable and exponent but the different positions are similar terms.	ST1	Students answer $9xy$ and $-15yx$ are not similar terms.
		Two monomials that have the same variable but have different coefficients that are similar terms of monomials.	ST2	Students answer $-x^3y^2$ and $-3x^3y^2$ are not similar terms
	Computational	Two monomials that have different forms but once simplified are similar terms of monomials.	ST3	Students answer $-3xyz$ and $\frac{-5x^2y^2z^2}{xyz}$ are not similar terms.
		Two monomials that have different forms but once simplified are not similar terms of monomials.	ST4	Students answer $\frac{6x^3y^4}{xy}$ and $7xy^3$ are similar terms.
	Conceptual	Two monomials are constants.	ST5	Students answer 15 and -10 are not similar terms.
		Two polynomials that have the same variables, but the exponent of the same variable is not equal are not similar terms of a monomial.	ST6	Students answer $16a^2b$ and $61ab^2$ are similar terms.
		Two monomials that have the same exponent, but different variables are not similar terms of a monomial.	ST7	Students answer $4z^2$ and $5y^2$ are similar terms.

For the topic of addition of polynomials, there are five common mistakes, called A1-A5, which are classified as careless error, computational error or conceptual error shown in Table 4. There were three careless errors, one computational error and one conceptual error. For each item, the percentage of students with an incorrect answer is subsequently calculated.

As presented in Table 4, in the topic of addition of polynomials, students made several mistakes. The mistake A1 shows that students subtracted two polynomials instead of adding two polynomials because they did not know whether two polynomials should be added or subtracted. The question uses the comma sign between two polynomials without providing the mathematical operator, so students got confused about the operation of two polynomials and gave an incorrect answer. The mistake A2 shows that students wrote the term in the dividend polynomial by writing $5x^3$ instead of $-5x^3$ because students were careless in doing their work. They incorrectly wrote the coefficient of x^3 so the result came out incorrectly.

The mistake A3 shows that students made an incorrect addition of the term $4a^3 + 6a^3$. The correct answer is $10a^3$ but students wrote only 10 because they were careless in writing their answers. The answer after simplifying is thus not correct. The mistake A4 shows that students were wrong in the calculation for adding the coefficients of a^2. The correct answer of $-8a^2 + 7a^2$ is $-a^2$, but students got the answer a^2 so that the answer given is not correct. The mistake A5 shows that students did not add two similar terms which led to the wrong answer as $-14ab^3$. This deviates from the polynomial addition principle where two similar terms are added into a simplified form.

Table 4. Type of mistakes and errors in addition of polynomials

Topic in polynomials	Type of errors	Mistakes	Code	Example of mistakes
Addition	Careless	Students misunderstood that the two polynomials were subtracted despite being added.	A1	Find the sum of $3xy - 4xz + 6xyz$ and $-7xyz - 5xy - 10xz$! Answer: $3xy - 4xz + 6xyz$ $\underline{5xy + 10xz + 7xyz}$ + $\underline{8xy + 6xz + 13xyz}$
		Students used wrong polynomials in calculation.	A2	Find the sum of $3 - 2x^2 - 5x^3 + 6x$ and $7x^2 + 4x$! Answer: $5x^3 - 2x^2 + 6x + 3$ $\underline{+ 7x^2 + 4x}$ + $\underline{5x^3 + 5x^2 + 10x + 3}$
		Students wrote incorrect answers.	A3	Find the sum of $4a^3 - 8a^2 + 5a + 3$ and $6a^3 + 7a^2 - 5$ Answer: $4a^3 - 8a^2 + 5a + 3$ $\underline{6a^3 + 7a^2 - 5}$ + $\underline{10 - a^2 + 5a - 2}$
	Computational	Students were unable to determine the sum of coefficients of two similar terms when combining two polynomials.	A4	Find the sum of $4a^3 - 8a^2 + 5a + 3$ and $6a^3 + 7a^2 - 5$ Answer: $4a^3 - 8a^2 + 5a + 3$ $\underline{6a^3 + 7a^2 - 5}$ + $\underline{6a^3 + 1a^2 - 2}$
	Conceptual	Students made a mistake when calculating two not similar terms.	A5	Find the sum of $a^3b - 10ab^3 - a^4 7b^4$ and $-5ab^3 + 3a^3b - 4b^4$ Answer: $a^3b - 10ab^3 - a^4 7b^4$ $\underline{3a^3b - 5ab^3 - 4b^4}$ + $\underline{8a^3b - 14ab^3 - a^4 7b^4}$

For the topic of subtraction of polynomials, there are five common mistakes, called S1-S5, which are classified as careless error, computational error or conceptual error shown in Table 5. There were four careless errors and one computational error. For each item, the percentage of students with an incorrect answer is subsequently calculated.

Table 5. Type of mistakes and errors in subtraction of polynomials

Topic in polynomials	Type of errors	Mistakes	Code	Example of mistakes
Subtraction	Careless	Students misunderstood that the two polynomials should be added even when they should be subtracted.	S1	Find the difference of $6a^2 + 8a - 5$ and $8 + 9a - 7a^2$ Answer: $6a^2 + 8a - 5$ $\underline{- 7a^2 + 9a + 8}$ + $\underline{-a^2 + 17a + 3}$
		Students used wrong polynomials in calculation.	S2	Find the difference of $8a^2 + 3$ and $9a^2 - 5a + 4$ Answer: $8a^2 + 3$ $\underline{- 9a^2 - 4}$ + $\underline{- a^2 - 1}$
		Students wrote incorrect answers.	S3	$(4x^2 + 6y + 9) - (7y + 3) - (5x^2 - 9y + 1)$ Answer: $4x^2 + 6y + 9$ $7y + 3$ $\underline{5x^2 - 9y + 1}$ - $\underline{-x^2 - y + 5 + 9x}$
		Students wrote the opposing polynomials incorrectly.	S4	Find the difference of $6a^2 + 8a - 5$ and $8 + 9a - 7a^2$ Answer: $6a^2 + 8a - 5$ $\underline{- 7a^2 - 9a - 8}$ + $\underline{- a^2 - a - 13}$
	Computational	Students were unable to calculate the difference in coefficients of two similar terms when subtracting two polynomials.	S5	Find the difference of $9 + 3x - 7x^2$ and $5x^2 - 13x$ Answer: $9 + 3x - 7x^2$ - $\underline{- 13x + 5x^2}$ $\underline{9 + 16x - 2x^2}$

As shown in Table 5, in the topic of subtraction of polynomials, the mistake S1 shows that students added two polynomials instead of subtracting because students thought that the previous question was adding polynomials and they were careless in doing their work. Thus, they considered adding two polynomials when they should subtract them, so the answer is not correct. The mistake S2 shows that students wrote the opposite polynomials incompletely as $-9a^2 - 4$ instead of $-9a^2 + 5a - 4$, because they were careless in doing their work. This resulted in a mistake after subtracting incomplete terms.

The mistake S3 shows that students wrote the wrong answer because they were careless in writing, making the wrong variable in their answer from y to x. By doing so, they were unable to add the term to get the correct answer that is $-x^2 + 8y + 5$. The mistake S4 shows that students wrote the sign of the opposite term incorrectly because they were careless when writing the sign of the subtrahend, making a mistake for subtracting two polynomials. Indeed, students made an error in writing the opposite sign for the coefficient of a^2 as $-8 - 9a - 7a^2$ instead of the correct term $-8 - 9a + 7a^2$, which contributes to mistake in subtraction. The mistake S5 shows that students subtracted the coefficient of x^2 incorrectly. The correct answer is $-7x^2 - 5x^2 = -12x^2$, but the students' answer is $-7x^2 - 5x^2 = -2x^2$ which is the incorrect answer.

For the topic of multiplication of polynomials, there are five common mistakes, called M1-M5 which are classified as careless error, computational error or conceptual error shown in Table 6. There was one careless error, three computational errors and one conceptual error. For each item, the percentage of students with an incorrect answer is subsequently calculated.

Table 6. Type of mistakes and errors in multiplication of polynomials

Topic in polynomials	Type of errors	Mistakes	Code	Example of mistakes
Multiplication	Careless	Students wrote incorrect answers.	M1	$(6x + 2)(3x^2 - 5)$ $= 18x^3 - 30x + 6x^2 - 10$ $= 18x^3 + 6x^2 - 30 - 10$
	Computational	Students could not multiply two monomial coefficients accurately.	M2	$(5x^2 - 3x)(2x^2 + 6x - 9)$ $= 10x^4 + 30x^3 - 45x^2 + 6x^3 + 18x^2 - 27x$ $= 10x^4 + (30x^3 + 6x^3) + (-45x^2 + 18x^2) + (-27x)$ $= 10x^4 + 36x^3 - 27x^2 - 27x$
	Computational	Students multiplied two polynomials incorrectly using the wrong indices properties.	M3	$(6x + 2)(3x^2 - 5)$ $= 18x^2 - 30x + 6x^2 - 10$ $= 24x^2 - 30x - 10$
		Students were unable to add two similar terms after multiplying two polynomials.	M4	$(5x^2 - 3x)(2x^2 + 6x - 9)$ $= 10x^4 + 30x^3 - 45x^2 - 6x^3 - 18x^2 + 27x$ $= 10x^4 + 24x^3 - 57x^2 + 27x$
	Conceptual	Students did add two unsimilar terms.	M5	$(6x + 2)(3x^2 - 5)$ $= 18x^3 - 30x + 6x^2 - 10$ $= 18x^3 + 6x^2 - 40$

From Table 6, the mistake M1 shows that students wrote some terms in the answer incorrectly because they were careless in writing the term $-30x$ to -30. So, the incorrect answer was given. The mistake M2 shows that students multiplied the term $-3x \cdot 2x^2$ incorrectly. The correct answer is $-6x^3$, but they got $6x^3$ and gave an incorrect answer. The mistake M3 shows that students multiplied the term $6x \cdot 3x^2$ incorrectly. The correct answer is $18x^3$; nevertheless, they got an incorrect answer as $18x^2$ which came from an error in applying the property of indices $a^m \cdot a^n = a^{m+n}$. The mistake M4 shows that students made a mistake in subtracting the coefficient of x^2. The correct answer is $-45x^2 - 18x^2 = -63x^2$ but they got $-57x^2$ which is an incorrect answer. The mistake M5 shows that students added two non-similar terms which were $-30x$ and -10, and gave the answer -40. They made mistakes in adding $-30x$ and -10 which are not similar terms. Indeed, the two terms cannot be combined since they are not similar and do not correspond with the principle of adding monomials, so the answer is not correct.

3. RESULTS AND DISCUSSION

3.1. Results

Based on students' work collected from the exercise books, the number of students who made mistakes is recorded for each incorrect item listed in Table 3 to Table 6. Then the percentage of incorrect answers from the total number of students is calculated for each topic of polynomials according to the three categories of errors, namely, careless error, computational error, and conceptual error. The average percentages of incorrect answers from the total number of students for each topic according to three types of errors are presented in Table 7.

Table 7. Average of percentage of errors in learners' exercise book in polynomials

No	Topic in polynomials	Mistakes	Type of errors	Percentage of incorrect
1	Similar terms	ST1, ST2	Careless	5.98
		ST3, ST4	Computational	17.86
		ST5, ST6, ST7	Conceptual	4.44
2	Addition	A1, A2, A3	Careless	4.78
		A4	Computational	10.88
		A5	Conceptual	0.46
3	Subtraction	S1, S2, S3, S4	Careless	4.80
		S5	Computational	12.04
4	Multiplication	M1	Careless	14.44
		M2, M3, M4	Computational	6.11
		M5	Conceptual	0.83

3.2. Discussion

It can be seen from Table 7 that computational error is the main source for mistakes in the topics of similar terms of monomials, addition, and subtraction of polynomials. However, for multiplication of polynomials, students tend to have careless errors more frequently. Further comments and reasons for errors made by students can be discussed.

3.2.1. Topic 1: similar terms

In similar terms of monomials, it can be seen that 17.86% of all students made computational errors as they used the properties of indices to simplify terms incorrectly. It caused them to believe that two monomials are not similar terms, even though they are. Conceptual errors were found in 4.44% of the total students. This can be divided into three categories. The first group is the students who misunderstand that two constants are not similar terms because they are different types of numbers, which corresponds to Seng [42] that discussed about two constants for positive and negative integers. After all, they were confused between the concept of negative integers and similar terms. The second group consists of students who incorrectly assume that two monomials with the same number of variables but different exponents are similar terms of a monomial. The result corresponds to [42], which found that students did not perceive the concept of like terms, and they made a common blunder of similar terms by comparing their coefficients rather than their variables. Two monomials are indeed similar if and only if the exponent of the same variables is equal.

The last group is the students who believe that two monomials with the same exponent but different variables are similar terms of monomials. Because they think that if any two monomials have the same exponent, the two monomials are similar terms. Then they immediately give answers which correspond to Ancheta and Subia [43], students did not recognize that the variables must be the same and that the corresponding exponent must also be the same for the terms to be similar. However, two monomials must have the same variables before considering the exponent of the same variables. This error may hinder specifying that two monomials are not similar terms. Finally, 5.98% of all students made a careless error when comparing any two monomials that are similar.

3.2.2. Topic 2: addition of polynomials

For the error analysis from students' work in the topic of addition of polynomials, it is discovered that 10.88% of all learners made computational errors because they added two integers by adding the coefficient of two similar terms incorrectly. This result corresponds to Makonye and Hantibi [44] that students made errors in addition between negative and positive numbers. For example, they added $65+45$ by assigning a subtraction sign. It may occur when they do not correctly understand the principles and rules about the operation between positive and negative integers [45]. Second, on average, 4.78% of students made careless errors because they hurried through their assignments. In other words, they already knew the answer but were negligent in obtaining it, resulting in an incorrect answer. Finally, it is learned that 0.46% of learners made conceptual errors by combining two non-similar terms, failing to follow the proper principle. This result agrees with Ferrer [46] that the students added $100x^3 - 5x + 93$ for a total of $98x^3$. Since the terms are not similar, this expression cannot be combined. Students frequently mistakenly believe x^3 and x to be similar terms, although they include different exponents, making them different terms. To summarize, most students comprehend the principle of adding two polynomials, but they make a mistake that is not involved in polynomials.

3.2.3. Topic 3: subtraction of polynomials

In the topic of subtraction of polynomials, it is found that 12.04% of students made computational errors because they subtracted the coefficients of two similar terms incorrectly. This result corresponds to

Seng [42] that this error happens more often when solving integer and simplifying algebraic calculations. Students had problems in subtracting with negative integers, indicating that they should review the subtraction of any two integers in every case. It contributes to the reduction of errors. Second, 4.80% of students made careless errors caused by three issues: writing the wrong problem, writing the wrong answer, and misunderstanding the operation of two polynomials. Students were careless in their work and became confused about the operation of two polynomials, which caused them to be careless with their work, resulting in mistakes and giving incorrect answers. In addition, students miswrote the opposite polynomials, and most students changed the sign in front of the term in the polynomial, causing the answer to be incorrect. This result corresponds to Marpa [47], which found that most students forgot to change the sign of the subtrahend before they proceeded to the addition. They directly proceed to the process without considering the operation. Finally, there are no students who made conceptual errors due to most students comprehend the principle of polynomial subtraction, yet they frequently make calculation errors and are hurry in writing the solution when subtracting two polynomials so that the answer is not correct.

3.2.4. Topic 4: multiplication of polynomials

For the error analysis from students' work in the topic of multiplication of polynomials, it is discovered that 14.44% of students had careless errors at the highest percentage because they made mistakes such as writing incorrect problems or answers. These errors occurred when they rushed to complete their tasks, lacked concentration, and failed to verify their solution to obtain the correct answer. Secondly, around 6.11% of students made computational errors due to incorrectly multiplying the coefficients for two integers in the case of the multiplication of any integers with negative integers. According to Daud and Ayub [48], when students multiply $-3x(2y - z)$, they fail to deal with the negative sign when performing algebraic multiplications and give the answer as $-6xy - 3xz$. This error resulted in an incorrect answer. In addition, they make errors in using the properties of indices to multiply two polynomials that correspond to Ulusoy [49]; this appears to corroborate the idea that students' knowledge of exponents is still procedural and it is not sufficient to accurately compute exponential expressions without understanding about number systems and the logic behind the computation. Indeed, these students are unable to comprehend the laws of exponents. The last is a conceptual error, for which 0.83% of students have made in this category. Students were wrong in adding two unsimilar terms after multiplying two polynomials which do not correspond to the principle of combining two monomials. The results show that most students already understand the multiplication principle of polynomials.

It should be noted that a clear understanding of both procedural and conceptual knowledge is required for students to successfully work on subsequent topics in polynomials. Students need to recognize similar terms of monomials in order to correctly compute addition, subtraction and multiplication of polynomials. They also require a previous understanding of principles and rules about the operation between positive and negative integers. These factors impact teachers, who must be aware of the need for precision in their answers at each step of the problem-solving process in order to reduce errors that have little bearing on other topics. Therefore, teachers should emphasize on correcting errors or instructing about errors before students could develop inaccurate computational procedures and concepts [50].

4. CONCLUSION

The topic of polynomials contains abstract contents which can be difficult for eighth grade students to visualize. This is a significant issue when solving polynomials. Students who have insufficient knowledge on polynomials tend to make errors and are not able to relate their knowledge to other topics in mathematics and other subjects. Error analysis aids teachers in identifying misunderstandings and providing additional information to increase understanding. This research identified mistakes that occur from students in four topics in polynomials according to three categories including careless error, computational error, and conceptual error. It is found that most students made mistakes in computational errors in similar terms, addition, and subtraction of polynomials accounted for 17.86%, 10.88%, and 12.04% respectively. However, in multiplication of polynomials, careless errors were the highest mistake which accounted for 14.44%. From the result, it shows that most students have computational errors but not conceptual errors.

From the findings and conclusion, it is recommended that teachers should design learning activities and strategies to improve understanding and visualizing of the operations with integers, which are the core foundation for students to perform the algebra of polynomials. This would help students to reduce computational error which is the major mistake in addition and subtraction of polynomials. Further investigation should be studied on the causes of mistakes and strategies to reduce them focusing on common mistakes found in this research.

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REFERENCES

[1] H. Julie, F. Sanjaya, and A. Y. Anggoro, "The students' ability in the mathematical literacy for uncertainty problems on the PISA adaptation test," in *AIP Conference Proceedings - The 4th International Conference on Research, Implementation, and Education of Mathematics and Science (4th ICRIEMS)*, 2017, vol. 1868, doi: 10.1063/1.4995153.
[2] Rusdawati, "The early childhood mathematics learning," in *Proceedings of the International Conference of Early Childhood Education (ICECE 2019)*, 2020. doi: 10.2991/assehr.k.200715.021.
[3] A. Esra, "Analyzing students views about mathematics teaching through stories and story generation process," *Educational Research and Reviews*, vol. 13, no. 7, pp. 249–259, Apr. 2018, doi: 10.5897/err2018.3498.
[4] P. Phonapichat, S. Wongwanich, and S. Sujiva, "An analysis of elementary school students' difficulties in mathematical problem solving," *Procedia - Social and Behavioral Sciences*, vol. 116, pp. 3169–3174, Feb. 2014, doi: 10.1016/j.sbspro.2014.01.728.
[5] G. P. Mingke and E. M. Alegre, "Difficulties encountered In mathematical word problem solving of the grade six learners," *International Journal of Scientific and Research Publications (IJSRP)*, vol. 9, no. 6, p. p9053, Jun. 2019, doi: 10.29322/ijsrp.9.06.2019.p9053.
[6] K. M. Themane and K. Luneta, "Misconceptions and associated errors in the learning of mathematics place value in South African primary schools: A literature review," *Preprint*, pp. 1–14, 2021.
[7] J. Enu and Z. Ngcobo, "Formative assessment: A tool for rectifying learners errors and misconceptions in mathematics," *Journal of Education and Training*, vol. 4, no. 3, pp. 48–52, 2020, doi: 10.31248/IJET2020.085.
[8] J. Voigt, "Negotiation of mathematical meaning and learning mathematics," in *Learning Mathematics*, Dordrecht: Springer Netherlands, 1994, pp. 171–194. doi: 10.1007/978-94-017-2057-1_6.
[9] E. Zahara, A. Murni, and N. M. Hutapea, "Development of mathematics learning tools by implementing numbered head together type cooperative models to improve students' mathematical understanding ability in matrix topic," *Journal of Educational Sciences*, vol. 4, no. 2, p. 250, Apr. 2020, doi: 10.31258/jes.4.2.p.250-260.
[10] K. P. Adhikari, "Difficulties and misconceptions of students in learning limit," *Interdisciplinary Research in Education*, vol. 5, no. 1–2, pp. 15–26, 2021, doi: 10.3126/ire.v5i1-2.34731.
[11] H. Harun, B. Kartowagiran, and A. Manaf, "Student attitude and mathematics learning success: A meta-analysis," *International Journal of Instruction*, vol. 14, no. 4, pp. 209–222, Oct. 2021, doi: 10.29333/iji.2021.14413a.
[12] R. Capuno, R. Necesario, J. O. Etcuban, R. Espina, G. Padillo, and R. Manguilimotan, "Attitudes, study habits, and academic performance of junior high school students in mathematics," *International Electronic Journal of Mathematics Education*, vol. 14, no. 3, May 2019, doi: 10.29333/iejme/5768.
[13] A. P. Makur, R. C. I. Prahmana, and B. Gunur, "How mathematics attitude of mothers in rural area affects their children's achievement," *Journal of Physics: Conference Series*, vol. 1188, p. 012009, Mar. 2019, doi: 10.1088/1742-6596/1188/1/012009.
[14] N. A. Albelbisi and F. D. Yusop, "Factors influencing learners' self-regulated learning skills in a massive open online course (MOOC) environment," *Turkish Online Journal of Distance Education*, vol. 20, no. 3, pp. 1–16, Jul. 2019, doi: 10.17718/tojde.598191.
[15] Y. AL-Rababaha, W. T. Yew, and C. C. Meng, "Misconceptions in school algebra," *International Journal of Academic Research in Business and Social Sciences*, vol. 10, no. 5, May 2020, doi: 10.6007/IJARBSS/v10-i5/7250.
[16] E. Warren, M. Trigueros, and S. Ursini, "Research on the learning and teaching of algebra," in *The Second Handbook of Research on the Psychology of Mathematics Education*, Rotterdam: SensePublishers, 2016, pp. 73–108. doi: 10.1007/978-94-6300-561-6_3.
[17] S. Şengül and İ. Üner, "What is the impact of the teaching 'algebraic expressions and equations' topic with concept cartoons on the students' logical thinking abilities?" *Procedia - Social and Behavioral Sciences*, vol. 2, no. 2, pp. 5441–5445, 2010, doi: 10.1016/j.sbspro.2010.03.887.
[18] C. L. Ying, S. Osman, D. Kurniati, E. S. Masykuri, J. A. Kumar, and C. Hanri, "Difficulties that students face when learning algebraic problem-solving," *Universal Journal of Educational Research*, vol. 8, no. 11, pp. 5405–5413, Oct. 2020, doi: 10.13189/ujer.2020.081143.
[19] M. Adnan *et al.*, "The ability and analysis of Students' errors in the topic of algebraic expression," *Journal of Physics: Conference Series*, vol. 1988, no. 1, p. 012049, Jul. 2021, doi: 10.1088/1742-6596/1988/1/012049.
[20] J. Sikombe, R. L. Sampa, and A. Musonda, "Grade 11 learners' misconceptions and inability to apply basic algebra operations: The case of selected secondary schools in Chilanga district, Zambia," *Journal of Education and Practice*, Nov. 2021, doi: 10.7176/jep/12-33-11.
[21] R. Oktaviani, "Mistake analysis of MA students solve mathematics on polynomial materials," *Journal Of Educational Experts (JEE)*, vol. 3, no. 1, p. 25, Mar. 2020, doi: 10.30740/jee.v3i1p25-30.
[22] H. E. Zuya, "Prospective teachers' conceptual and procedural knowledge in mathematics: The case of algebra," *American Journal of Educational Research*, vol. 5, no. 3, pp. 310–315, 2017, doi: 10.12691/education-5-3-12.
[23] D. Salim Nahdi and M. Gilar Jatisunda, "Conceptual understanding and procedural knowledge: A case study on learning mathematics of fractional material in elementary school," *Journal of Physics: Conference Series*, vol. 1477, no. 4, p. 042037, Mar. 2020, doi: 10.1088/1742-6596/1477/4/042037.
[24] H. Roselizawati Hj Sarwadi and M. Shahrill, "Understanding students' mathematical errors and misconceptions: The case of year 11 repeating students," *Mathematics Education Trends and Research*, pp. 1–10, 2014, doi: 10.5899/2014/metr-00051.
[25] M. A. Al-Mutawah, R. Thomas, A. Eid, E. Y. Mahmoud, and M. J. Fateel, "Conceptual understanding, procedural knowledge and problem-solving skills in mathematics: High school graduates work analysis and standpoints," *International Journal of Education and Practice*, vol. 7, no. 3, pp. 258–273, 2019, doi: 10.18488/journal.61.2019.73.258.273.

[26] L. A. Curry, "The effects of self-explanations of correct and incorrect solutions on algebra problem-solving performance," *Proceedings of the Twenty-Sixth Annual Conference of the Cognitive Science Society*, 2004, p. 1548.
[27] R. S. Siegler, "Microgenetic studies of self-explanation," in *Microdevelopment*, Cambridge University Press, 2009, pp. 31–58. doi: 10.1017/cbo9780511489709.002.
[28] S. W. D. Pomalato, L. Ili, B. A. Ningsi, F. Fadhilaturrahmi, A. T. Hasibuan, and K. H. Primayana, "Student error analysis in solving mathematical problems," *Universal Journal of Educational Research*, vol. 8, no. 11, pp. 5183–5187, Oct. 2020, doi: 10.13189/ujer.2020.081118.
[29] N. N. Riantini W., I. N. Suparta, and I. G. P. Sudiarta, "Analysis of student errors in learning circle based on the Watson error category," *International Journal on Emerging Mathematics Education*, vol. 4, no. 1, p. 31, Dec. 2020, doi: 10.12928/ijeme.v4i1.15840.
[30] J. Ryan and J. Williams, *Children's mathematics 4–5: Learning from errors and misconceptions*. McGraw-Hill Education, 2007.
[31] K. Luneta and P. J. Makonye, "Learner errors and misconceptions in elementary analysis: A case study of a grade 12 class in South Africa," *Acta Didactica Napocensia*, vol. 3, no. 3, pp. 35–45, 2010.
[32] J. Baidoo, "Dealing with grade 10 learners' misconceptions and errors when simplifying algebraic fractions'," *Journal of Emerging Trends in Educational Research and Policy Studies*, vol. 10, no. 1, pp. 47–55, 2019, doi: 10.10520/EJC-17aa794007.
[33] N. Agustyaningrum, A. M. Abadi, R. N. Sari, and A. Mahmudi, "An analysis of students' error in solving abstract algebra tasks," *Journal of Physics: Conference Series*, vol. 1097, p. 012118, Sep. 2018, doi: 10.1088/1742-6596/1097/1/012118.
[34] H. T. Lestiana, S. Rejeki, and F. Setyawan, "Identifying students' errors on fractions," *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, vol. 1, no. 2, pp. 131–139, Aug. 2017, doi: 10.23917/jramathedu.v1i2.3396.
[35] M. Oktaviani, "Analysis of students' error in doing mathematics problem on proportion," in *2nd Asian Education Symposium*, 2018, pp. 172–177. doi: 10.5220/0007300601720177.
[36] Y. Makhubele, P. Nkhoma, and K. Luneta, "Errors displayed by learners in the learning of grade 11 geometry," in *ISTE International Conference on Mathematics, Science and Technology Education*, vol. 12, 2015, pp. 26–44. [Online]. Available: https://uir.unisa.ac.za/handle/10500/22450.
[37] R. Herholdt and I. Sapire, "An error analysis in the early grades mathematics – a learning opportunity?" *South African Journal of Childhood Education*, vol. 4, no. 1, p. 19, Jul. 2014, doi: 10.4102/sajce.v4i1.46.
[38] P. Muthukrishnan, M. S. Kee, and G. K. Sidhu, "Addition error patterns among the preschool children," *International Journal of Instruction*, vol. 12, no. 2, pp. 115–132, Apr. 2019, doi: 10.29333/iji.2019.1228a.
[39] C.-D. Lee, "Worksheet usage, reading achievement, classes' lack of readiness, and science achievement: A cross-country comparison," *International Journal of Education in Mathematics, Science and Technology*, vol. 2, no. 2, 2014, doi: 10.18404/ijemst.38331.
[40] H. Cooper, *The battle over homework: Common ground for administrators, teachers, and parents*. United States: Corwin Press, 2007. doi: 10.4135/9781483329420.
[41] M. F. Mortensen and K. Smart, "Free-choice worksheets increase students' exposure to curriculum during museum visits," *Journal of Research in Science Teaching*, vol. 44, no. 9, pp. 1389–1414, Nov. 2007, doi: 10.1002/tea.20206.
[42] L. Kok Seng, "An error analysis of form 2 (Grade 7) students in simplifying algebraic expressions: A descriptive study," *Electronic Journal of Research in Education Psychology*, vol. 8, no. 20, Nov. 2017, doi: 10.25115/ejrep.v8i20.1398.
[43] C. M. D. Ancheta and G. S. Subia, "Error analysis of engineering students' misconceptions in algebra," *International Journal of Engineering Trends and Technology*, vol. 68, no. 12, pp. 66–71, Dec. 2020, doi: 10.14445/22315381/IJETT-V68I12P212.
[44] J. Makonye and N. Hantibi, "Exploration of grade 9 learners' errors on operations with directed numbers," *Mediterranean Journal of Social Sciences*, Sep. 2014, doi: 10.5901/mjss.2014.v5n20p1564.
[45] A. N. K. Rosyidah, M. A. Maulyda, I. S. Jiwandono, I. Oktaviyanti, and G. Gunawan, "Misconceptions and errors in integer operations: A study in preservice elementary school teachers (PGSD)," *Journal of Physics: Conference Series*, vol. 1779, no. 1, p. 012078, Feb. 2021, doi: 10.1088/1742-6596/1779/1/012078.
[46] G. A. Ferrer, "Error analysis in the operations of algebraic expressions of grade 8 Students at Kalinga state university laboratory high school," *International Journal of English Literature and Social Sciences*, vol. 5, no. 6, pp. 2494–2504, 2020, doi: 10.22161/ijels.56.89.
[47] E. P. Marpa, "Common errors in algebraic expressions: A quantitative-qualitative analysis," *International Journal on Social and Education Sciences*, vol. 1, no. 2, pp. 63–72, Sep. 2019, doi: 10.46328/ijonses.11.
[48] M. Y. Daud, and A. S. Ayub, "Student error analysis in learning algebraic expression: A study in secondary school Putrajaya," *Creative Education*, vol. 10, pp. 2615–2630, 2019, doi: 10.4236/ce.2019.1012189.
[49] F. Ulusoy, "Serious obstacles hindering middle school students' understanding of integer exponents," *International Journal of Research in Education and Science*, vol. 5, no. 1, pp. 52–69, 2019.
[50] G. Nelson and S. R. Powell, "Computation error analysis: Students with mathematics difficulty compared to typically achieving students," *Assessment for Effective Intervention*, vol. 43, no. 3, pp. 144–156, Jun. 2018, doi: 10.1177/1534508417745627.

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