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Making sense of students' errors in solving problems related to measures of dispersion

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ABSTRACT

The importance of statistical learning has been widely discussed. However, little effort has been made to understand the difficulties of students in learning measure of dispersion, which is the key component to statistical learning. The present study sought to identify the reasons behind the errors committed in solving problems related to measures of dispersion, by examining students' errors in the diagnostics tests, followed by in depth interviews to elicit their thinking and understanding. There were 85 grade-11 high school students involved in the first phase of the quantitative research and 10 students with weak performance were subsequently selected for the second phase qualitative research. The interviews were conducted using the contingent teaching model. The findings indicated that students' committing errors in solving problems related to measure of dispersion due to lacking statistical vocabulary knowledge, weak symbol sense, rote learning, low statistical reasoning and statistical thinking ability. The results of the study and proper remedials are discussed.

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1. INTRODUCTION

Statistics is the art of gaining insight through data analysis. The relevance of studying statistics has come into the spotlight as more data is generated and collected than ever before, humans are continually overwhelmed with statistics from a variety source [1]. This is largely attributable to the emergence of data science, which is associated with big data and the so-called fourth industrial revolution [2]. Unforeseen health-related catastrophes, such as COVID-19, have revealed significant gaps in people's statistical and probabilistic skills in interpreting mathematical and stochastic models, which could result in poor individual or societal decision-making with dire consequences for the entire population [3]. However, past research showed that statistics and probability are still given insufficient classroom time [4]. This deprives students of the statistical literacy necessary to appreciate the claims made in contemporary issues, such as COVID-19.

Although many statistics classrooms in the world nowadays are experiencing the integration of technology and practicing creative instructions to teaching statistics, past research show that students have difficulties related to the learning of statistics in measures of central tendency, variability and distribution, and statistical reasoning [5]–[8]. Teachers and researchers of statistics often overestimate school students' mastery of fundamental concepts in statistics and underestimate their difficulties in the same subject [9]. Some students found statistical concepts to be complex and difficult [10], lack of statistical knowledge [11], maintain negative perception towards statistics [12], and lose interest in the subject [13], [14].

Statistical variation is integral to all aspects of statistical problem-solving and is at the heart of statistics [15], [16]. An understanding of statistical variation or measures of dispersion is needed to understand complex concepts such as sampling distribution, inference and ρ-values [17], [18]. Prior to recent years, there was a paucity of studies on the extent to which students understood the concept of variation [19], [20] notwithstanding the importance of the notion to statistics [16], [21]. Past studies [15], [20] showed that there is an evident conceptual gap in student knowledge of variation. Another studies [22], [23] have been undertaken on how different teaching methods affect students' grasp of sampling variability. Further investigations entailed understanding the statistical linkages needed to reason about variation [20], [24]; students' statistical reasoning on variation [25], [26]; expository writing presents viewpoints needed to understand variation [27]; analysis of student's understanding on variation [28]; and frameworks for understanding variation [21], [29].

Without variation, neither statistics nor statisticians would exist [16]. A misconception and incomplete understanding of the measures of dispersion many cause errors in students solving statistical problems and further limit students' understanding of learning more advance statistical topics [19]. Keeping a misconception is a natural part of learning and obtaining more right concepts (or expert comprehension) may require students to maintain two or more competing conceptions simultaneously [29]. To rectify these misconceptions, teachers must learn what students know and believe. The perception that many teachers lack experience with statistics [30] adds urgency to the need for teachers and students to gain a grasp of measures of dispersion by explaining the causes of students' errors. Moreover, past studies [31], [32] ascribed the negative attitude towards statistics to the students' poor performance in the subjects. Despite many facets of measures of dispersion were well documented, little is known regarding the high school students' grasp of measures of dispersion, in particular their difficulties in solving problems involving measures of dispersion, and the causes of their errors or misconceptions. Hence, this study seeks to fill in the gap by focuses on gaining insights into the difficulties and reasons of high school students committing errors in solving problems related to measures of dispersion. This may provide teachers with some direction as they seek to improve their students' understanding of and facility with statistical concepts in the classroom.

2. LITERATURE REVIEW

Measures of dispersion is the tool used to determine "the extent to which the magnitudes or attributes of the objects differ," or the degree of diversity [33]. Numerous instruments, including the range, interquartile range, sample variance, and standard deviation, have been used to characterize dispersion. It is essential to notice the broad diversity of terms used in the literature to explain measures of dispersion, which include variability, variation, fluctuation, spread, scale, and scatter [33], [34]. Some researchers assert that "variation" and "variability" can be used interchangeably [21]. The understanding of variation begins with recognizing that variability is omnipresent, that it can be observed everywhere and in everything [35]. Reading and Shaughnessy [20] however evaluated them differently, with variability being the apparent attribute of the thing and variation pertaining to displaying or evaluating the attribute. There is no agreed-upon or consistent way to make this distinction in terminology thus far [34].

The significance of measures of dispersion as a foundation for statistical education in schools has increased since the explanation of "uncertainty" in connection to chance and data [36]. Measures of dispersion adds intrigue to statistics and enables us to comprehend, analyze, and anticipate based on data. The Guidelines for Assessment and Instruction in Statistics Education (GAISE) report identifies comprehension of data variability as the most fundamental introduction concept to statistical reasoning [37]. In addition, the report recommends statistics instructors to prioritize conceptual grasp over procedural memory. Gould [27] stated that variation is the fuel for statistical imagination, which according to Wild and Pfannkuch [38] is one of the eight attributes of statistical thinkers. Although statisticians have acknowledged the importance of variation to statistics [38], [39]; this fact is rarely emphasized in statistics courses [23]. School curriculum tends to place more emphasis on measures of center tendency rather than on measures of dispersion [15], [24]. It was claimed that young students found it easier to calculate the mean or a basic probability than the standard deviation [15].

Leading statistics educators [40] claimed that despite the effort in reformation in the learning and teaching of statistics, statistics education is still considered as a challenging discipline. Students with varying background and abilities could intensify the challenges in teaching statistics [32]. Another reason could be that statistical education in school focuses on the procedural and computational aspects of statistics rather than on developing conceptual understanding. The traditional emphasis on skills development has resulted in many students not being able to think or reason statistically [39]. This study reveals the five main reasons that causes the errors committed by grade 11 students in solving problems related to measures of dispersion.

2.1. Statistical vocabulary knowledge

Students' fluency of mathematical language is important to the development of one's conceptual understanding of content knowledge and skills [41]. Previous study [42] showed that focused instruction on mathematical vocabulary may help the low performing students in learning mathematics. Rubenstein and Thompson [43] claimed that there are at least 11 categories of difficulties associated with learning the language of mathematics. As for statistics, statistical language consists of a blend of general English, mathematical English, and statistics-specific English, often known as statistical English [44]. Rangecroft [45] and Rothery [46] identified six categories of words used in statistics based on the meaning of the words in terms of general English, statistical English and mathematical English. Dunn *et al.* [44] further described the difficulties students encounter when learning statistics due to the terminology used. Some "lexically ambiguous words" that have a more specific meaning in statistics than in general English are said to generate difficulty and confusion in the learning of statistics by students [47]–[49]. Without proper vocabulary instruction, students were confused with the application and definition of the statistical vocabulary especially when these terms are abstract.

2.2. Rote learning

Mayer [50] categories learning into three scenarios i.e. no learning, rote learning, and meaningful learning. Students who have attended to the materials but cannot understand the relevant information is characterized as rote learning. In the meaningful learning, a student not only possesses the relevant knowledge, but is able to transfer that knowledge to solve problems and understand new concepts. Rote learning in mathematics is the mastering of a rule or procedure through the process of repeated learning without understanding the reasons that make it work. Students' misconception and low achievement in solving and reasoning mathematics problems could be due to inefficient rote learning [51]. When mathematic curricula is rigid and emphases academic achievement rather than the process of learning, students will tend to memorizing procedures, instead of seeking solution; memorizing formulas instead of observing patterns; and doing exercise instead of formulating conjectures [52].

2.3. Symbol sense

Symbols are the component of the mathematics language that enable the communication, manipulation and reflection upon abstract mathematical concepts. Arcavi [53] defined symbol sense as the skill to appreciate the power of symbols, the right application and manipulation of symbols in a range of context. Past studies [43], [54] revealed that students often struggle and confuse over the symbolic representations in mathematics mainly due to the conciseness and abstraction. Students often perceived their personal meaning to symbols, and failure to manipulate and understand mathematics symbols has attributed to students' difficulties in mathematics learning [55], [56]. Rubenstein and Thompson [43] categorized the challenges related to learning mathematical symbols into three areas, namely: i) Verbalization challenges (i.e. the translation of symbols into spoken language); ii) Reading challenges (i.e. conceptual understanding of the symbols); and iii) Writing difficulties (i.e. producing symbols). These challenges are complex and often occur simultaneously.

2.4. Statistical reasoning

Garfield and Gal [26] defined statistical reasoning as a way of reasoning with statistical ideas and understanding statistical information. Other researchers defined statistical reasoning as making sense of the statistical information [17], interpretating statistical results, summarizing statistical data, and draw conclusion from data [19], [57]. In addition, previous researchers [58], [59] characterized students' reasoning across four levels: idiosyncratic, transitional, quantitative and analytical. At the idiosyncratic level, students' reasoning is narrowly and consistently bound to idiosyncratic or subjective reasoning. Students provide irrelevant information and often focused on personal experiences or subjective beliefs. At the transitional level, students began to reasoning quantitatively, but are inconsistent in their use of such reasoning. At the quantitative level, students' reasoning is consistently quantitative and they can identify the problem but do not necessarily make sense and apply the relevant mathematical ideas in solving the problem. At the analytical level, students are able to represent the multiple aspects of a problem into a meaningful structure such as creating multiple data displays, or making a reasonable prediction.

2.5. Statistical thinking

Statistical thinking are the cognitive actions that students engage in during the data handling processes of describing, organizing and reducing, representing, and analyzing and interpreting data [59]. This definition is different from the definitions emerged from the statisticians which focuses more on the practical experiences and reflections. Wild and Pfannkuch [38] categorized statistical thinking into general types of thinking which seeking explanations and applying techniques; and fundamental statistical thinking which involves recognition of need for data, consideration of variation, and reasoning with statistical models. In Garfield middle school student statistical thinking (M3ST) framework [26], he characterized middle school students' thinking in statistical situation into four processes: i) Describing data; ii) Organizing and reducing data; iii) Representing data; and iv) Analyzing and interpreting data. The four statistical processes are closely interrelated, and determining students' ability to analyze and interpret data.

3. RESEARCH METHOD

3.1. Participants

There were two phases of data collection which consists of a quantitative approach in the first phase, while a qualitative approach in the second phase. The first phase of the study involved 85 grade-11 students at a private school in Penang, Malaysia. All the students were taught about the calculation and simple application of measures of dispersion such as range, interquartile range, quartile deviation, variance, and standard deviation before participating in this study. In the second phase, purposive sampling was used for the case study, where 10 students with weak performance were selected to undergo in-depth interviews. The student's test scores were used to identify the target group for this research question. The researcher has chosen to use purposive sampling in view that purposive sampling involves the researcher selecting individuals who have knowledge of the phenomena studied or deemed potential information rich cases [60].

3.2. Research design

This study adopted a mixed method methodology. In the first phase, a diagnostic test with no formula given (Appendix 1) was used to explore the students' errors in measures of dispersion. In the second phase, interviews were conducted with 10 students selected from phase one. During the interview, students were questioned to elicit their thinking in the errors committed using the contingent teaching method adapted from past research [61]. Following the step-by-step modelling of contingent teaching, the researcher first referred to the answers that students had written in the test and diagnose students' current or actual understanding. Second, the researcher checked the diagnosis by using why and how questions to elicit the student's thinking and strategy in solving the problems. Third, the researcher supported the student contingently, using the gathered information and probes/hints. Finally, the researcher checked the student's new (potential) understanding of the learning. The researcher made detailed field notes during each interview. The interviews were audiotaped and transcribed verbatimly.

3.3. Data analysis

The data analysis used in this study was adapted based on the three phases described by Kurasaki [62]. The phase one involved the codebook development where the researcher familiarized with the data and identified potential themes. The purpose of this phase was to identify themes in the text and to refine the themes into codes that form the basis of a codebook. In the phase two, intercoder reliability was established to ensure the reliability of the coding. Both the intercoder agreement [63] between two coders and the Cohen's kappa results were satisfactory at 93.2% and 88.5% respectively. In the phase three, the researcher applied the codebook systematically to the data set using the agreed-upon themes.

4. RESULTS AND DISCUSSION

4.1. Results

In the first phase, the degree of understanding of measures of dispersion concepts among students, was analyzed. There were altogether 11 items in the test with no formulas provided (Appendix 1). The total scores for the test were 20 marks. Each item was allocated 1 to 3 marks. From the analysis, it showed that the range of the marks among the 85 students was 18, where the lowest was zero mark and the highest marks was 18. The mean mark of the 85 students was 10.64, with standard deviation of 5.173. This revealed that students on average scored 53.2% out of the total 20 marks and the spread of the marks around the mean is relatively big, as evidenced by the coefficient of variation of 48.6%. Figure 1 shows the distribution of students' marks and the overall test results are presented in Table 1. In the second phase, 10 students with test scores of lower than the mean mark were selected to undergo in-depth interview with the researcher. Analysis of the interview transcripts revealed five major causes of students' errors in solving problems related to measures of dispersion.

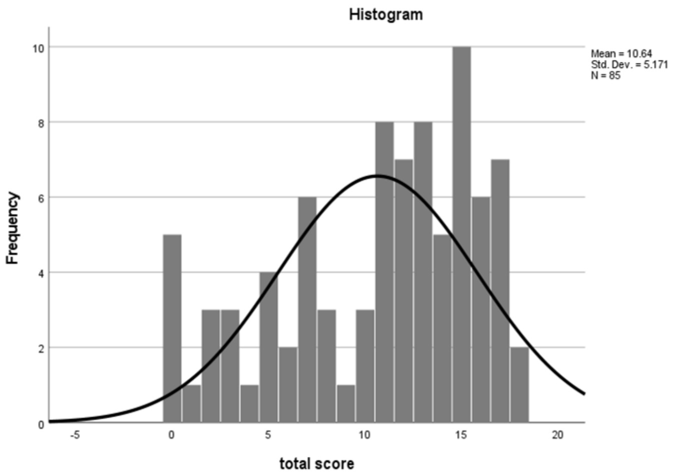

Figure 1. Students' performance on diagnostic test

Table 1. Summary statistics of the diagnostic test (N=85)

Indicator		Value
Mean		10.64
Median		12.00
Mode		15
Standard Deviation		5.173
Range	Minimum	0
	Maximum	18
Percentile	25	7.00
	50	12.00
	75	15.00

4.1.1. Lack of statistical vocabulary knowledge

The analysis of the responses of interviews with the students showed student's inadequate grasp of the language of statistics. When asked by the researcher, some students were not able to tell the definition of the statistical terms such as standard deviation, interquartile range, quartile deviation and range. The abstraction in meaning and the difficulties in expressing some terms explicitly in ordinary language (i.e., standard deviation) have hindered students from understanding and applying the statistical vocabulary and the related concepts in solving problems. The interview transcripts revealed students lacking statistical vocabulary knowledge (T=teacher; S=student).

Interview transcript 1 (S44)

> "*Now, let's look at the next question (Question 2.a.iii), standard deviation. You had left the answer blank. Why?*" (T)
> "*I don't know how to do*" (S)
> "Can you tell me the symbol of standard deviation?" (T)
> "*(Student points at the symbol "σ " that she had written on the answer sheet)*" (S)
> "*Yes, this is the symbol. Do you know what is standard deviation?*" (T)
> "*… I don't know …*" (S)

Interview transcript 2 (S44)

> "*What is the meaning of "range"? For example, we have number 1 to 10, what is the range of these numbers?*" (T)
> "*Range are these numbers*" (S)
> "*What do you mean? Can you please explain?*" (T)
> "*Erm … range are these numbers, all these, 1 to 10*" (S)

Interview transcript 3 (S72)

> "*Do you know what is range?*" (T)
> "*Highest frequency minus lowest frequency*?" (S)

Interview transcript 4 (S53)

"*Now, the second part (Question 1.ii). We look for the interquartile range. Can you please tell me what is interquartile range?*" (T)
"*The Q1, Q2…*" (S)
"*Yes, what else?*" (T)

Interview transcript 1 and 2 revealed that S44 was unable to answer the questions because the student has no idea of the definition of standard deviation (although he knew the symbol that represents standard deviation "σ ") and range. Instead, the student had a misconception on range as indicated by the language used. Another student (S72) had a false definition on range, which led the student to the wrong workings and thus wrong answer (Interview transcript 3). As shown in interview transcript 4, student S53 can relate the interquartile range with Q1 and Q2 but cannot give the definition of the term interquartile range. Students may struggle to grasp the underlying principles if they are unfamiliar with the language used to describe them [44]. Students find the test questions using unusual and specific mathematical words with ambiguous meaning more challenging [64].

4.1.2. Rote learning

In this study, some students were found to have memorized and applied the formulas correctly in the test items without understanding the meaning of the procedures. The interview transcripts reveal students' weaknesses in understanding measures of dispersion due to rote learning.

Interview transcript 5 (S52)

"*Now we come to the last question (Question 3.a.i). You need to fill in the necessary information in Table 3 to find the mean and the standard deviation of the mass of class A (pointing at student's answer). Look at your answer. You had found the mid values here. Can you please tell me why do you need to look for the mid values? What are the mid values for?*" (T)
"*Erm … I don't know. I just follow the format*" (S)

In answering question 3.a.i (Interview transcript 5), student S52 filled in the information required in the table but was unable to make sense of the purposes of the procedures to find the mean and standard deviation from the grouped data. The student just followed a process with different numbers rather than understanding how the sequence of action produces an answer. A key concept of standard deviation is the arithmetic mean [19]. A conceptual understanding of standard deviation requires more than procedural or symbolic knowledge of the method for calculating the mean.

Interview transcript 6 (S61)

"*Do you know the formula of quartile deviation?*" (T)
"*Q3 minus Q1, divided by 2 (i.e., $\frac{Q3-Q1}{2}$)*" (S)
"*Yes. You had written the formula in your answer, but you got the wrong Q3 and Q1. So, can you tell me why do we need to divide the interquartile range by 2?*" (T)
(Student shakes her head) (S)

Interview transcript 7 (S61)

"*Now, come to standard deviation. You had written the formula correctly. But the answer is incorrect. The formula you had written here is … (i.e., $\sigma=\sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n}}$)*" (T)
"*Do you understand why the formula is as such?*" (T)
"*No*" (S)
"*How do you know the formula?*" (T)
"*I memorized*" (S)

Interview transcript 8 (S53)

"*Let's look at question (ii), mean. How did you get the mean? Can you please tell me?*" (T)
"*Sum of fx divided by sum of f (i.e., $\frac{\Sigma fx}{\Sigma f}$)*" (S)
"*Ok, good. You had gotten this answer correctly too. What about the standard deviation?*" (T)
"*Err … square root, sum of fx squared over sum of f, minus sum of mean squared ($\sqrt{\frac{\Sigma fx^2}{\Sigma f}-\bar{x}^2}$)*" (S)
"*Alright. Do you understand why the formula is as such?*" (T)
"*I try to understand but I don't really know why it is like that. I just memorized*" (S)

When asked to find the quartile deviation and standard deviation, student S61 could remember the formulas correctly but was unable to transfer the knowledge to solve the problems (Interview transcripts 6 and 7). Student S61 has attended to relevant information but has not understood it and thus, cannot use it. The inability to solve the questions was due to the fact that these students had not yet understood the topics' formal principles. In another instance, student S53 was able to answer the questions regarding standard deviation correctly with the correct formula. However, the student admitted that she did not understand the formula of standard deviation and merely memorized the formulas. Knowledge of a computational rule does not necessarily imply a comprehensive understanding of the underlying concept and may inhibit conceptual learning. Understanding the fundamental concepts is more important than memorizing a formula [5].

4.1.3. Lack of symbol sense

In the study of statistics where statistical symbols often have multiple layers of meaning, understanding when and how to employ symbols is an essential ability [65]. Students must also identify statistics as variables and differentiate statistics and parameters [66]. In this study, students were found to have great difficulties correctly associating symbols with concepts. The interview transcripts reveal students' errors in measures of dispersion due to lacking of symbol sense.

Interview transcript 9 (S79)

"*Now, please try to write down the formula of standard deviation. What symbol do we use to represent each data? If we use bar x ($\bar{x}$) for mean, what do we use for each data?*" (T)
"*(Student is thinking and not responding, while teacher is waiting)*" (S)
"*Bar x ($\bar{x}$) represent mean, which show that the mean is the average of x. So, what is the symbol for each data?*" (T)
"x" (S)
"*It's x_i*" (T)

Interview transcript 10 (S52)

"*Now, do you think you can write down the formula of the standard deviation?*" (T)
"*Yes (Student writes* $\sigma=\sqrt{\frac{\Sigma(x_i-\bar{x})}{x}}$*)*" (S)
"*Look at this symbol, what is this? (Teacher is pointing at "x"). What is the total number of data over here?*" (T)
"*8*" (S)
"*Yes. What is 8? What symbol do you use to represent the total number of data? We use "n", not "x", alright?*" (T)

Interview transcript 11 (S61)

"*Alright. Look over here. What is this symbol? (Teacher is pointing at "Σ")*" (T)
"*This is sum*" (S)
"*Yes, correct. This one? (Teacher is pointing at "x_i")*" (T)
"*… I don't know*" (S)
"*This one? (Teacher is pointing at "$\bar{x}$")*" (T)
"*Mean*" (S)
"*Mean, yes. What about this? (Teacher is pointing at "n")*" (T)
"*... I don't know*" (S)

Interview transcript 12 (S44)

"*Can you try to write down the formula of mean for grouped data, based on your understanding?*" (T)
"*(Student writes* $\bar{x} = \frac{fx_i}{f}$*)*" (S)
"*Yes, but we need the sum of it*" (T)
"*the sign … "E" (Student writes "Σ")*" (S)
"*Yes. This is summation or sigma*" (T)
"*(Student writes* $\bar{x} = \frac{\Sigma f x_i}{\Sigma f}$*)*" (S)

S79 was confused with the symbol "x_i" and "x". This could be due to the two symbols are used for two different though related ideas. Besides, "x_i" represents each of the in the ungrouped data, while the symbol "x_i" in grouped data represents the mid values of each group. Another student, S52 was confused with the symbol "x_i" that represents each data and "n" that represents sample size in ungroup data. There were similar findings [67] that some students treated the symbol "x" as a specific unknown that representing a fixed number while some students almost never treated "x" as variable to represent multiple values or as variable representing a range of values at the same time. As for student S61, she was unable to name and explain the concepts of the symbols "x_i" and "n" used even though she had written the formula of the standard deviation for ungroup data correctly in question 2a(iii). Students might be able to understand and accurately identify statistics symbols but had trouble linking them with the relevant notion [11]. In another instance, student S44 was not able to pronounce or name "Σ" properly and referred the symbols as "E".

Interview transcript 13 (S51)

"*Yes. So, what is the value of Q1?*" (T)
"*24.5*" (S)
"*No (Teacher checks on student's work). See how you work out the answer. You keyed in (i.e., using calculator) like this, 15 plus 19 divided by 2 (i.e., 15+19÷2), the calculator will take your instruction as 15 plus, 19 divided by 2 (i.e., 15+(19÷2)). But is this the correct operation?*" (T)
"*... Erm ... No*" (S)

In interview transcript 13, student S51 used the incorrect syntax (did not perform grouping) when translated the information into calculator. The student only realized that the implicit grouping must be made explicit when prompted by the teacher. Although this error was not directly linked to statistical symbols, it shows that low performing students confront more obstacles when studying statistics, from order of operation to a process-oriented view of functions.

4.1.4. Lack of statistical reasoning

In this study, students were found lacking of the statistical reasoning skills in descriptive statistics. In question 2, many students had done correctly in finding the values of mean and standard deviation of the ungrouped data but were unable to answer or provide statistical reasoning to item 2b. Similar to question 3, many students did not provide proper answer to item 3b even though they had coped well with item 2a. The interview transcripts reveal the lack of statistical reasoning skills among the students.

Interview transcript 14 (S44)

"*If you were the farmer, which condition do you think is better for your plant?*" (T)
"*A*" (S)
"*Why?*" (T)
"*Because the mean is higher*" (S)
"*Yes, and ...*" (T)
"*The standard deviation is smaller*" (S)
"*What does it tell you when the standard deviation is smaller?*" (T)
"*It is better*" (S)
"*Why better?*" (T)
"*Because the difference ... the difference ... is smaller*" (S)
"*Can you please explain?*" (T)
"*... teacher, I don't know how to say ...* (S)

Interview transcript 15 (student S44)

"*Yes, good. What does bigger differences tell you about the mass?*" (T)
"*... some are very heavy some are very thin…I don't know ...*" (S)

Interview transcript 16 (student S51)

"*Now, we will discuss about the last question (Question 3b). You need to compare the mass of the student in Class A and Class B. You didn't answer the question*" (T)
"*The means are the same*" (S)
"*Yes, correct. What about the standard deviation?*" (T)
"*The standard deviation of class B is smaller*" (S)
"*Ok. What does it tell you if the standard deviation of Class B is smaller?*" (T)
"*(Student is thinking and not responding, while teacher is waiting)*" (S)
"*Are the mass of the students closer to the mean or further from the mean?*" (T)
"*Closer*" (S)
"*Yes, so, what does it tell you if the data is closer to the mean?*" (T)
"*... I don't know ...*" (S)

S44 and S51 were unable to explain what "bigger or smaller differences", that is the wider or narrower spread between each data and the mean implies. When asked by the teacher, student S44 used words, such as heavy and thin, instead of statistical description such as consistent, spread, or vary to describe the data. These students were unable to reason with statistical concepts and make sense on the information given. The capacity to reason about statistical measures is understanding what measures of center tendency, measures of dispersion, and position disclose about a data set, when to utilize them, and how they reflect a data set [26]. The constructs of statistical reasoning in [68] also include "organizing data", which refers to the ability to summarize data in terms of measures of tendency and measures of dispersion.

4.1.5. Lack of statistical thinking

In solving question 1, students were found to have confused between frequency and value of distribution in item 1(i). Some students failed to retrieve the correct information from the graph which shows the lack of ability in graph reading. Besides, students were also found to have confused between the quartile position and the quartile values in items 1(ii) and 1(iii). Some students had wrongly taken the difference between the Q3 and Q1 positions as the interquartile range, which shows that these students did not understand the concept of interquartile range. Another common error committed by students were found in items 3a(ii) and 3a(iii), where group frequencies were not taken into consideration when calculating the group mean and standard deviation. These mistakes were not due to carelessness but is a proof of misconceptions among the students, as evident by the interviews with the students. These students did not understand the element of group frequencies in calculating the descriptive statistics in group data. The interview transcripts show the lacking of students' statistical thinking ability when answering questions related to measures of dispersion:

Interview transcript 17 (S72)

"*Now, please find the interquartile range*" (T)
"*54 minus 18?*" (S)
"*No. 54 and 18 are the position, not the value. Are we looking for the position or the value?*" (T)
"… I don't know …" (S)

Interview transcript 18 (S79)

"*Alright. You had written here Q3 minus Q1, and you wrote 60 as Q3 and 20 as Q1. How did you get 60 as Q3 and 20 as Q1?*" (T)
"*… from the graph*" (S)
"*Can you please explain?*" (T)
"*I took 80 times one over four and get 20 (i.e.,* $80 \times \frac{1}{4} = 20$*). Then I took 80 times three over four, and get 60 (i.e.,* $80\times\frac{3}{4}=60$*)*" (S)
"*I see. Why did you take 80?*" (T)
"*80 is the total frequency*" (S)
"*Is 80 the total frequency? Please read the graph carefully. See where the curve ends*" (T)
"*Oh … is 72*" (S)

In answering question 1(b), both student S72 and S79 revealed lack of ability in graph reading. Student S72 was unable to differentiate between the value and position of the quartiles. Student S72 wrongly took the difference between the Q3 position and Q1 position as the interquartile range. As for student S79, she showed no awareness of the data displayed in the graph and wrongly took 80 as the total frequency. This findings are consistent with past studies [69], [70] who found that students have a tendency to confuse frequency with data value when data are presented graphically.

Interview transcript 19 (S79)

"*Now let's look at how to work out the mean. Can you tell me how to work out the mean?*" (T)
"*35 plus 45 plus 55 plus 65, divided by 4 (i.e.,* $\frac{35+45+55+65}{4}$*)*" (S)
"*Look at your solution, how many data are there?*" (T)
"*4?*" (S)
"*Do you only have 4 students here?*" (T)
"*… No*" (S)
"*How many students do we have?*" (T)
"*… 30*" (S)

"*Yes. So, what should be the sum of the data be divided by?*" (T)
"*30*" (S)
"*Yes, 30. But over here, we only have 4 data (pointing at* $\frac{35+45+55+65}{4}$*). So, what shall we do?*" (T)

In answering question 3.a.ii, student S79 was unable to identify the group frequency in the table displayed, despite teacher's prompts. Student 79 did not realize that she had to multiply group mid values with the group frequency. It was claimed that students sometimes overlook the frequency of each interval when determining the mean [71].

Interview transcript 20 (S61)
"*Now we move on to the second question (Question 2.a.i). The heights, in centimeter, of 8 seedlings grown under condition A are this much. Part (i), find the interquartile range. You had written here 25 minus 13, equals to 12 (i.e.,* $25 - 13 = 12$*). Your answer is not correct. Why did you write your answer as such?*" (T)
"*Highest value minus lowest value*" (S)
"*Alright. But before you take the highest value minus the lowest value, what should you do? Do you know why you got the answer wrong?*" (T)
"*… I didn't rearrange the data*" (S)

In answering question 2a(i), student S61 did not rearrange the ungrouped data before looking for the value of Q3 and Q1. Student S61 was lack of the awareness of the importance of order of the data set in finding interquartile range. In a study [72] investigating students' conceptions of the median, roughly half of the students perceived the median as the middle point of the ungrouped data before they were arranged. Order statistics is said to be computationally and conceptually complex because the algorithm of median, quartiles, and percentiles is taught differently for grouped and ungrouped data [5], and many of the algorithm's steps are frequently not fully articulated or comprehended [73].

Interview transcript 21 (S53)
"*Do you know why we need to find out the differences between each data and mean?*" (T)
"*No, teacher. (Student shakes her head)*" (S)
"*That will give us the spread of the distribution, how far is each data from the mean. For example, there are 6 students in group A, whose marks are 50, 50, 40, 30, 60 and 70. And we have another six students from group B, whose marks are 10, 20, 30, 70, 80 and 90. (Teacher writes the data on the white board). Can you tell me what is the average marks for group A*?" (T)
"*(Student calculates …) 50*" (S)
"*Yes. What about group B?*" (T)
"*(Student calculates …) 50*" (S)
"*Yes, also 50. So, which is the mean that better represent the actual situation of the students' performance?*" (T)
"*… both …*" (S)

Student S53 was unable to tell the reason of looking for the difference between each data and the mean, that is, the spread of the distribution. Despite teacher's explanation and prompting, student S53 was unable to make inferences of the means between the two data sets given. Campbell [74] noted that ignoring the spread of data is a common error among students. The study of frequency should go beyond the focus of averages since two data sets with the same average may have significant variation [5].

4.2. Discussion

Statistics is an expansive subject with its own vocabulary, syntax, notation, and symbols that must be taught and acquired [45]. The findings of this study showed that low-performing students struggle through the process of statistical problem solving, particularly in the topic of measures of dispersion. Some of the causes of difficulties identified include lack of the knowledge of statistical vocabulary, rote learning, symbol sense, statistical reasoning, and statistical thinking. The findings suggest that instructions that focuses particularly on these areas may improve the learning and problem-solving proficiency of students and thus can serve to inform the weaknesses of students learning in statistics.

In the study, lack of statistical vocabulary is one of the primary causes of students' errors in calculating measures of dispersion. The process of learning definitions of statistical terms is complicated and difficult in view of the abstraction in nature and multiple meanings. Students struggle in numerous areas of mathematics and statistics instructions, due to the absence of a standardized language used in the subjects

[44]. Moreover, most students only utilize statistical language in class [75], therefore do not get familiar with the language of statistics. As such, the teaching and learning of statistics should emphasis the discussion of the various meanings and definition of statistical vocabulary. Definitions are useful for initial learning [44], since it formalizes or extends a concept logically [76], and influence their grasp of statistical concepts [48]. Teachers have a statistically significant impact on students' capacity to define lexically problematic words in statistics [49]. To help students in acquiring the language of statistics, teachers must first recognize the linguistic obstacles they confront and be adaptive in the classroom. Several researchers provide suggestions for teaching mathematical and statistical vocabulary which include focusing on selection of vocabulary [77], providing student-friendly definitions to vocabulary [78], identify the general English words that have technical statistical meanings, as well as giving implicit/explicit instruction involving statistical vocabulary in daily learning [44].

Another cause of error identified was lack of symbol sense. Symbolic knowledge and fluency are crucial in understanding and applying algebra [53], while algebraic proficiency does help students in their grasp of statistical concepts [79]. Kim, Connelly, and Cook [80] however claimed that although students can apply algebraic or functional knowledge to statistics, they may lack symbol sense since statistical notions such as mean, standard deviation, and sample require an object understanding. In this study, the students were found to have difficulties in the verbalizing, reading, and writing of statistical symbols. In some instances, even if the statistical formulas were applied correctly, students were confused and had no idea of the concepts of the symbols used. This was in line with the past finding [43] where symbolic language is often a cause of great confusion for students; and some students failed to use the statistical symbols correctly even though they can identify the symbols [11]. One reason for the confusion could be due to students applying personal meaning to symbols [43]. Verbalizing and reading symbols are receptive processes, while producing symbols is a generative process. Students must be able to read statistical symbols to decode, produce and transform them when solving problems. Instructions should not emphasize teaching a formula and performing computations. Teachers can instead focus on the symbolic aspects by helping students to identify and differentiate the numerous statistical symbols through classroom activities and discussion. For example, students can be reintroduced to the verbalizations and interpretations of statistical symbols through problem solving, quizzes and project works.

Ausubel [81] refers rote learning as new knowledge that is arbitrarily and non-substantively incorporated into cognitive structure. In this study, questions related to standard deviation revealed the most errors. Statisticians [15], [20] claimed that the standard deviation is computationally complex and rarely intuitively appealing as a measures of dispersion. The emphasis on calculations and procedures doesn't promote a conceptual knowledge of standard deviation [19]. The finding shows that some students were found to have memorized and applied the formulas correctly in the test items without understanding the meaning of the procedures. These can be seen from the students' works and further confirmed via interviews. The researcher noticed that one of the possible factors that attributed to students choose to memorize the formulas without understanding is because the formula itself cannot explain the meaningful procedures. This is very clear when students are taught to use alternative methods of calculating the standard deviation. Both the first method and the alternative method (Appendix 2) are equivalent.

The calculations of standard deviation are somewhat complex and the risk of making mistakes is high. The learning of the first method (i.e., $\sigma = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n}}$ for ungrouped data; $\sigma = \sqrt{\frac{\Sigma f(x_i-\bar{x})^2}{\Sigma f}}$ for grouped data) will give us insight into how standard deviation really works, which is a procedure of getting the squared root to the average squared differences between each data and the mean. However, both the alternative methods (i.e., $\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \bar{x}^2}$) for ungrouped data; $\sigma = \sqrt{\frac{\Sigma f x_i^2}{\Sigma f} - \bar{x}^2}$ for grouped data) do not carry any meaning in the procedure and thus students are not able to understand how the concepts of standard deviation is related to the formulas. Students are further confused by the variety of the formulas and thus hinder them from learning the concept. This could be one of the reasons students choose to use rote learning strategy in calculating standard deviation, because they simply cannot understand what they were doing.

These findings also reveal students lack of statistical reasoning and statistical thinking ability. This is further evident by a study [82] where 412 10th grade students from Malaysia secondary schools revealed poor level of statistical reasoning ability. Another research [83] analyzed 122 postgraduates in Malaysia shows that some postgraduates have poor statistical reasoning skills. The findings showed that majority of the students had difficulty in interpreting standard deviation in a given context although they could infer mean. Statistical reasoning and thinking involves interpretation and making inferences of the combined ideas about data and chance. It is thus important to ensure students gaps the important statistical conceptual understanding such as distribution, central limit theorem, measure of dispersion, association, uncertainty, randomness, and sampling. Promotion of a statistical reasoning learning environment can help to improve students' statistical thinking and reasoning [84], as well as positive attitudinal changes towards statistics learning [85]. Some of the activities include using real world data, integration of appropriate technological tools, focus on statistical ideas rather than procedures, promote classroom discourse, and using proper assessment to measure and monitor the development of students' statistical learning.

Teachers are essential for students to be statistically literate. Past research however reveals teachers' pedagogical statistical knowledge are inadequate and need to be enhanced to improve students' statistical learning [86], [87]. Pedagogical statistical knowledge entails a teacher's awareness of potential student difficulties with statistics, techniques to improve student learning, and strategies for including students in a statistical investigative cycle [88]. Teacher education in statistics should focus on five fundamental principles which include developing fundamental statistical concepts, investigating, and learning from data, developing statistical arguments, understanding students' reasoning, and implementing formative assessment [89]. Appropriate assessment in statistics is critical to examine students' understanding of statistical concept [40]. It is thus important to develop proper tools to assess students' conceptual understanding in learning statistics despite the use of traditional large-scale assessment that emphasis on accuracy of procedures and computation [90]. This study suggests that despite being time-consuming and demanding a great deal of effort, interview could be included as an effective assessment tool to reveal the performance of students in statistical learning. Through interview, teachers can elicit and foster insight into students' thinking and reasoning, as well as understand the strategies students apply in solving the statistical problems. This will thus help teacher to remediate the appropriate teaching pedagogies to improve students' statistical conceptual understanding and ability.

5. CONCLUSION

This study reveals that students, in particular the low achievers, encounter substantial challenges in learning measures of dispersion. Teachers should pay attention to the causes of errors in order to develop teaching strategies that will address the displayed difficulties. As indicated in the findings, a teacher's observational skills and reflective practice enable them to better understand students' difficulties in learning and make important proper adjustment in the teaching instruction. Eliciting students' thinking in committing the errors can be taken as an opportunity to facilitate other students in getting deeper understanding in the learning of mathematics. Future research could focus on the teaching pedagogies and remedials to overcome students' difficulties from the aspect of statistical vocabulary, rote learning, symbol sense, statistical reasoning and statistical thinking. More extensive research in the same area could be conducted involving students in multiple domains.

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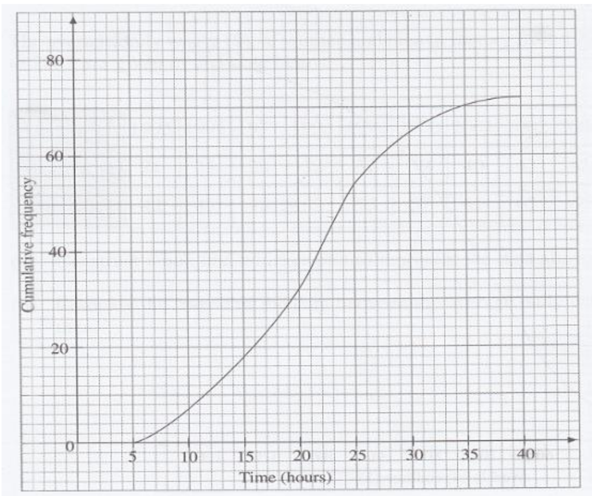
APPENDIX 1

Measures of dispersion diagnostic test

1. **The cumulative frequency polygon in Graph 1 shows the number of hours a group of 72 people spend watching television each week**



Use the graph to estimate:
i) The range of the distribution
Answer: …
ii) The interquartile range
Answer: …
iii) The quartile deviation
Answer: …

2. The heights of 8 seedlings grown under condition A are: 15, 24, 21, 19, 20, 19, 13, 25 (centimeters)
a) Based on the heights of the 8 seedlings grown under condition A,
 i) Calculate the interquartile range
 Answer: …
 ii) Calculate the mean height
 Answer: …
 iii) Calculate the standard deviation
 Answer: …

b) The mean and standard deviation of another 8 seedlings of the same plant species grown under Condition B are 16.4 cm and 5.21 cm respectively. Compare Condition A and Condition B, which condition is more suitable for the growth of the seedlings? State the reason (s).
 Answer: …

3. The masses, in kilograms, of the 30 students from Class A are given:

Table 2. The mass (in kg) of Class A

Mass (m kg)	$30 < m \leq 40$	$40 < m \leq 50$	$50 < m \leq 60$	$60 < m \leq 70$
Frequency	3	14	11	2

a) Based on the information given in Table 2,
 i) Complete the necessary information in Table 3 to find the mean and standard deviation of the mass of Class A.
 Answer: …

Table 3

Mass (m kg)	Frequency
$30 < m \leq 40$	3
$40 < m \leq 50$	14
$50 < m \leq 60$	11
$60 < m \leq 70$	2
Total	

 ii) Calculate the mean mass of Class A
 Answer: …
 iii) Calculate the standard deviation of the mass of Class A.
 Answer: …

The mean and standard deviation of the mass (in kg) of students in Class B are given in Table 4.

Table 4

Mean	Standard deviation
49	6.55

b) Compare, briefly, the masses of the students in Class A and Class B.
 Answer: …

APPENDIX 2

For ungrouped data: $x_1, x_2, x_3, \ldots\, x_n$
And mean: $\bar{x}$
The standard deviation is: $\sigma = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n}}$ (First method)
Arrange as:

$$\sigma^2 = \frac{1}{n}\Sigma(x_i-\bar{x})^2$$

$$\sigma^2 = \frac{1}{n}\Sigma(x_i^2-2x_i\bar{x}+\bar{x}^2)$$

$$\sigma^2 = \frac{\Sigma x_i^2}{n}-2\bar{x}\frac{\Sigma x_i}{n}+\frac{n\bar{x}^2}{n}$$

$$\sigma^2 = \frac{\Sigma x_i^2}{n}-2\overline{xx}+\bar{x}^2$$

$$\sigma^2 = \frac{\Sigma x_i^2}{n}-2\bar{x}^2+\bar{x}^2$$

$$\sigma^2 = \frac{\Sigma x_i^2}{n}-\bar{x}^2$$

$$\sigma^2 = \sqrt{\frac{\Sigma x_i^2}{n}-\bar{x}^2} \text{ (Alternative method)}$$

For grouped mid values: $x_1, x_2, x_3, \ldots\, x_n$
And mean: $\bar{x}$
The standard deviation is: $\sigma = \sqrt{\frac{\Sigma f(x_i-\bar{x})^2}{\Sigma f}}$ (First method)
Arrange as:

$$\sigma^2 = \frac{1}{\Sigma f}\Sigma f(x_i-\bar{x})^2$$

$$\sigma^2 = \frac{1}{\Sigma f}\Sigma f(x_i^2-2x_i\bar{x}+\bar{x}^2)$$

$$\sigma^2 = \frac{\Sigma f x_i^2}{\Sigma f}-2\bar{x}\frac{\Sigma f x_i}{\Sigma f}+\frac{\Sigma f\bar{x}^2}{\Sigma f}$$

$$\sigma^2 = \frac{\Sigma f x_i^2}{\Sigma f}-2\overline{xx}+\bar{x}^2$$

$$\sigma^2 = \frac{\Sigma f x_i^2}{\Sigma f}-2\bar{x}^2+\bar{x}^2$$

$$\sigma^2 = \frac{\Sigma f x_i^2}{\Sigma f}-\bar{x}^2$$

$$\sigma = \sqrt{\frac{\Sigma f x_i^2}{\Sigma f}-\bar{x}^2} \text{ (Alternative method)}$$