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Rethinking strategy on developing students' levels of geometric thinking in Sokoto state, Nigeria

Muhammad Nasiru Hassan[1], Abdul Halim Abdullah[2], Norulhuda Ismail[2]
[1]Department of Science Education, Faculty of Education, Sokoto State University, Sokoto, Nigeria
[2]School of Education, Faculty of Social Sciences and Humanities, Universiti Teknologi Malaysia, Johor Bahru, Malaysia

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ABSTRACT

Geometric thinking skills remained a topical issue in mathematics education. The purpose of this research is to investigate the van Hiele levels of geometric thinking skills of the students in Sokoto state to provide a clear picture of the students' levels for the appropriate development of learning activities, and better understanding. The study involves three mathematics teachers and 200 students (100 students each of basic and senior secondary school students). The samples of the teachers were purposely selected and students were randomly selected. There were two instruments used in the study; interviews for the mathematics teachers while a van Hiele test for geometric thinking was adopted to collect data for the students' van Hiele levels of thinking. Thematic analysis (for teachers' interview), descriptive, and Mann-Whitney U test for the analysis of students' van Hiele levels of thinking was used. The result shows that all the teachers indicate that the traditional approach is the dominant method used in teaching and learning and that students in the state lack basic skills in school geometry. Also, the result indicated that the majority of students sample among Basic and Senior secondary schools in Sokoto state were operating at level 0 respectively. Furthermore, a significant difference between the two independent groups was found in favor of senior secondary school students. It is hoped that in the future, educational institutions could use the present research as a guide for the development and design of modules, learning activities based on the van Hiele levels to bridge the gap in the state.

Corresponding Author:

Muhammad Nasiru Hassan
Faculty of Education, Department of Science Education, Sokoto State University
Brinin Kebbi Road, (Near Airport), Sokoto State, Nigeria
Email: mohammad.nasiruhassan@ssu.edu.ng

1. INTRODUCTION

Mastery of School geometry among mathematics topics continues to be difficult for many students in the world. Literature bound that, students in many countries across the globe, such as Turkey, South Africa, Indonesia, and Malaysia among others continue to have low performance in geometry [1]–[5]. Geometry is one of the topics of mathematics that students find difficult to master globally. The problem in Malaysia is alarming and discovered that majority of the students are found on the low level of van Hiele's level of thinking [6]. Students had difficulty with the definition of geometric figures, the relationship between shape and properties, and class inclusion [1]. problems for the student's difficulties were affirmed among Indonesian students [7], [8] and a low level of geometric thinking for van Hiele's levels [9]. Like other students over the world, students in Nigeria also experience a similar problem of low basic skills and achievement in geometry. It is evidenced that students in Nigeria continue to demonstrate low basic skills

and achievement in geometry [10]. The worse performing topic that contributed to the poor performance in mathematics is geometry in Nigeria [11], [12]. However, van Hiele's theory continues to be one of the outstanding theories used to investigate students' levels of geometric thinking and also provide a model for achieving the levels [13]. Understanding students' levels of geometric thinking are imperative for better selection of learning strategy, materials, and development of lesson activities [14].

In spite of the importance of geometric thinking skills, students continue to demonstrate a lack of basic skills in geometry and difficulties in learning school geometry in Nigeria. Atebe and Schäfer [15] affirmed this in both Nigeria and South Africa. In a qualitative investigation, the research indicated that Nigerian senior secondary school students lack conceptual understanding of identifying rhombuses with different orientations; use imprecise reasoning to describe geometric figures, and that 9 out of 18 (50%) students were at level 0 of the van Hiele's levels [16]. This situation indicates that Nigerian students' levels of geometric thinking skills are lower than the needed geometric reasoning as specified in the Senior secondary school mathematics curriculum. Most of the students in Nigeria show a negative attitude towards geometry, they do not answer questions on geometry, many lack an understanding of the basic knowledge of geometry, and many cannot correctly apply the theorem [17], [18]. Similarly, Ogan and George [19] investigated the problems of difficult concepts in the Senior Secondary School Mathematics curriculum in Nigeria, some of the difficulties identified are problems based on circle theorem, which include, the angle at the center of the circle, chord theorem, and angles among others. Previous research [20] discovered that students in Kwara State, Nigeria perceived plane geometry & circle; geometrical construction; and coordinate geometry as difficult topics with 57.65%, 77.55%, and 77.54% respectively.

Moreover, the worst-performed topic in mathematics in both internal and external examinations in Nigeria that contribute to the overall poor performance in Mathematics is geometry [12]. Several studies had shown that some of the reasons for these problems are associated with the nature of teaching and learning which is the use of conventional strategies [21]–[24]. This mode of instruction and learning strategy (conventional) failed to meet the needs of society [25]. A study by previous researchers [21] explained that students who learn geometry through the conventional process often failed to recognize geometric figures, their characteristics, and the relationship between the figures. It is evident that the Nigerian students are weak in the area of geometry [26]. A plethora of studies were carried out in Nigeria with a view to solving the problems of difficulties with school geometry [27]–[31].

Despite such efforts, the problem of students' difficulty and lack of basic skills in geometry persisted in the context of Nigeria, [32], [33]. However, learning school geometry requires a careful selection of activities that develop students' understanding. Also, the use of more specific pedagogical-psychological theories to design and develop a specific instructional approach to teaching mathematics is required. It is imperative to know the geometric thinking levels (GTLs) of the learners for better instruction [34]. Teachers who do not understand their students' hierarchy of level and language, could easily misinterpret the GTL of their students [34]. In other words, understanding the GTLs of students could provide information about the needs, problems, or difficulties, appropriate language to use at each level for the organization of content, and the selection of learning strategies for a better understanding of geometry [35]–[37]. This indicates that appropriate instructions and learning strategies in school geometry could best determine if the GTLs of the students are known. Looking for the need for specific strategy virtually at all levels of schooling and lack of research studies on van Hiele levels, particularly in Sokoto state. The researchers deemed it imperative to conduct a preliminary investigation, on the students' levels of geometric thinking skills. Thus, an attempt to provide insight into the problem of students' difficulties and the level of geometric thinking skills in Sokoto state for better development of a learning strategy was considered.

This research considered the use of van Hiele's theory of geometric thinking as appropriate because of the investigative need of the present research to provide the students' levels of geometric thinking skills. Thus, van Hiele's theory is one of the best and most well-articulated theories that can describe how learners learn geometry and offers a better explanation of their levels of geometric thinking, better understanding of geometry, and develops higher levels of thinking the learners in geometry through instruction based on the phases. However, the theory was developed on the grounds of the difficulties encountered by the students with geometry by the husband and wife, Purre van Hiele and Dana van Hiele – Geldof in their doctoral thesis. After the death of his wife Purre, van Hiele clarified the formulation of theory in geometric thinking involves levels of thinking in geometry [36], [38]. The levels begin with the visualization level and improve to more sophisticated levels of description, analysis, abstraction, and rigor or proofs as described in both old and recent studies of geometric thinking [39], [40]. Thus, the present research investigates the teachers' perception of the learning of school geometry, and the level of students' geometric thinking, also to determine the van Hiele levels of basic and senior secondary school students and hypothesized that there is no significant difference between basic and senior secondary school students based on the van Hiele levels.

2. RESEARCH METHOD

The researchers considered a quantitative and qualitative approach (exploratory sequential) to explore the view of the participants qualitatively and follow up with the quantitative for better clarification and understanding of the research problem [41] and that a comprehensive evaluation in research can be best done if the two methods are combined, this because when the two methods are used in-depth explanation can be obtained to explain the problem [42]. Thus, the teachers' viewpoints about learning school geometry and students' levels of thinking skills were explored qualitatively. More so, a follow-up with the quantitative investigation based on the students' geometric thinking levels was carried out, the pictorial representation is shown in Figure 1.

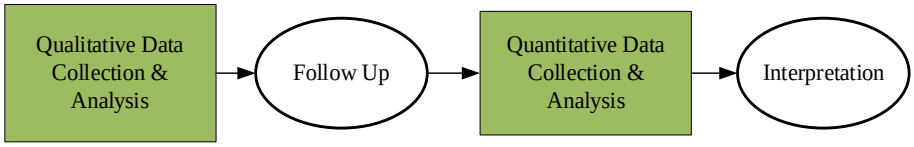

Figure1. Exploratory sequential method

The participant involves in this study include the selection of three Mathematics teachers and secondary school students in the public secondary school in the Sokoto metropolis. The selection of the teachers is based on their experience. A total number of 200 Upper Basic and Senior secondary school students were involved. 100 students were randomly selected from each Basic Secondary School (BSS). and Senior Secondary School (SSS) in the selected school. The interview was done to explore the teachers' views about learning school geometry and students' levels of geometric thinking skills of students in the state. The researcher administered van Hiele's test of geometric thinking to the selected students to investigate van Hiele's levels of thinking students. To explore the teachers' views on the learning of school geometry, and the level of students' geometric thinking interview questions were developed by the researcher. For the students' levels of geometric thinking, the van Hiele test from Cognitive Achievement in Secondary School Geometry (CDASSG) was adopted with permission based on the objective of the present research. The researcher scrutinized the content and objectives of the New Nigerian Basic and Senior Mathematics Curriculum and textbooks used in Nigeria and see if it goes along with the content of the van Hiele geometric test and found the questions agreed with the content and objectives of the curriculum.

Furthermore, a discussion was performed with experts to further ascertain the agreement of the content and objective. In this regard percentage of absolute agreement (PAA) of 80.4% was found. Their observations based on the discussion indicated that the adopted test is a replicant of what was found in the content of the New Mathematics Curriculum, the textbooks, and the content covered in the present research. More so, the test was also adopted and used in Nigeria [16]. Thus, a reliability index of 0.89 was found for the van Hiele test. The criterion for marking, grading, and determination for the levels of geometric thinking was adopted as provided [39]. Thus, the results of the qualitative data obtained were analyzed using thematic analysis and descriptive statistics were used to answer research objective two. To answer the hypothesis Mann-Whitney U test was used because the van Hiele levels scores are ordinal. Therefore, non-parametric statistics are appropriate for the analysis [43].

3. RESULTS

3.1. Teachers' perceptions about learning school geometry and students' levels of geometry

It is evident that students in Nigeria lack the conceptual ability to recognize rhombuses in different orientations, and the use of imprecise explanations for describing geometric figures was found, with students in Nigerian samples and the majority were at the pre-recognition level [16]. In the present study, the result of the interview conducted with the teachers in Sokoto State with teaching experience of 15–25 years, marking and coordinating both internal and external examinations indicates that geometry is one of the key topics in the mathematics curriculum in Nigeria. The traditional approach was found to be the current learning strategy as all (100%) of the participant indicated in Sokoto State. Furthermore, lower levels of geometric thinking continue to be a problem in the state, where teachers indicate that students lack basic skill of geometric ability. Teachers also indicate the need to develop a new strategy that can provide a solution to the problem and recommend the use of workshops to improve the experience. The result was obtained from the feedback of the teachers involved.

3.2. Analysis for the distribution of students' van Hiele levels of thinking

In this section, analysis of the students' geometric thinking was done based on the established criteria. The result indicates that a total of 75 (75%) and 42 (42%) in both basic and senior secondary school students respectively failed to achieve van Hiele level 1. However, out of the 100 students in each school level, 5 and 8 students respectively obtained the sum of scores of 2 marks skipped level 1, and were assigned level 0 (pre-recognition) based on the forced van Hiele's level table [41]. A total of 24 (24%) and 47 (47%) students in both basic and senior secondary school students achieved level 1 by scoring sum scores of 1 marks, although 4 out of 24 students in basic secondary school obtained sum scores of 5 by skipping level 2 and were assigned at level 1 based on the forced van Hiele table. There were eight students achieved levels 2, 1 (1%), and 7 (7%) in the basic and senior secondary school students respectively with a sum score of 3 marks. Only four students representing 4% of the senior secondary school students achieved the sum scores of 7 marks and were assigned level 3 based on the forced van Hiele's table [41]. Table 1 provides a summary of the results. However, to further provide a detailed explanation the scores were subjected to non-parametric analysis to answer the research hypothesis.

Table 1. Distribution of students' geometric thinking levels

van Hiele's level	Sum of scores	Levels 1	2	3	Three out of five correct answer criteria BSS	Total	SSS	Total
0	0				70	75(75%)	34	42(42%)
	2		*		5		8	
1	1	*			20	24(24%)	47	47(47%)
	5	*		*	4	-		
2	3	*	*		1	1(1%)	7	7 (7%)
3	7	*	*	*	-	-	4	4 (4%)
4	-	-	-	-	-	-	-	-
5	-	-	-	-	-	-	-	-
Total					100	100%	100	100%

3.3. Analysis of the students' van Hiele level of thinking

The analysis of data obtained shows that the sample of senior secondary students obtained a higher mean rank (118.05) than the sample of basic secondary school students with a mean rank of 82.95. this suggests that the sample of Senior secondary school students is generally having a higher level of geometric thinking skills. Figure 2 provides the pictorial presentation of the mean rank of the sample students among basic and senior secondary schools in Sokoto state.

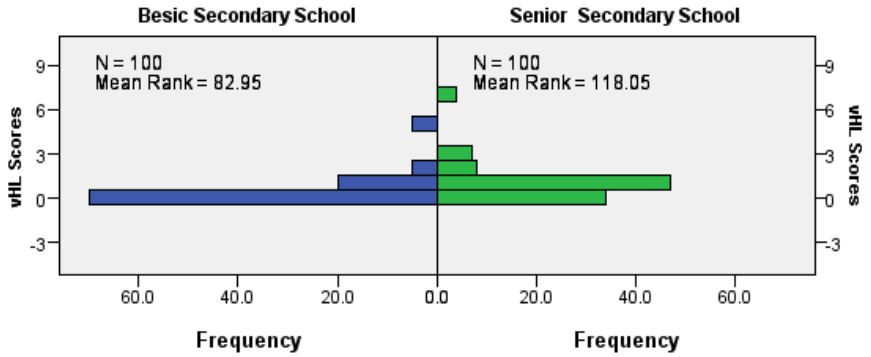

Figure 2. Chart representation of the mean rank of two independent groups

However, Table 2 provides the result of the hypothesis to test significant differences if any between basic and senior secondary school students based on the van Hiele levels of geometric thinking skills using the Mann-Whiney U test. The result of the two independent groups indicates a significant difference with U=6755.000, P-value (0.000). Since $p<0.05$ level of significance, it indicates that the null hypothesis is rejected and concludes that there is a significant difference between the two independent groups (basic and senior secondary school students) based on the van Hiele levels of geometric thinking skills.

Table 2. Mann-Whiney U test result for students' van Hiele levels based on the school level

Levels	N	Mann-Whitney U	Wilcoxon W	Test statistics	Stand. Error	Standardized test statistics	Sig.
Basic Secondary School	100	6,755.000	11,805.000	6,755.000	370.941	4.732	.000
Senior Secondary School	100						
Total	200						

4. DISCUSSION

In this section possible interpretations are provided from the findings of the result on teachers' interpretations of the learning of school geometry and students' levels of thinking skills. The analysis of the results indicates that all the participating teachers' views show that the current strategies used in learning geometry in Sokoto State are conventional, this agreed with previous studies conducted in Nigeria that the nature of students learning remained traditional [15], [30]–[32]. The strategies do not provide a real-life application, that involves hands-on activity, is student-center, and is not in line with van Hiele's phases of learning geometry. Atebe [16] explained that the approach is parallel with the objective of the mathematics curriculum. Also, the result of the interview affirmed that most of the students do not answer questions from geometry and that those answers questions do not get reasonable scores or marks because of the low level of geometric thinking skills demonstrated. This also agreed with Hassan, Abdullah, and Ismail [10] consistently indicated that Nigerian secondary schools' student's performance in geometry is very poor and that students lack basic geometric skills. More so, the worst performed topic that contributes to the overall poor performance in mathematics is geometry [12]. Thus, this could be a result of the nature of the learning strategy used which is conventional (traditional) where the learners cannot see the real-life application of geometry in relation to their environments and are not given the opportunity to discover learning experiences by themselves.

Studies had confirmed that the use of conventional strategies is one of contributing factors that create a gap in students' understanding of geometry and a higher level of geometric thinking skills [21]. Abdullah and Zakaria [38] affirmed that learning school geometry using a conventional (teacher-centered) strategy makes students have very weak reasoning skills and lead to difficulties in learning geometry with low acquisition of geometric thinking levels. The result of the second research question indicates that the majority of sample students in both basic and senior secondary school are operating at pre-recognition. The findings also accord with the finding of previous research [16] in the comparative study between Nigerian secondary school students and South African secondary school students, where the majority of the Nigerian secondary school students failed to achieve the first level of van Hiele's levels of geometric thinking. It indicates that students operate between pre-recognition level and analysis. Similar results were found in a survey conducted based on van Hiele's levels of grade 10 students in South Africa where the majority of students are at the pre-recognition level [44]. In another recent study, students were found operating at the pre-recognition level of geometric thinking [1], [5]. Rote learning as the dominant practice in learning school geometry in Nigeria contributes to preventing students from obtaining high levels of geometric thinking skills [23].

5. CONCLUSION

In view of the results obtained in the present research, the results concluded that the nature of teaching and learning in Sokoto state is traditional and that majority of the sample students in secondary school were operating at a low level of van Hiele levels. Thus, based on this it is recommended, that special training and workshop for teachers can be given consideration to develop the pedagogical knowledge of the teachers in the state as the current training in teaching is the use of specific instruction in teaching. More so, further studies should be conducted to cover a large sample of students for generalization.

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BIOGRAPHIES OF AUTHORS

Muhammad Nasiru Hassan is a lecturer at the Sokoto State University, Sokoto, Nigeria. HE was appointed as Assistant lecturer in the university in 2015 and promoted to the rank of lecturer 1 went on to pursue his graduate studies in Mathematics Education (PhD.) at the Universiti Teknologi Malaysia from 2018 to 2021. Dr. Hassan's research interests lie in Mathematics education, geometric thinking, teacher and teacher education. He can be contacted at email: nasiruhassan@graduate.utm.my; mohammad.nasiruhassan@ssu.edu.ng.

Abdul Halim Abdullah is currently an Associate Professor in School of Education, Universiti Teknologi Malaysia. Associate Professor Dr. Abdul Halim Abdullah's research interests include Mathematics Education, Higher order thinking skills and geometric thinking skills. He can be contacted at email: p-halim@utm.my.

Norulhuda Ismail She is currently a Senior Lecturer in School of Education Universiti Teknologi Malaysia. Dr. Norulhuda Ismail's research interests include Mathematics education, Mathematics reasoning, Motivation, and attitude towards mathematics. She can be contacted at email: p-norulhuda@utm.my.