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Collaborative mathematics learning management: Critical thinking skills in problem solving

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ABSTRACT

This qualitative-ethnography study described the critical thinking skill based on indicators of interpretation, analysis, evaluation, and inference in solving mathematical problem. The subjects of the study were the headmaster, math teachers, and ninth grade students of a public junior high school in academic year of 2020/2021. The data were collected through observation, field notes, interview, and documentation. Triangulation of sources and technique were used to ensure the data validation in this research. Data analysis technique was done inductively with continuous process along with the data collection. The results of the study showed high initial skills in mathematics become the basic need for the students in developing critical thinking skills based on indicators of interpretation. The improvement of critical thinking based on the indicator of interpretation is accustomed through problem-based learning. Critical thinking process based on the indicator of analysis is cultivated through collaborative learning. In this indicator, the students need special attention from the teacher. There is always the possibility for every student from each level to face difficulties in solving some problems without collaborating with others. The teachers can make more effort to optimize the students' basic holistic skills in line with their knowledge and experience in order to improve students' critical thinking skills based on the indicator of inference. In this indicator, teachers should guide the students to improve their critical thinking skills.

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1. INTRODUCTION

The development in information and communication technology (ICT) nowadays influences the changes in academic performances. The academic performances are demanded to adjust the learning management in order to prepare the 21st century students to later take part in the era of industrial revolution 4.0. ICT becomes a suitable communication facilitator in learning. In the other words, students' success depends on how ready their skills are to face the contemporary world. The skills needed in 21st century mathematics learning are critical thinking, problem solving, communication, and collaboration [1]–[3].

Between those skills, the ones that are still needed to be optimized are critical thinking and problem solving [4]. Problem solving is understanding the problem, planning the strategy to solve the problem, then

rechecking it [5], [6]. Problem solving in mathematics needs more critical thinking and appropriate steps. The critical thinking includes having views from all sides of the problem, making choices to solve the problem, and choosing the best solution in the end. Critical thinking skills will help students to communicate their idea, analyze, synthesize, and evaluate the information as well as to adjust with their own knowledge [7]–[9].

Therefore, each student needs to develop their critical thinking skills in learning mathematics. This will help the students to understand the concept and be more sensitive with the problem given to them [10]. The students who have high critical thinking skills view a problem from many sides and are more open minded, are more rational, make decisions that are supported by the evidences, draw a conclusion based on the facts provided, and finally solve the problem [1], [8].

Students have good critical thinking skills when their characteristics can fulfill these indicators: solving a problem with a simple answer, arranging basic procedures for problem solving, drawing a conclusion, elaborating the answers, and planning and arranging the strategies to decide the best result [11]. Other opinion stated that there are six indicators for critical thinking, such as interpretation, analysis, evaluation, inference, explanation, and self-regulation [12]. Some factors related to students' low critical thinking skills can be divided into two which are external and internal. Mursari [13] stated that the external factors are including family, school environment, and society while the internal factors are independent learning, learning motivation, health, knowledge, and learning style.

The results of the observation and early interview with the teachers and some students showed that the learning management in SMP Negeri 1 Colomadu Karanganyar is still not optimal. The teaching and learning process tend to use conventional lecture method only without any creativity and innovation from the teachers. For that reason, the alternative solution that can be done is "collaborative mathematics learning innovation". This learning innovation enables to stimulate critical thinking characters in problem solving. The existence of collaborative learning innovation is expected to help students understand the material because students will help each other in the learning process. This innovation will certainly be related to the aspects of 21st century skills.

Collaborative learning in mathematics can help students to think more critically because of the interpersonal skills development. Collaborative learning has great influence in interpersonal skills and emotional intelligence to form students' competences [14], [15]. Besides, collaborative learning also involves students to think more critically in reflective learning by sharing ideas with others [16]. The implementation of collaborative learning demands teachers to be more clever, creative, and innovative in order to develop critical thinking skills in solving mathematical problem.

Teachers' performance in collaborative mathematics learning requires ideas and commitment from both the teachers and the principle. This is supported by previous studies [17], [18] that in super team-based learning innovation there are some changes in the process of deciding and selecting the idea. This, hopefully, can build student-teacher cooperation in collaborative mathematics learning management that can develop critical thinking in problem solving. Collaborative learning is a transformation from individual with critical thinking skills that can build motivation and responsibility in learning process [19], [20]. In shorts, collaborative learning aims to: i) Create a more natural and humanist cooperation between students; ii) Build critical thinking and problem-solving skills; iii) Create social communication; and iv) Build students' enthusiasm and motivation. Through collaborative learning, students can experience the three concepts which are individual accountability, group benefits, and equal achievement of success [21], [22].

Based on the explanation, critical thinking skills in solving mathematical problem is students' skills and abilities in understanding, analyzing, and making conclusion and decision to solve mathematical problem. This study of critical thinking in solving mathematical problem will be focusing on four indicators: i) Interpretation which is understanding the problems by writing down the information and questions correctly; ii) Analysis which is identifying the relation between the questions and answers to find the solution of the problem; iii) Evaluation which is implementing the solution correctly; iv) Inference or conclusion, which is using different alternative solution to check on the result and draw a conclusion based on the result.

Therefore, there are four objectives of the study in this article. First, to describe critical thinking skills based on the indicator of interpretation in solving mathematical problem. Second, to describe critical thinking skills based on the indicator of analysis in solving mathematical problem. Third, to describe critical thinking skills based on the indicator of evaluation in solving mathematical problem. Lastly, to describe critical thinking skills based on the indicator of inference in solving mathematics problem.

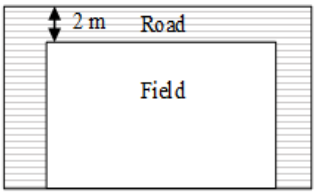
2. RESEARCH METHOD

The type of this research is research and development. According to previous study [23], research and development is a process of modifying existing products and resulting to a new product that can be accounted in terms of efficiency, effectiveness and strength. Research design in this article is qualitative-

ethnography. Qualitative research aims to understand the social phenomenon from the perspective of the participants and when their turn comes to develop the theory. Ethnography is the research strategy in which the researcher learns about the habit of critical thinking in solving mathematical problem on middle school students academic years of 2020/2021, for eight months (September 2020–April 2021) through observation and field notes, interview and assessment analysis [23], [24]. The process of this research is flexible and developed contextually in response to the reality of mathematics learning in the research area.

The object of this research is the critical thinking skills in solving mathematical problem which was analyzed from students' assessments and the results of the interview with participants. The students' assessments consist of four mathematics problems provided by the teacher. To analyze the assessment, the researcher arranged a rubric for the answers of every question based on the indicators of critical thinking skills in solving mathematical problem, such as interpretation, analysis, evaluation, and inference. The example of the rubric for question number 3 can be seen on the Figure 1.

3. A rectangular field. It is known that the length of the field is twice its width. On the outer edge of the three squares a road 2 meters wide was made. If the entire road (shaded area) is 168 m^2, then the area of the square is....



Solution:

a. Interpretation
For example, the length of the rectangle be p and the width of the rectangle is l. It is known in that problem p = 21

b. Analysis
$L_{rectangle} = p \times l = (2l+4)(l+2) = 2l^2 + 8l + 8$
$L_{field} = 2l \times l = 2l^2$
$L_{road} = 168\ m^2$

c. Evaluation
Field area can be determined by:
$L_{field} = L_{rectangle} - L_{road}$
$2l^2 = 2l^2 + 8l + 8 - 168$
$8l = 160$
$l = 20$ m
So, $L_{field} = 2l \times l = 2l^2 = 2(20)^2 = 800\ m^2$

Figure 1. Rubric for answer number 3

The subjects of this research were the headmaster, the mathematics teachers, and ninth grade students of SMP Negeri 1 Colomadu, Karanganyar, Central Java, Indonesia, academic year 2020/2021. The headmaster was a man who has more than 10 years experiences in teaching. The two mathematic teachers, a man and a woman, both have more than five years experiences in teaching. Class IX B consisted of thirty students with four male (13.33%) and 26 female (86.67%). Based on the results of students' assessments in the beginning, the students were divided into high, average, and low initial skills as presented in Table 1. According to the table, the researcher took one answer from each category (High initial skills/Subject 1; Average initial skills/Subject 2; Low initial skills/Subject 3) and analyzed their skills in critical thinking.

Table 1. Students' initial skills

Subjects	Initial ability value interval	Category	Frequency	Percentage (%)
1	59–88	High	21	70.00
2	29.5–58.5	Medium	8	26.67
3	0–29	Low	1	3.33
	Total		30	100

Exploration and analysis for the research data were collected through observation and field notes, interview, and documentation [25]. Observation and field notes aim to observe and take notes on the mathematics learning activities which was done through online study. Interview with the headmaster, the mathematics teachers, and the students was done to investigate more on both learning paradigm and critical thinking activities in solving mathematical problem on each level of initial skills. Documentation was done to analyze the archive of subjects' initial score and the answers on the mathematics problem.

The data validity was analyzed through triangulation of sources and technique [23]. Triangulation of sources was to verify the data credibility by checking the data collected from the participants. The data was then described and categorized into three different types, same data, different data, and specific data. Afterwards, the researcher drew a conclusion before asking the subjects' approval for the data.

Triangulation of technique was done to verify the data credibility by applying different techniques on the same subjects (the headmaster/the teachers/the students). For example, the data from document analysis was then checked through interview, observation and field notes. When the three techniques showed different results, the researcher needed to discuss more with the subjects to decide the correct one, or all considering the difference in point of views.

The data were analyzed inductively [23], [26]. The process was done continuously along with the data collection. The field notes consisted of two parts, with the first part being the data description, data resources, data collection techniques, and the environmental situation of the research setting. The second part was about the research reflection on the results and the next step. The reflection was aimed to relate back to the initial conclusion or theory about the research findings and the exploration and understanding of the data at the next research. This part was where the field notes were hopefully developed so that the researcher already got the conclusion of the findings when the research ended. Data analysis consisted of data collection, data reduction, data presentation, and conclusion. These activities are shown in Figure 2.

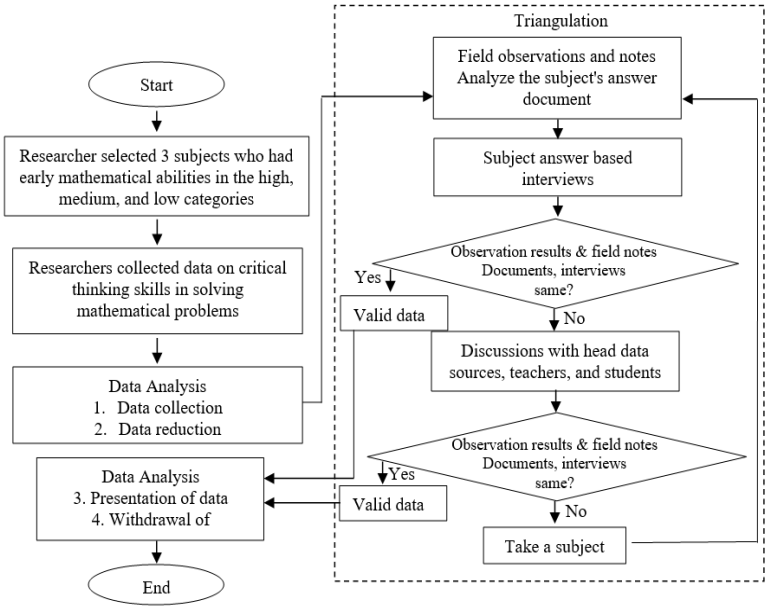


Figure 2. The activities of data collection to data analysis

3. RESULTS AND DISCUSSION

Critical thinking skills in solving mathematical problems contributed positively to good attitude, respecting others, curiosity, also improving the interpretative and calculating skills. It was similar to what was said by the two mathematics teachers on their interview which was supported by the headmaster's statement:

> "*The critical thinking skills of the students on solving mathematical problems was monitored since the introduction, main activity to closing. The mathematics teachers always show examples of 5S (Senyum, Salam, Sapa, Sopan, Santun/Smile, Greeting, Polite*)."

According to the teachers, those habits are intended to teach students how to respect others (between teachers and students, and students and students). Moreover, these habits may influence students to understand to always think positively and politely in learning; be more curious; improve their abilities in learning mathematics. This study focused on four indicators of critical thinking skills in solving mathematical problems: i) Interpretation which is understanding the problems by writing down the information and questions correctly; ii) Analysis which is identifying the relation between the questions and answers to find the solution of the problem; iii) Evaluation which is implementing the solution correctly; iv) Inference or conclusion, which is using another different alternative solution to check on the result and draw a conclusion based on the result.

3.1. Critical thinking skills based on the indicator of interpretation

Based on the assessment analysis, Subject 1 (high initial skills) can think critically based on the indicator of interpretation. In question number 1, students are expected to show their understanding on the problem by drawing a right triangle and write down the known information such as x=base, y=height, and 2x+y=24 or in other formula y=-2x+24. Hence, for question number 2, the first step is to write down the height equation of the bullet which is h(t)=5t2+80t+10. As for number 3, students should draw a field based on the information. Then they write down the width of the figure (2l+4) m which is the results of adding the length of the field with twice the width of the road, and the width (1+2) m from adding the width of the field with the width of the road. For question number 4, Subject 1 understands the problem by writing down the perimeter of the rectangle which is 40. The example of Subject 1's answer can be seen at Figure 3. The transcript of an interview with Subject 1 about solving quadratic equations can be seen as (A=researcher; B=Subject 1):

A: *Do you understand the problem, about the information and the question?*
B: *Yes, of all four questions I understand everything, what was the information and what was the question.*
A: *What did you do to understand that?*
B: *I tried to find some information from the problem then wrote down everything into mathematical symbols. As for the questions, I understand the sentence and calculated them with mathematical formulas.*

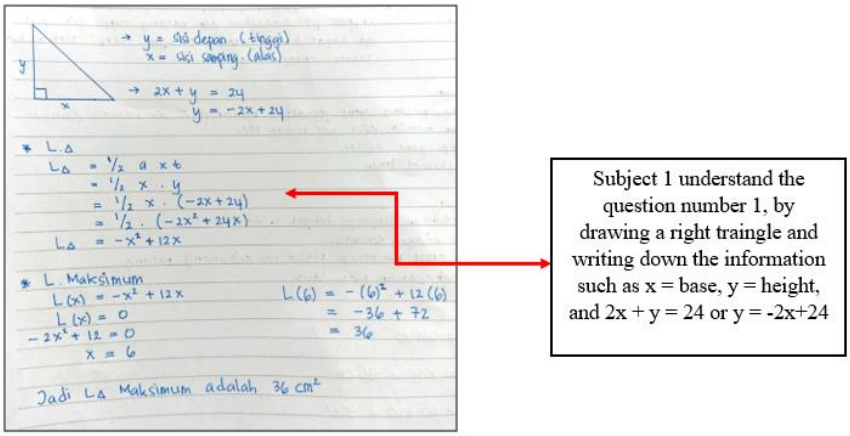


Figure 3. Subject 1 answer for question number 1

Based on the assessment analysis, Subject 2 (average initial skills) is still not used to think critically based on the indicator of interpretation. It can be seen by how Subject 2 does not bother to write down the detailed information but gets some correct answers. For question number 1, Subject 2 finds the correct answer without listing the detailed information. For number 2, Subject 2 cannot find the correct answer and also does not list any detailed information. Number 3 has some information down such as the length of the field=21. For the last number 4, no detailed information is written. The example of Subject 2's answer can be seen at Figure 4. The transcript of an interview with Subject 2 about solving quadratic equations can be seen as (A=researcher; C=Subject 2):

A: *Do you understand the question, about the information and the question?*
C: *I could only understand some of them but I am not used to writing everything in detail for the answer.*
A: *What did you do to understand that?*
C: *I looked for the right formula and applied them on the data from the questions, then calculated everything.*

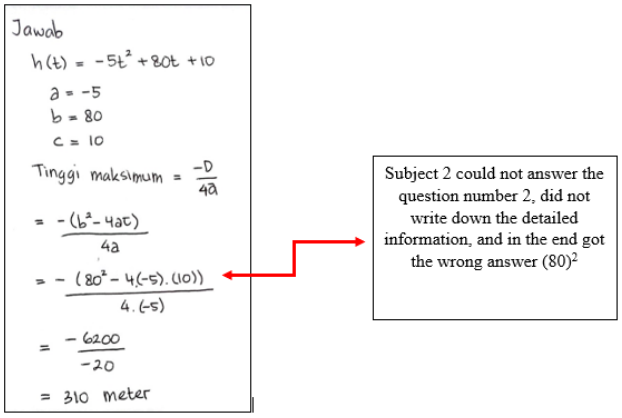


Figure 4. Subject 2's answer for question number 2

This fact is also confirmed by one of the mathematics teachers in an interview as he said that most students with average skills in mathematics is not used to write down any detailed information before calculating everything which make them tend to miss out something from the questions. In the end, they get the wrong answer. The other mathematics teacher and the headmaster also stated the same thing as each student has their own characteristics but most student in this school has good mathematical ability (Table 1).

Based on the assessment analysis, Subject 3 (low initial skills) is still not able to think critically based on the indicator of interpretation. It can be seen from the short answer to the question. The example of Subject 3's answer can be seen at Figure 5. The transcript of an interview with Subject 3 about solving quadratic equations can be seen as (A=researcher; D=Subject 3):

A: *Do you find any difficulties in solving the quadratic function problem?*
D: *Yes, I couldn't understand the questions.*
A: *Which one do you find the most difficult?*
D: *(silence)*
A: *Can you read the question number one for me?*
D: *Sure.*
A: *So, any information from number 1?*
D: *The leg and the hypotenuse of the triangle.*

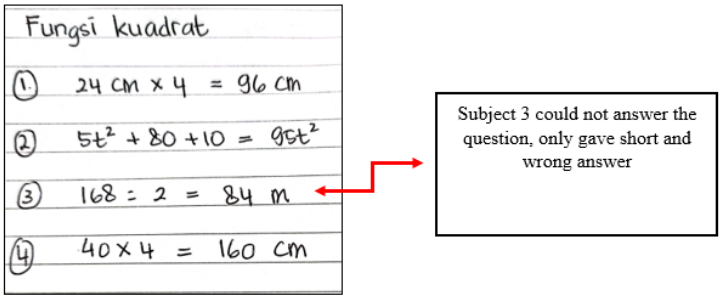


Figure 5. Subject 3's answer

This fact is also confirmed by the two mathematics teachers. In an interview they stated that the students with low initial skills in mathematics need extra attention continuously. The helps from the teachers and fellow students are expected to make them happy and be more comfortable so they can improve their learning performance.

Based on the result of the assessment analysis on Figure 3 and the interview, Subject 1 understands the problem to get the information and the question then takes them as keywords that indicate their skills to think critically based on indicator of interpretation. Related to those keywords, subject with high initial skills in mathematics successfully solve the problem which shows their good result in learning mathematics. This is proven by the research findings which show that students who are confidence they can solve any mathematics questions will really get positive results [27], [28]. This also means that high initial skills in mathematics is the basic attitude needed in order to develop students' critical thinking skills based on indicator of interpretation.

Based on the result of the assessment analysis on Figure 4 and the interview, Subject 2 did not categorize the data into information and the question which means Subject 2 is still not able to think critically based on the indicator of interpretation. Related to the inability to think critically on indicator of interpretation, subject with average initial skills in mathematics mostly finds difficulties in both the process and final answer of solving the mathematical problem which affects their learning result in mathematics. Therefore, the students who are still not used to the systematical process of problem solving needs to practice more through concept development learning. This is proven by the research findings which show that the implementation of problem-based learning and concept development learning can improve students' interpretation skills [29], [30]. This also means that improving students' critical thinking skills based on indicator of interpretation can be done through problem-based learning on mathematics including concept development, controlled practiced, or individual practice.

Based on the result of the assessment analysis on Figure 5 and the interview, Subject 3 gives short and wrong answers only which means Subject 3 is not capable to think critically based on the indicator of interpretation. That is why subject with low initial skills in mathematics finds many difficulties at solving the mathematical problem which resulting on the bad result in learning mathematics. Students in this level need to do collaborative learning. This is supported by the findings which concludes that students with low initial skills tend to be less careful, have low motivation in learning mathematics, and find difficulties in understanding the question that cause them to not know how to summarize the problem [31], [32]. This also means difficulties in understanding the question is the main factor in failing to summarize the problem, the information and the question, is mainly caused by low motivation in learning or students' lack of accuracy.

3.2. Critical thinking skills based in the indicator of analysis

Based on the assessment analysis, Subject 1 (high initial skills) can think critically based on the indicator of analysis. In question number 1, Subject 1 can write the process on how to solve the problem properly and writes down the correct formula for area of triangle=-x2+12, x as the base and -2x+24 as the height. In question number 2, Subject 1 can analyze the height equation of the bullet and find a=5, b=80 and c=10 as the answer. For question number 3, Subject 1 can arrange the steps in solving the problem using the relation between the information of length and width of the field and found the formula of area of the road=total area–area of the field. For question number 4, Subject 1 could substitute the information into the formula of perimeter of a rectangle and found 2(x+y)=40, x as the width and y as the length, then it was simplified into x+y=20.

This result is also confirmed by the mathematics teachers in an interview. The teachers stated that the students with high initial skills in mathematics tend to be more independent and can solve any mathematical problems by themselves. In a discussion for material development to find the concept or formulas, those students can become the facilitators for other students.

Based on the assessment analysis, Subject 2 (average initial skills) is still not used to think critically based on the indicator of analysis. It can be seen from the way Subject 2 answers the questions without writing down the systematic process which affects some of the final answers to be wrong. In question number 1, Subject 2 writes down the formula for area of a triangle which is L=-(x2)+12x correctly. In question number 2, Subject 2 cannot analyze the height equation of the bullet which resulting on the wrong answer. In question number 3, it should be about finding the width through the area of the road and changed the p variable into 2l, but Subject 2 analyzes them wrong. In question number 4, Subject 2 can find the relation between the length of the wire and the perimeter of the rectangle. In an interview, the two mathematics teachers stated that the students with average initial skills tend to be a bit lazy in learning mathematics. While diligence and discipline are basic attitude in learning mathematics, the situation causes the students to fail to show their best in learning mathematics.

Based on the assessment analysis, Subject 3 (low initial skills) is still not able to think critically based on the indicator of analysis. It can be seen from the way Subject 3 almost does not finish solving all the mathematical problems. This result is also supported by the mathematics teachers in an interview. The teachers stated that students with low initial skills in mathematics need extra attention from the teachers, fellow students, and their parents at home.

Based on the result of the assessment analysis and an interview with the mathematics teachers, subject with high initial skills is able to find the relation between statements and questions then able to write them into the systematical process of solving the problem. The students at this level have the ability to think critically based on the indicator of analysis thus have the ability to become the facilitator for other students in learning mathematics. This is supported by the research findings which show that the teachers need to help practicing the process of critical thinking to improve students' analytical skills by giving group or individual task to help them understand notation and skilled in arithmetic [33], [34]. Thus, the process of critical thinking based on the indicator of analysis needs to be implemented through collaborative learning and problem solving.

Based on the result of the assessment analysis and an interview with the mathematics teachers, subject with average initial skills can barely find the relation between statements and questions and cause them to unable to make the right systematical process of the problem solving. The research findings from [35] explained that understanding the concept of a problem does not guarantee students to always have the ability to implement their knowledge to solve the problem. Therefore, students with average initial skills need more practice for critical thinking through collaborative activities including concept development learning, controlled practice, or individual practice. This is supported by the research finding [36] which showed that numerical literacy on the aspect of analyzing information on concept development activity, controlled practice, or individual practice can help students to think systematically and structured in understanding the process of mathematics learning.

Based on the result of the assessment analysis and an interview with the mathematics teachers, Subject 3 is unable to find the relation between statements and questions because they find it difficult to understand the questions. That means Subject 3 cannot fulfill the indicator of analysis in critical thinking skills. Therefore, subject with low initial skills needs extra attention from the teachers, fellow students, and their parents at home. This is supported by the research findings [37] which showed that improving students' critical thinking skills based on the indicator of analysis can start by planning the lesson with something the students are already familiar with then connecting them with the new material and most importantly staying to pay attention to them.

3.3. Critical thinking skills based in the indicator of evaluation

Based on the assessment analysis, Subject 1 (high initial skills) can think critically based on the indicator of evaluation. In question number 1, Subject 1 can evaluate problem solving correctly. In question number 2, Subject 1 uses the formula for maximum height=b2-ac/-4a and gets the result for 330 m. In question number 3, Subject 1 can solve the problem and finds the final answer for the area of the field correctly by using the formula of area of the road=8l+8 and applying 168=8l+8 then the width (l) was 20 cm. In question number 4, Subject 1 does the first step correctly with L=xy but does not substitute y and fails to obtain the quadratic function thus fails to calculate the maximum area too. Those explanation means in term of indicator of evaluation, Subject 1 is considered not skilled at critical thinking yet related to a question like number 4. The example of Subject 1's answer for question number 4 can be seen at Figure 6.

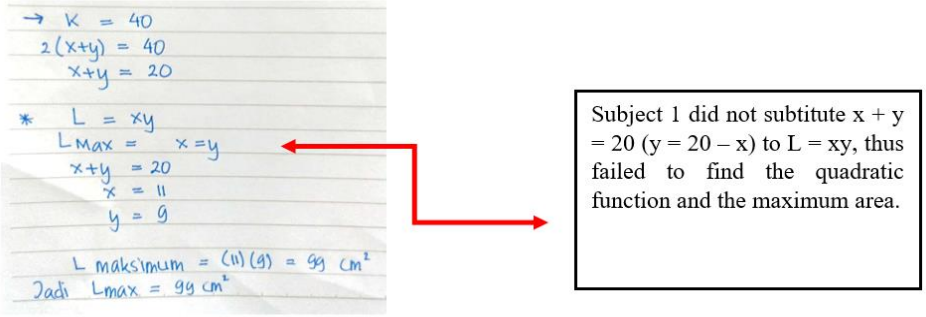


Figure 6. Subject 1's answer for question number 4

This result is also supported by the mathematics teachers' statement in their interview. The teachers agreed that the students with high initial skills often make mistake on the type of question like number 4 because they tend to do it hastily and forget the concept of maximum/minimum inversion of the quadratic function. Based on the assessment analysis, Subject 2 (average initial skills) is still not used to critical thinking based on the indicator of evaluation. It can be seen from the way Subject 2 answers the questions without writing down the systematic process which affects some of the final answers to be wrong. In question number 1, Subject 2 does not write down the formula for maximum area. In question number 2, Subject 2 can answer the maximum height of the bullet without writing down the formula. In question number 3, there are mistakes caused by no systematic process written and the answer for the width and the length are wrong. In question number 4, Subject 2 also fails to solve the problem about explaining an area become maximum if the equation is width=length.

In an interview, Subject 2 admitted to not used to story problem and maximum formula of quadratic function. Both mathematics teachers also agreed that most students with average initial skills tend to have difficulties in understanding the concept and other mathematical formulas. However, the teachers always remind them about it that understanding the concept and other mathematical formulas are very important in order to solve any mathematical problem.

Based on the assessment analysis, Subject 3 (low initial skills) is not capable to think critically based on the indicator of evaluation. It can be seen from the way Subject 3 does not finish solving all the questions and gives no answer. Especially for question number 4, none of the three students of Subject 3 fulfils the indicator of evaluation for critical thinking skills when both Subject 1 and Subject 2 are at least giving an answer even though some were wrong. An interview with the two mathematics teachers confirms the analysis result of Subject 3. The teachers stated that students with low initial skills in mathematics need extra attention from the teachers, fellow students, and their parents at home.

Based on the result of the assessment analysis of Subject 1 on Figure 6 and an interview with the mathematics teachers, Subject 1 fails to fulfill the characteristics of critical thinking skills based on the indicator of evaluation as there is one type of problem that subject with high initial skills is unable to solve. This was supported by the research findings from [38] which showed that some students make mistake in drawing the conclusion because of wrong calculation. The research from [39] mentions some students with high initial skills also find difficulties in drawing conclusion. That means there is always possibility for every student from each level to face very difficult problem to solve without collaboration with others.

Based on the result of the assessment analysis of Subject 2 and an interview with both Subject 2 and the mathematics teachers, it is obvious that subject with average initial skills in mathematics can barely understand the concept of story problem and are not mastering the formula for quadratic function yet thus cannot fulfill critical thinking skills based on the indicator of evaluation. This result is supported by the research findings from [40] which show that the mistake in problem solving evaluation is usually related to the wrong formula. The research findings from [41] mention that to solve the problem correctly the students need to use all concepts and lessons that have been taught. So, in order to fulfill the indicator of evaluation in students' critical thinking skills, students need to get used to the discipline of exploring mathematical concepts and implementing them.

Based on the result of the assessment analysis of Subject 3 and an interview with the mathematics teachers, Subject 3 is unable to think critically based on the indicator of evaluation as the subject does not answer the question. There are many factors related to low initial skills students but the most important one comes from their own selves that is why they need extra attention. This result is supported by the research findings which show that numerical literacy skills through continuous mentoring will improve the pedagogic competence of both students and teachers [42]. Thus, the efforts to improve critical thinking skills based in the indicator of evaluation can be done by doing fun collaborative learning.

3.4. Critical thinking skills based in the indicator of inference

Based on the assessment analysis, Subject 1 successfully draws a conclusion on question number 1 by stating that the final answer for the maximum area of the triangle is 36 cm^2. For question number 2, Subject 1 is able to check the calculation that has been done and concludes the maximum height of the bullet as 330 m. For question number 3, Subject 1 is able to find the width=20 m and length=40 m correctly and concludes the area of the field as 800 m^2. For question number 4, there is a mistake in the solving process that causes the wrong conclusion at the end.

In an interview, Subject 1 admitted the mistake in number 4 was because subject failed to think thoroughly when deciding the formula. Both mathematics teachers also agreed to the assessment analysis result. The teachers stated that the student with high initial skills tend to be less careful when reading the question that caused them to miss some information.

Based on the assessment analysis, Subject 2 does not make any conclusion for question number 1. Subject 2 also made wrong conclusion for question number 2 while does not conclude the final answer in number 3. The last in number 4, Subject 2 writes down some systematical process but fails to conclude it in the end. In an interview, Subject 2 admitted to just focused on the answer directly without drawing any conclusion first. Both mathematics teachers also agreed that most students with average initial skills tend to have difficulties in understanding the questions.

Based on the assessment analysis, Subject 3 (low initial skills) is not capable to think critically based on the indicator of inference. It is shown from the way Subject 3 does not finish solving all the questions and gives no answer. In an interview, Subject 3 admitted that it is difficult to transform story problems into mathematical form and thought mathematics was a difficult lesson. An interview with the two mathematics teachers confirmed the analysis result of Subject 3. The teachers stated that students with low initial skills in mathematics need extra attention and guidance in learning mathematic.

Based on the result of the assessment analysis of Subject 1 and an interview with the mathematics teachers, subject with high initial skills in mathematics can draw a conclusion from the solving process on number 1, 2, 3, but fails at number 4. In other words, subject with high initial skills is still not optimal in critical thinking based on the indicator of inference. This result was supported by an opinion [11] stated that students with high initial skills can elaborate their answers by induction and deduction but that does not necessarily mean they can decide the right activity to interact with others. Therefore, teachers' effort to improve students' critical thinking skills by optimized their basic ability to be in line with their knowledge and experiences is very important.

Based on the result of the assessment analysis of Subject 2 and an interview with both Subject 2 and the mathematics teachers, it can be concluded that subject with average initial skills in mathematics never make any conclusion in the process to solve mathematical problems. This result is supported by the research findings [43] which stated that numerical literacy skills in mathematics is closely related to interpreting contextual problems into the language of mathematics that can be interacted with other people which is rarely done on average skills students. Therefore, students at this level need to get used to more assistance in critical thinking training based on indicator of inference through collaborative activities.

Based on the result of the assessment analysis of Subject 3 and an interview with both Subject 3 and the math teachers, it can be concluded that Subject 3 cannot make any conclusion in the solving process thus is considered unable to think critically based on indicator of inference. Students with low initial skills in mathematics tend to have this mindset that mathematics is a difficult subject since the beginning that make them lose motivation and effort in learning. Thus, guidance and extra attention are needed to help the students at this level to be mutual with students of other level. This result is supported by the research findings [44] which concluded that students who have metacognition knowledge usually have better grade and performance than the students that do not. Therefore, teachers should never give up in guiding the students for them to not be left behind.

4. CONCLUSION

High initial skills in mathematics are the basic for students in order to develop their critical thinking skills based on the indicator of interpretation. Improving the critical thinking skills based on this indicator can be done through problem-based mathematics learning in concept development, controlled practice, or individual practice. Difficulties in understanding the question is the main factor in failing to summarize the problem, the information and the question, and is mainly caused by low motivation in learning or students' lack of accuracy.

Critical thinking process based on the indicator of analysis still need more practice through collaborative learning in problem solving. For students with average initial skills in mathematics, collaborative learning in concept development, controlled practice, or individual practice are some ways to improve the critical thinking skills based on indicator of analysis. On the other hand, students with low initial skills in mathematics need extra attention from the teachers, fellow students, and their parents at home.

There is always the possibility for every student from each level to face difficulties in solving some problems without collaborating with others. In order to fulfill the indicator of evaluation in students' critical thinking skills, students need to get used to the discipline of exploring mathematical concepts and implementing them. Thus, the efforts to improve critical thinking skills based in the indicator of evaluation can be done by doing fun collaborative learning. The teachers can make more effort in optimized the students' basic holistic skills in line with their knowledge and experience in order to improve students' critical thinking skills based on the indicator of inference. Students with average initial skills need to get used to more assistance in critical thinking training based on indicator of inference through collaborative activities. Teachers also need to be extra attentive to the students with low initial skills.

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