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# Enhancing creative reasoning through mathematical task: The quest for an ideal design

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**ABSTRACT**

This study aimed to: i) identify the types of completion and ways of completing mathematical tasks carried out by students based on the type of their educational institution (junior high school/SMP and junior Islamic high school/MTs); and ii) explore the tendency of the type of reasoning students use in completing the tasks. This study involved 93 students at grade 8 of prominent SMP and MTs in Sidoarjo Regency, East Java, Indonesia. Variety of ways and solutions to mathematical tasks and the tendency to type students' reasoning were explored through creative reasoning tests. Interview techniques by telephone were implemented to further explore the types of student reasoning: local creative reasoning (LCR) or global creative reasoning (GCR). The results showed that in comparison to MTs students, SMP students have more varied answers and ways to solve mathematical tasks. However, in certain cases MTs students show some unique answers. The type of creative reasoning that students tend to use is of the LCR. These findings indicate the importance for mathematics teachers to design mathematics tasks that develop GCR-type creative reasoning. Examples and exercises in mathematics textbooks should also be directed at developing this type of creative reasoning.



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## 1. INTRODUCTION

Reasoning is a fundamental component in mathematics [1], [2]; because with reasoning, students' creative thinking can develop, and mathematics learning becomes effective [3]. Students will also understand and master the mathematical concepts they learn more meaningfully, because they do not only memorize them but use them in context [4], [5]. Considering the importance of reasoning in mathematics, national mathematics education curricula and standards in various countries, such as Singapore, Finland, Australia, Canada, Ireland, USA, Switzerland, South Africa, and India, pay particular attention to this reasoning aspect [6]–[8].

Singapore's mathematics framework-for example-places reasoning as a key element of students' mathematical problem-solving abilities [9]. Finland also places mathematical logical reasoning and thinking in its mathematics curriculum as important and fundamental provisions for students to participate in society in the future [10]. Such changes in the global arena also influence curriculum policies in Indonesia that then places mathematical reasoning explicitly in a series of competencies formulated in the 2013 Curriculum [11]. Mathematical reasoning is directed to connect formal mathematical concepts and the real-world problems, namely the real world outside the classroom.

Students' mathematical reasoning abilities can be seen from the way students solve mathematical problems [12]. Based on their answers, they can be classified in such a way to get a picture of their reasoning ability in solving the problem [1]. Students who have good mathematical reasoning can arrange answers in a complete and orderly manner and provide logical reasons for a given mathematical problem [13]. They can make connections between concepts and draw the right conclusions or decisions to solve the given mathematical problems [14].

A study found that mathematical tasks made a significant contribution to students' reasoning abilities [5]. Previous researchers [5], [6] analyzed the relationship between the type of task and the mathematical reasoning students use. When faced with tasks commonly encountered in textbooks, they tend to try to recall existing facts, formulas, or algorithms. In contrast, tasks given by the teacher and rarely found in textbooks are completed using mathematical reasoning [1], [15], [16]. In this context, according to Lithner [17], mathematics teachers are required to have the ability to choose or design mathematical tasks that challenge students' mathematical reasoning abilities.

Such reasoning abilities can be designed through curriculum and syllabus. Curriculum and syllabus mandating mathematical reasoning in Singapore result in Singapore as the top performing country in Trends in International Mathematics and Science Study (TIMSS) and Program for International Student Assessment (PISA) [9], [18]. Also, such tasks on reasoning can be incorporated in textbooks. Previous research [5] found that more than half of junior high school students in Australia, Finland, Singapore, Switzerland, South Africa, and Canada use mathematics textbooks as their main reference in learning. In USA, mathematics textbooks are also a reference for more than 48% of students. In some other countries, including Indonesia, the percentage of using mathematics textbooks as the main reference is more than 90% [18]. This fact indicates that mathematics textbooks are an important component in the mathematics learning process, especially for students. For teachers, mathematics textbooks also greatly influence what teachers teach, how to teach them and what tasks to give to students [19], [20].

A study conducted by Jäder, Lithner, and Sidenvall [5] on student textbooks in Australia, Canada, India, Finland, Ireland, Nepal, Scotland, Singapore, South Africa, Sweden, Tanzania, and USA found that 79% of tasks presented can be solved by imitating the exemplified formula or procedure, 12% can be solved by applying the given procedure with slight modifications, and only 9% need reasoning. Similar conditions also occur in the Netherlands, where the tasks presented in mathematics textbooks only require the application of formulas or procedures [21]. Previous studies [22], [23] found that only 10% of tasks in mathematics textbooks in Indonesia can encourage students to think or reason. The rest of the tasks only invite students to apply routine formulas or algorithms to finish [24].

The aforementioned description on the important role of mathematics textbooks and tasks indicates the importance of designing mathematical tasks that can stimulate students' reasoning. Tasks must also be able to provide challenges and require students' persistence in the completion process [5], [25]. Challenging tasks are needed to trigger students' motivation to find the completion of these tasks [16]. The active involvement of students in completing the challenging and demanding tasks is very much needed to develop students' mathematical reasoning [25].

Learning mathematics using reasoning can develop more effective students' solutions to the task [26], [27]. Thanheiser [28] identified the impact of tasks on understanding mathematical concepts. Previous researchers [17], [29] examined reasoning in the presentation of the completion of mathematics tasks given by the teacher. The findings from the studies indicate that mathematical tasks in the context of reasoning have attracted a number of researchers, highlighting the significance of the issue.

The researchers divided mathematical reasoning into two categories, imitative reasoning, and creative reasoning [29]–[31]. The first type of reasoning involves imitating previously studied completion steps, through sample questions in textbooks or recalling existing formulas/algorithms. This reasoning tends to only encourage students to use memorization in learning mathematics. Creative reasoning is a mathematical thinking activity with completion strategies that students develop themselves or they modify the steps from previously learned concepts, formulas, or algorithms. The students need this type of creative reasoning to equip them with the skill to face the increasingly complex and dynamic challenges in life [32].

Bergqvist [29] categorized creative reasoning into two types: local creative reasoning (LCR) and global creative reasoning (GCR). LCR is a type of reasoning used to solve mathematical problems; most of the solutions are using formulas/algorithms that are commonly encountered by students, but there is one unfamiliar solution step [1]. On the other hand, in the GCR type of reasoning, students need to think hard when solving a mathematical problem because all the necessary solving steps are not familiar to them [4]. Mathematical tasks that can be completed using GCR are completely new tasks for students [29]. These two types of reasoning will be explored in more detail in this paper, then analyzed based on the creative reasoning indicators developed [17], [29], including mathematical foundation, novelty, plausibility, and flexibility as concisely explained.

Mathematical foundation is the involvement of the intrinsic nature of mathematics in the selection and implementation of strategies and arguments presented [29]. In the context of the task, this indicator targets the ability to mention information that is known and asked for on the task, determine and implement strategies to complete the task [33]. The novelty that is meant is the ability to complete tasks that are unusual for the level of students' knowledge in general or to use the old method of completion that many other students have forgotten [12]. Plausibility is the ability to provide logical arguments about the strategies or solutions used in such a way that it provides sufficient reasons for why the resulting conclusions make sense [1]. The final indicator, flexibility, is the students' ability to present several methods used to complete the given task.

This study took up this issue of creative reasoning which has not gained much attention in previous studies. It aimed to produce a design for mathematical assignments that can encourage the development of students' LCR and GCR by elaborating the hypothetical learning trajectory (HLT) during the task design process. HLT is developed from three main components: learning goals, learning activities, and hypothetical learning process [34], [35]. Learning objectives as the first component of HLT become important parts containing the objectives that students must achieve after working on assignments. The mathematical tasks developed based on the objectives can strengthen and deepen students' understanding of the materials. The learning process can be carried out into several activities to provide various mathematical problems according to specified objectives. Furthermore, the hypothesis of the students' learning process contains the teacher's assumptions about the possible responses of the students when they are asked to complete mathematical tasks [36]. So, the objectives of this study are: i) to identify the types of completion and ways of completing mathematics tasks carried out by students based on the type of educational institutions (junior high school/SMP and junior Islamic high school/MTs); and ii) to explore the tendencies of the types of reasoning students use in completing mathematical tasks.

## 2. RESEARCH METHOD

This study employed three phases design research: i) preparation and design; ii) design experiment; and iii) retrospective analysis [37] with details. In the preparation and design stage, the HLT design was carried out intensively with SMP and MTs mathematics teachers. The teachers were involved in this design phase because they are more aware of the real conditions of the flow of the thinking process of grade 8 students. In the design experiment phase, the tasks were tried out to two different schools, one state SMP and one state MTs. In the final phase, retrospective analysis, data from the results of the try out were analyzed in terms of the relationship between the HLT that has been designed and the students' actual answers.

### 2.1. Participants

A total of 93 students (33 male and 60 female) in grade 8 of two state SMP and MTs in Sidoarjo Regency, East Java, Indonesia participated in this research. The two schools were selected because they represent the leading schools in Sidoarjo Regency, so that students may be given mathematical reasoning tasks in the LCR or GCR types. Both schools also follow the national curriculum, and the students participating in the research had previously learned the material of number patterns. So, the students in the two schools get relatively the same learning material.

However, most sample questions, exercises, and homework given by the teachers mostly honed routine math skills, which, in order to complete the task, the students can just copy the steps exemplified by the teachers or textbook. Previous studies also focused more on instilling procedural knowledge as experienced [38] than on developing students' conceptual understanding and reasoning.

### 2.2. Research instrument

The instruments of this research were a creative reasoning test and an interview guide. The creative reasoning test was developed with reference to a textbook published by the Ministry of Education and Culture of the Republic of Indonesia with some adjustments to the principles of designing tasks according to creative reasoning through three stages [17]. In the first stage was developing the question grid as the basis for compiling four question items in the form of non-objective. The grid is also completed with a scoring guideline, which then was used as a reference in developing HLT. There was also an intensive discussion with the mathematics teachers to develop the relevant HLT.

In the next stage, the four questions and the scoring guideline were validated by seven mathematics teachers of grade 8. This process was carried out by relying on the grid that was developed in the previous stage in order to analyze the content validity of the questions [39]. The content validity of the four items was calculated by using the Aiken formula [40]. It was found that the validity coefficient (V) was between 0.76 to 0.83 (with a mean of 0.79) and with the V table at $p=0.05$ the value is 0.75 for the seven validators. This means that the four items are valid in terms of the content. In the third stage, the validated reasoning test was then checked for similarity using Turnitin software. This test was carried out with a consideration that the students were not assigned the task directly in the classroom but through google classroom. This assignment model can allow students to search through the internet to find solutions to a given task, and teachers would find it difficult to control. For this reason, any items with a high level of similarity (more than 20%) were removed from the test. After the deletion, the similarity index was less than 9% for the remaining three question items. These three items were ultimately used as the instrument for creative reasoning.

In addition to the test, the interview guideline was developed with reference to indicators of creative reasoning consisting of elements of mathematical foundation, novelty, flexibility, and plausibility. The interviews were conducted over a distance by eliminating physical contact between the researcher and the students participating in the study to provide more comfort for students being interviewed during this pandemic [41], [42]. The interviews were conducted online by telephone [43]. There were nine students interviewed (five SMP students and four MTs students).

### 2.3. Data analysis

The test results were analyzed by comparing them with the HLT previously designed. The results were tabulated to get the percentage of students who have successfully reached a certain level according to the indicators [30] in each type of creative reasoning (LCR and GCR). The results of the creative reasoning tests that matched the criteria (LCR and GCR) were used as a reference for conducting the interviews. The interview data were analyzed using the stages of reduction, data presentation, and verification. The use of a combination of creative reasoning test results with deepening data through interviews was intended to test the validity of the data through technical triangulation.

## 3. RESULTS AND DISCUSSION

### 3.1. The types of completion and methods for completing mathematical tasks performed by students in terms of the types of educational institutions (SMP and MTs)

Varieties of answers and uniqueness of the methods used were identified from a sample of 93 students (48 SMP students and 45 MTs students). The results show that five students from SMP and four from MTs had answers relevant to these provisions. The rest of the students had similar or even the same answers and methods. These students were categorized as having one type of solution. Table 1 presents the proportion of subjects selected from each type of educational institution along with the types of methods. The SMP and MTs were further compared on the grounds that there is still a perception held by many that students in SMP have better abilities compared to those in MTs [8]. MTs are considered to have too much content on religious subjects and do not prioritize general subjects.

Table 1. The proportion of subjects selected from SMP and MTs

| Type of institution | Class | Number of students | Selected subject | Variants of methods of completion |
|---|---|---|---|---|
| SMP | 8-1 | 32 | 3 | 23 |
| | 8-7 | 16 | 2 | |
| MTs | 8-1 | 22 | 2 | 19 |
| | 8-2 | 23 | 2 | |
| Total | | 93 | 9 | |

Table 1 provides information that the variety of answers by SMP students is not different from that of MTs students. However, it becomes more interesting when compared to the data on the potential diversity of answers given by students of both types of educational institutions for each task as presented in Table 2. The data show that in general SMP students have more varied types and methods of completion than MTs students. However, it must be acknowledged that for certain cases MTs students have unique answers.

An example for such a unique case is found in the completion of task number 1. MTs students combine the odd number pattern formula with that of the even number pattern. This idea of comes from their observations on the formed number pattern. They then divided it into two different types of number patterns. This method has never been found before from all the available answers, because they construct their own ideas. One of them said:

> *"At first, I was just trying to divide the number pattern into two. After I observed and compared it with the answer from the first method, I concluded that to find the 6th day, I can use the odd number pattern formula with n=3. Meanwhile, to find the 7th day I can use an even number pattern formula with n=4."*

Variations of answers and more diverse methods by the students as presented in Table 2, for example, occur in completion of the task of “determining the nth pattern formula based on a certain arrangement of patterns from a picture.” In responding to this task, the two groups of students came up with four types of methods, but the methods used by the SMP students came from more diverse sources. They do not only depend on what the teacher has explained during learning. This fact reflects what Bergqvist and Lithner [44] have proposed. They stated that students' ways of thinking and reasoning when they complete assignments do not only depend on the characteristics of the task, but also on the student's social context. Students' social settings will affect the reasoning used.

Table 2. The potential variants of completion by students of SMP and MTs

| No. Task | Variants of completion of SMP | Variants of MTs |
|---|---|---|
| 1. | The answers and methods vary | The answers vary and there is one unexpected and unique method |
| 2. | The answers and methods vary | No variants in the answers but the methods vary |
| 3. | The answers and methods vary | The answers vary but the methods do not |

An example of such an influence of social setting to students’ reasoning is the method used by one junior high school student. The method came from his own creativity; he was inspired from the mathematics textbooks he read. Another SMP student stated that his method to answer the question was very much influenced by the tutoring he had and from his reading on the internet. He reflected *“I have never used this formula before; it was still trial and error and it works. Because I've been taught by my tutor and have seen it on the internet.”* On the other hand, the methods used by MTs students tended to imitate what their mathematics teacher had given them during the lesson.

This finding is in line with previous researchers’ [5], [21]. When students are often faced with tasks commonly encountered in textbooks, they tend to recall formulas or algorithms commonly given by the teacher to work on new assignments and imitate the steps or procedures of completion. In fact, most of the tasks in the mathematics textbook (90%) tend to encourage students to apply routine formulas or algorithms when they are required to complete them [24]. This condition may occur in the mathematics assignments given to MTs students.

The results of HLT identification with a variety of student answers are also in line with and support the findings. This identification is important because HLT functions as a prediction of the possibilities for students’ answers to the given task [36]. HLT has also gone through a long formulation process and involved teachers in both institutions. Hence, it is relevant to present HLT in this context. Table 3 presents the various answers of the students (SMP versus MTs students) in relation to HLT.

Table 3. Proportion of suitability of various answers with HLT

| No. Task | Topic of the task | Proportion and percentage | |
|---|---|---|---|
| | | SMP | MTs |
| 1 | Finding the next term from 3 sequences of number presented | 33/48 (69%) | 39/45 (87%) |
| 2 | Determining a certain term from a mathematical problem presented in a story | 41/48 (85%) | 34/45 (76%) |
| 3 | Determining the $n^{th}$ pattern from an object configuration presented | 2/48 (4%) | 5/45 (11%) |
| | Average | 25/48 (52%) | 25/45 (58%) |

Table 3 illustrates that SMP students have a lower tendency than MTs students in terms of the suitability of HLT formulated with the various answers given. However, this does not happen in task number 2. In this assignment, most SMP students used arithmetic formulas which were the most familiar method for them as initially thought as included in HLT. These findings indicate that MTs students tend to use more predictable solutions and tend to follow what the teacher has taught them.

Further analysis indicates that the patterns of communication and interaction between students and teachers in MTs seem to have played a role. This is influenced by the Islamic boarding school (*pesantren*) culture inherent in MTs (which the long history of the birth of madrasa cannot be separated from *pesantren* [45]). In this case, a student's commitment to his teacher in *pesantren* culture is to maintain obedience and loyalty to his teacher so that he is blessed with his knowledge [46]. They believe even a little knowledge will benefit many people when it is accompanied with blessing. However, this formality and loyalty in certain contexts are often so excessive that students can lose their critical power [47], [48]. Students may be afraid of being different from their teachers, including in doing subject assignments such as mathematics.

### 3.2. The tendency of the type of reasoning students uses in completing math tasks

As much as 60% of the tasks were completed correctly. This 60% includes 24% of the tasks of the LCR type, and only 5% of the tasks of the GCR type, while 31% of the tasks do not show LCR or GCR types but can be completed correctly. To further explore the students' tendency of the type of reasoning, an analysis was carried out for each sub-task. Table 4 presents all the tasks for LCR and GCR reasoning types that the students can complete correctly.

Of the three tasks, the second most successful sub-task was completed by students. As many as 87% of students successfully completed this task well. This can be because to answer the second sub-task the students do not fully require new methods. Although this type of task has never been encountered before, most students can understand the information which was well presented. They can also relate it to the methods they are already familiar with. According to previous study [49], assignments like this fall into the category of repetition even though they are not visible. This repetitive behavior eventually forms the students' habit. Reasoning that is done repeatedly will lead to the formation of their thinking schemes. A student stated, *"The part that has not been taught is to look for the jumping pattern. For this part, I used the arithmetic formulas previously taught by the teacher."* This way is also confirmed by other students.

Table 4. The correct answer by students for each type of reasoning (LCR and GCR)

| No. Task | Sub task | The correct answer | Type of creative reasoning LCR | GCR |
|---|---|---|---|---|
| 1. | Finding the next term in sequence of 3 presented | 74% (69/93) | 3% (3/93) | 1% (1/93) |
| 2. | Determining a certain term from a mathematical problem presented in a story | 87% (81/93) | 57% (53/93) | 11% (10/93) |
| 3. | Determining the $n^{th}$ pattern in a configuration of object presented | 18% (17/93) | 10% (9/93) | 4% (4/93) |
| | Average | 60% (56/93) | 24% (22/93) | 5% (5/93) |

While the second subtask can be well completed by the students, the first subtask ranks second as the task that the students successfully completed. There are 74% of students who completed this task well. Most likely possible cause of such failure to complete this task is their inability to correctly identify the type of sequence of numbers. Some students used a method that was not in accordance with the number sequence presented. No students failed to complete this task due to errors in doing calculations, although sometimes it was a bit complicated. Of the 24 incorrect answers, 14 students grouped the numbers they faced into arithmetic sequences or assumed that the difference value was always one.

The third sub-task becomes a burden for the students. Only a small number of the students (18%) successfully completed this task. This can be because of the lack of concepts mastery and understanding of information and assignment questions. This can greatly affect the way students complete the task. The failure of students to complete this type of task can be caused by the students' lack of understanding of concepts to determine the $n^{th}$ pattern. This form of assignment seems to have never been encountered by students so that they did not know how to complete it. A student said, *"because previously the teacher had never given such an assignment. So, at first, I was confused."* The confusion is reflected in the improper application of the method. The students tend to do calculations until a certain term is obtained from the object configuration presented instead of looking for the $n^{th}$ pattern.

Analyzed from the elements of reasoning (mathematics foundation, plausibility, novelty, and flexibility), it can be highlighted that the students' failure to complete mathematical tasks can be mainly because of the novelty element. The empirical data show that 69% of the answers do not present new things. This was mostly found in the first sub-task (32%), then the third sub-task (27%), and finally the second sub-task (10%). This failure is due to the students' difficulty in developing their creativity or they tend to use methods that have been used frequently. Only a small number of students use unique or uncommon methods for their level. A student recalled, *"In my completion, there is nothing new because the formula I use has been taught by the teacher."*

In terms of the flexibility element, students also seem to have significant constraints. This can be seen from the methods the students use in which as many as 53% of the answers were not completed with the use of alternative methods. This is because the students already know the exact way to complete the task. Another possible cause is that students do not fully understand the task so they do not develop alternative methods as reflected by a student, *"I cannot complete the task other than using the Fibonacci formula."*

The students also experienced less constraints when they were asked to fulfill the mathematical foundation elements. The data show that only 40% of the answers are not based on the intrinsic nature of mathematics. This problem was mostly found in the third sub-task (27%), then the first sub-task (9%), and the second sub-task (4%). The mathematical foundation element is blocked because the students develop methods that are not relevant to the information presented by the assignment, and they cannot apply the methods according to the intrinsic nature of mathematics.

The various data and findings of this study suggest that the second type of mathematical task is relevant to the Indonesian context. Given the fact that all schools in Indonesia use the same textbook as their main reference, this finding can be generalized to the majority of Indonesian students. This task contains several correct methods, unfamiliar to students, but still involves the methods the students have learned. Apart from encouraging students to think creatively, mathematical assignments of this type can also improve students' conceptual understanding [49]. This is because students will develop solutions based on the intrinsic nature of mathematics, not only based on the existing methods without understanding the logic of using these methods.

If students are given challenging mathematical tasks similar to this second type and according to their level of thinking ability. According to Simon and Tzur [36], students will develop all their potentials to complete the task. If most of the time students in the class are devoted to completing tasks like this, then the students' opportunities to develop their creative thinking skills will increase [1].

On the other hand, a task that has the characteristics of requiring several alternative correct methods requires efforts and struggles from the teacher to make the students get used to it. Tasks like this can provide challenges and require students to be persistent to complete [5]. Tasks with these characteristics are very important in order to trigger student motivation in finding solutions [16]. Students' resilience in completing this assignment model is needed in developing students' mathematical reasoning. In this context, textbooks containing assignments with these characteristics are indispensable to develop students' creative reasoning, both of LCR and GCR types.

## 4. CONCLUSION

Some important points of this research are highlighted. The study found that general school (SMP) students' answers and the methods used were more varied compared to those of *madrasa* students (MTs). However, it must be admitted that in certain cases, unique answers were found among *madrasa* students. The type of creative reasoning that students tend to use is the LCR type, and the GCR type reasoning is still not familiar among students, so it is still a burden for them. These findings indicate the importance for mathematics teachers to design mathematics tasks that develop GCR-type creative reasoning. Therefore, examples and tasks in mathematics textbooks should also be directed at developing this type of creative reasoning. Considering the findings from the design research reported here, the three mathematical tasks developed and analyzed in this research can serve as a model for other development of mathematical tasks targeting to develop students' LCR and GCR type of mathematical reasoning. Hence, development for mathematical tasks that adopt, adapt or are similar to this model should be further recommended.

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