International Journal of Evaluation and Research in Education (IJERE)
Vol. 11, No. 1, March 2022, pp. 182~192
ISSN: 2252-8822, DOI: 10.11591/ijere.v11i1.21171

Critical thinking skills: Error identifications on students' with APOS theory

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Article Info

Article history:

Received Feb 11, 2021
Revised Dec 2, 2021
Accepted Dec 16, 2021

Keywords:

APOS theory
Critical thinking skill
Error identifications

ABSTRACT

Identifying students' errors in solving cases of critical thinking skills from two variables of linear equation (TVLE). This was a qualitative study using a descriptive exploratory approach. The participants of the study were first-year students of mathematics education. The method used in this research is a test, interview, and triangulation. The findings showed that the students have low critical thinking skills; therefore, they could not complete the task correctly. Based on the Action-Process-Object-Schema (APOS) theory, students' mistakes in completing math problems consist of four elements, namely: i) The errors in interpreting; ii) The errors in understanding the concept; iii) The error in the procedures; and iv) The error in technical things. The student's response in this study was in the theoretical of APOS, so that they could not reach a correct schema. The study results are expected to be beneficial in developing the activities in teaching TVLE, so that the students will make less errors in completing critical thinking skills problems in mathematics. Therefore, further study in developing a teaching model for mathematics teaching to improve students' critical thinking skills is highly recommended.

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1. INTRODUCTION

Critical thinking skills become fundamental in the 21st century, that educators teachers begin to develop these skills [1], [2]. This situation is mainly because critical thinking skills are an essential prerequisite for continuing higher education [3], [4]. Moreover, society has considered critical thinking skills as the most augmented intellectual activity occupied in decision making [5]. The study has shown that someone can generate self-motivated knowledge in civilization if one has a critical thinking skills [6]. Having good critical thinking skills can lead one to have cognitive competence to reach the desired objective [7]–[9]. Therefore, one of the purposes of the educational system has put critical thinking skills as the one of utmost objective in education [8]. Undoubtedly, everyone needs to develop this skill to make reasonable and reliable decisions to help him cope with life problems [10].

One of the ways to cope with the problems is through critical thinking skills developed in mathematics. This is in line with the statement that developing critical thinking skills has been the primary agenda in the curriculum of mathematics teaching all over the world [2]. This has also been the concern of the Indonesian government to develop critical thinking skills in mathematics, which is an essential

competency in the curriculum for students to be successful in the future [1], [11]. Since joining programme for international student assessment (PISA) in 2000 to 2015, Indonesian students have always been in the low rank for their mathematic achievement [12]. In the Trends in International Mathematics and Science Study (TIMSS) survey, Indonesian students' mathematics achievement is alarming [13]. One contributing factor to the low PISA and TIMSS results is that students are not accustomed to solving mathematics problems that demand critical thinking skills [14]. Uzel and Uyangor [15] argued that the critical thinking skills in mathematics must be developed to help students complete their mathematical cases. Therefore, mathematics and critical thinking must not be separated in creating meaningful learning activities [6], [16].

Critical thinking in mathematics has aroused several expertise to set some indicators for this skill. Ennis [17] proposed that one can be categorized as having the critical thinking skills when six indicators of good critical thinking skills has fulfilled, which cover focus, reason, inference, situation, clarity, and overview. Focus means being able to identify problems of decision-making. Reasoning entails being able to provide arguments based on the evidence required for decision making. Inference means being able to correctly infer or conclude based on relevant opinions to support the inference made. Situation entails using all of the information required based on a specific circumstance for the problems being faced.

Clarity entails being able to provide additional explanations for the conclusions reached. Overview means conducting a review of each of the steps in decision making. University students of the mathematics education department have somehow neglected these essential yet crucial indicators. The evidence shows that the critical thinking skills of university students in Indonesia are still not entirely satisfactory [13], [18]. They still have a low degree of support for having critical thinking activities in education [19]. This was caused by several factors, including the students, the instructors, and the learning environment during the teaching and learning process. It was assumed that students who have a shortfall in their critical thinking were due to their lack of understanding of the mathematics concepts. Therefore, they still made numerous errors in completing the mathematical tasks related to their critical thinking skill. It is advised that students must have an excellent conceptual understanding of mathematics. It is proposed that when the students have an incorrect interpretation of their basic mathematics concepts, it will be challenging to develop their critical thinking. The understanding the basic concepts in mathematics are to ease them in expanding their knowledge of mathematics itself.

The reality has revealed that numerous students struggle to have complete the conceptual understanding [20]. Umam *et al.* [21] urged that the students were unsuccessful in identifying the causes of misconception, which leads them to the wrong ways of finding practical solutions to ease the misconception itself. The misconceptions that occur among students are misinterpretation, procedural errors, and technical errors. The mistakes in interpretation (reversal) refer to translating or deducing the student's word choice in prescribing variables into numbers. It tends to be about the object rather than the numbers [22]. Previous studies affirmed that the conceptual (structural) errors show the failure to understand a concept within a problem [23], which also means a failure to value the relationships within a situation [23], [24]. The procedural (arbitrary) errors occurred when a student failed in manipulation or an algorithm, even if the student has understood the concept [23], [25]. The errors are related to incorrect choices of extrapolation [26], [27]. For these reasons, it is essential to check the students' mental process in completing mathematical tasks related to critical thinking skills.

There are plenty of ways to check students' mental processes in critical thinking mathematics, and one of them is by employing the Action-Process-Object-Schema (APOS) theory. Arnon *et al*. claimed that the APOS theory covers four phases, actions, process, object, and schema [28]. In the action phase in the APOS theory, the transformation is carried out by an individual using a mathematical concept in relation to an explicit algorithm and is guided by external stimulation [28]–[30]. The process phase of the APOS theory covers repeated actions and reflections to alter from being dependent on external clues to internal clues [28], [31], [32]. The object phase of the APOS theory relates to one's ability to realize that a particular process and transformation of action are considered one unity to explicitly establish the shift [33]. The schema phase is a set of conception of actions, processes, objects, and schemas that previously established, synthesized to form a mathematics structure for solving mathematics problems [28], [34]. The APOS theory discovers what happens in a person's mind when they are about to learn a mathematical concept, as well as the success and failure they encounter when completing mathematical tasks.

Several experts have shown some studies related to critical thinking skills. Yasir and Alnoori have concluded that teachers were unaware of the prerequisites necessary for developing the students' critical thinking skills [35]. Meanwhile, Aktaş and Ünlü discovered that the critical thinking skills of the first-semester university students is still in the mid-level scale [36]. It was stated that the factor which has caused the biggest obstacle in developing the critical thinking skills is within the students themselves [37]. Previous study [38] added that students have difficulty developing their critical thinking skills because they face adversity in analyzing possible errors and overcoming mental impairment when confronted with the impairments of answers in geometry tasks. Kumar and James [39] stated that the degrees of critical thinking

skills viewed from the gender and participants or majors of study in the university level have varied indicators. There are five variables related to the views: inference, assumptions, deduction, interpretation, and argumentation. University students possess lower description ability in their critical thinking skills, since most of them did not implement the skills they have during the training [39].

The studies have revealed that the critical thinking skills of students, university students, and teachers have not yet been developed optimally. However, studies on the factors that contribute to their critical thinking skills are still not optimally designed, if at all, and have never been conducted. This has prompted the authors to conduct research in this field to uncover and identify the errors made by university students in the mathematics department when completing assignments involving critical thinking skills for two variables of linear equation (TVLE) utilizing the APOS theory. This research aimed to seek students' errors in completing critical thinking skills using TVLE. The authors chose APOS theory because it can elaborate on how students' mental activities construct mathematical concepts. Furthermore, this initial identification is essential to establish students' critical thinking skills so that appropriate learning models can be designed based on the research findings.

2. RESEARCH METHOD

2.1. Research design

The study took qualitative research using a descriptive exploratory approach. This approach obtains verbal data in student expressions when students complete a written test in a math problem. The uses of open inquiries and investigation is highlighted in this study approach. It provides opportunities for the research participants to respond using their answers [38]. Kumar and James [39] argued that researchers use a qualitative approach to understand the function of a study in discussing the problem in question. The issue discussed in this study is students' errors in solving mathematical task using critical thinking skills.

2.2. Participant

All the participants are students in the first semester of the mathematics education department from a private university in Indonesia. The participants were selected using the purposive sampling technique because not all the students were chosen as participants. There were 40 students who had learned the material system of two-variable linear equations. They are voluntarily requested to solve six mathematical problems that can explore their critical thinking skills in the allotted time. All the answers sheets were collected and checked based on critical thinking indicators. Students' who answered excellent were not chosen as participants. On the other hand, students who answered sheet contains an error in solving problems involving TVLE were considered as participant. Participants were strongly encouraged to have good communication skill, so that the data responding to critical thinking skills explored and discovered easily.

2.3. Research instruments

The research instrument employed in this study includes a test and interviews. The primary purpose of the test was to discover various mistakes made by students. The given test contained six problems that could be used to assess the students' mathematical critical thinking skills. All the problems made in daily stories form. Hopefully, students were encouraged to find out the critical information before they respond to the problems. The entire test had fulfilled six critical thinking indicators [17], which covered focus, reason, inference, situation, clarity, and overview.

The goal of the interview was to elicit additional information from the students' written sheet. The interview format was an unstructured interview, in which the researcher did not plan the questions to be asked to the participant. The interview process will be conducted in a comfortable manner to encourage communication without pressure. Hopefully, students revealed the reasons related to their answers. Interview questions are based on the answer sheets of students who are being interviewed. The researchers confirmed the students were interviewed about the test that had been done.

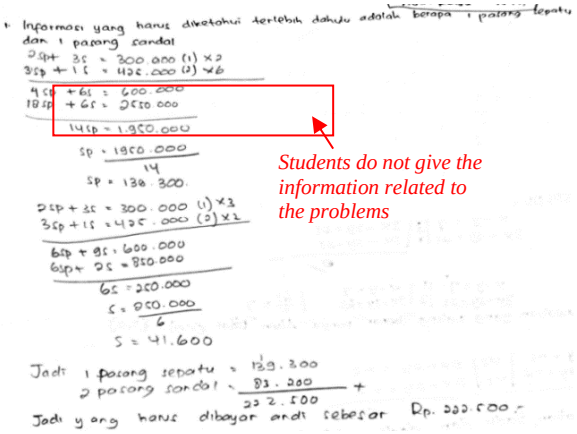
2.4. Data analysis

Initially, 40 university students were assigned to take a test consisting of six essay-style questions about mathematical critical thinking skills. They were given 60 minutes to complete the test and submitted the answer sheets once the time was over. The next step was that the researcher selected one of the students' answers based on the assumptions of the errors the student had made. This was discussed further in the discussion section of this study. After the research participants were chosen, an interview was held to confirm additional responses to the students' written responses. The interview took place for 45 to 60 minutes. The interview was recorded and transcribed. The interview for this study was conducted using the APOS theory to determine the errors made by the students while completing the task of critical thinking skills in

mathematics cases. The researcher then analyzed the students' works independently. The next step was to triangulate the data by comparing the written test results on the critical thinking skills of mathematics cases to the development of the interview the researcher has had with the student. Afterwards, the researchers reached a conclusion from the triangulation results.

3. RESULTS AND DISCUSSION

Figure 1 shows the test results and interviews on the student's critical thinking skills were presented based on Ennis' six indicators of critical thinking skills [17]. Figure 1(a) shows the student's answer in Indonesian and Figure 1(b) shows student's answer in English. The test results showed that in completing the task, the student had completed the whole task, which means that the students did not comprehend the task that they were working on. Because of the student's incomprehension, the signs of critical thinking abilities were not understood. It was assumed to be the reason why the student could not comprehend the questions given in the task. As a result, an interview was conducted in order to determine the student's faults in accomplishing the critical thinking mission in mathematical instances utilizing APOS theory.



(a)

The information to know in advance is how many pairs of 1 shoe and 1 pair of slippers

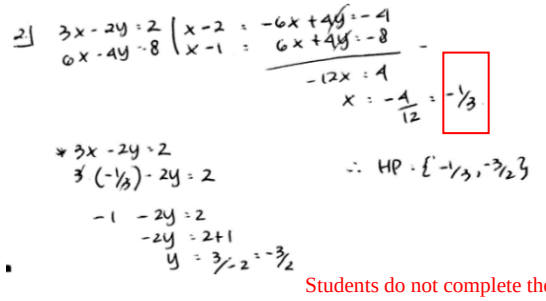
$2sp + 3s = 300.000\ (1) \times 2$
$3sp + s = 425.000\ (2) \times 6$
$4sp + 6s = 600.000$
$18sp + 6s = 2.550.000$
$14sp = 1.950.000$
$sp = \frac{1.950.000}{14}$
$sp = 139.300$
$2sp + 3s = 300.000\ (1) \times 3$
$3sp + s = 425.000\ (2) \times 2$
$6sp + 9s = 600.000$
$6sp + 2s = 850.000$
$6s = 250.000$
$s = \frac{250.000}{6}$
$s = 41.600$

So, 1 pair of shoes=139,300 and 2 pair of slippers=83,200
To be paid is Rp 222,500

(b)

Figure 1. Student's answer on indicator focuses in (a) Indonesian and (b) English

The interview results show that the student was distracted by the questions to finish the answer of a particular question ultimately. Figure 2 shows that this has caused them to have incorrect interpretations of that specific question. Figure 2(a) shows the student's answer in Indonesian and Figure 2(b) shows student's answer in English. This result demonstrated that the student only completed the task without providing any arguments in response to the questions. A lot of reasons or arguments were needed to make it well and correctly answered. It has shown that the students have failed to fulfil the indicator of critical thinking skills. An assumption was drawn that the student had made errors in identifying the questions of the task. As a result, an interview based on the APOS theory was held to determine the student's mistakes.



(a)

$3x - 2y = 2 \mid \times -2 \mid\ -6x + 4y = -4$
$6x - 4y = 8 \mid \times -1 \mid\ 6x + 4y = -8$
$-12x = 4$
$x = -\frac{4}{12} = -\frac{1}{3}$
$3x - 2y = 2$
$3(-\frac{1}{3}) - 2y = 2$
$-1 - 2y = 2$
$2y = 2 + 1$
$y = -\frac{3}{2}$
So HP $= \{-\frac{1}{3}, -\frac{3}{2}\}$

Students do not complete the solution with explanation in every step

(b)

Figure 2. Student's answer on indicator reason in (a) Indonesian and (b) English

The interview results revealed that the student had made errors in scripting the TVLE presented to interpret the errors. This has led them to make technical errors in calculating the TVLE. Furthermore, when asked about the concept of TVLE, which has no solutions, the student failed to respond, causing him to make conceptual errors. Other than that, when the student was assigned to complete the task using a graph, the student drew the diagram of TVLE, but the student could not manage to determine the solutions of the graph. It was evidence that this particular student has failed to understand the concept of TVLE, which does not have any solutions that can be seen in Figure 3. Figure 3(a) shows the student's answer in Indonesian and Figure 3(b) shows student's answer in English.

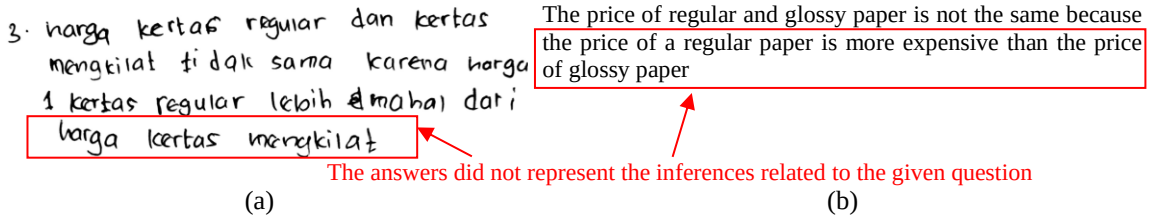


Figure 3. Student's answer on indicator inference in (a) Indonesian and (b) English

The student's responded by only drawing the conclusion and argumentation of the questions in the task without providing any evidence, which led the student to incorrect conclusions and arguments. As a result, the student has failed to meet the indicator of inference in the critical thinking skill. Assuming that the student made mistakes in completing the task, an interview based on the APOS theory was conducted to identify the student's errors in achieving critical thinking skills for the mathematics cases.

According to the interview results, the student did not understand the purpose of the questions and interpreted them incorrectly. Besides, when the student was given a question about the concept of TVLE, which has many solutions, the student could not answer it. It can be assumed that the student has made conceptual errors. Other evidence can also be seen in how the student provides a graph of the TVLE and cannot determine any solutions for the graph the student has completed. Figure 4 reveals that the student did not understand the concepts of TVLE, which has numerous answers. Figure 4(a) shows the student's answer in Indonesian and Figure 4(b) shows student's answer in English.

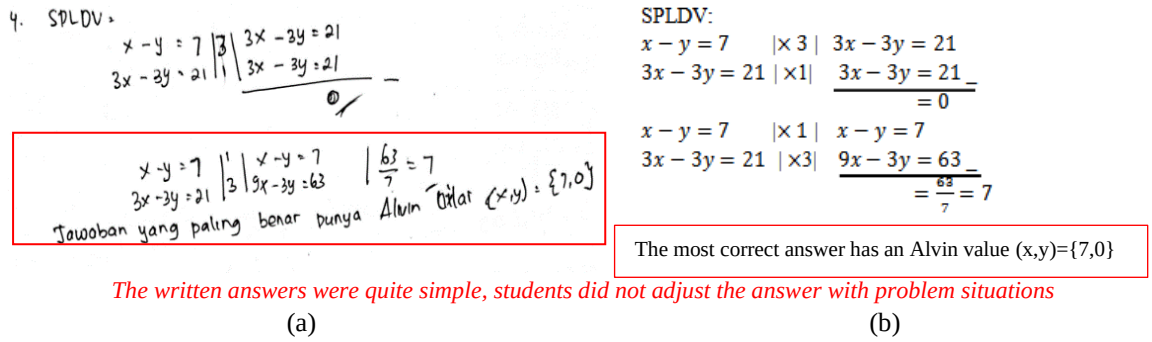
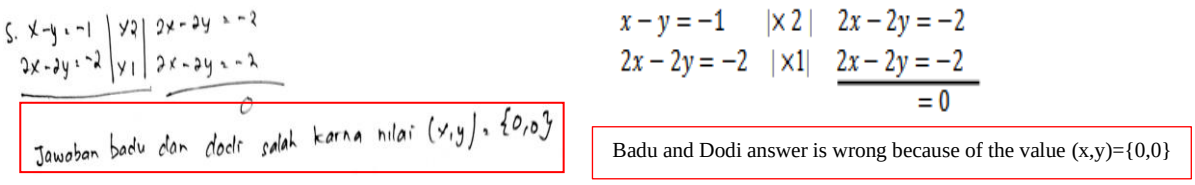


Figure 4. Student's answer on indicator situation in (a) Indonesian and (b) English

These have demonstrated that the student only provided evidence for one particular question in the task and that Alvin's proven to be the correct answer. However, the student has neglected the situation in the question, not to provide the exact arguments or specific justification why Alvin's was the correct answer. This indicated that the student could not fulfil the indicator of critical thinking skills and was assumed that they had made errors in completing the task. To detect the student's error in completing the task on critical thinking abilities in mathematics on the TVLE, an interview based on the APOS theory was conducted.

According to the interview results, the student had committed procedural errors. These errors occurred only when the student could prove one answer to the given questions so that the answer of Alvin has become the correct answer. In addition, the student has also made technical errors in doing the division

and calculation. As can be seen in Figure 5, the results presented with questions about types of solutions in mathematics, the student could only answer a small part of it. As a result, this has caused him to make conceptual errors. Figure 5(a) shows the student's answer in Indonesian and Figure 5(b) shows student's answer in English.



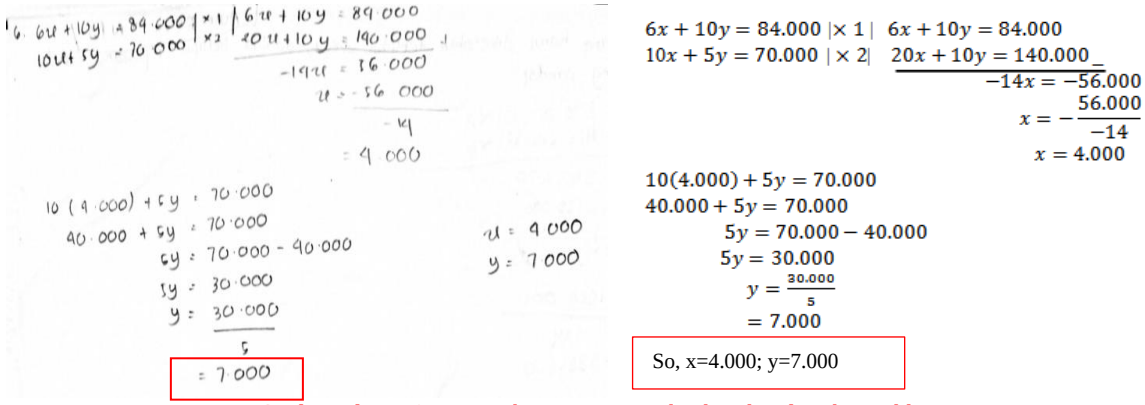
Students did not clarify his answers completely

(a) (b)

Figure 5. Student's answer on indicator clarity in (a) Indonesian and (b) English

These results have proven that Badu and Dodi's answers were based on using elimination. However, the operations done were incorrect and led to assuming that Badu and Dodi's answers were incorrect. Moreover, the students were unable to provide any alternative solutions to the question. As a result, the students have been unable to meet the critical thinking skill's indicator of clarity. Therefore, an interview utilizing the APOS theory was conducted based on the assumption that the students have made errors in completing the task.

According to the interview, the students have made conceptual errors because they assumed that 0 is a solution. Figure 6 shows that the students also did misinterpret in concluding a particular question. Figure 6(a) shows the student's answer in Indonesian and Figure 6(b) shows student's answer in English. When they were asked about the concept, they had used in solving the problem, they answered that they had used elimination.



Students do not interpret the answer completely related to the problems

(a) (b)

Figure 6. Student's answer on indicator overview in (a) Indonesian and (b) English

This response provided evidence that the students were unable to correctly fix the question, resulting in some errors in the steps of completing the problems. It showed that the students could not manage to fulfil the overview indicator in the critical thinking skills. To identify the students' errors in completing the problems, an interview implementing the APOS theory was held to respond to the assumption that the students have made errors in completing the task.

The interview results showed that the students misinterpreted due to the inability of the students to understand the purpose of the questions. Therefore, they made mistakes in concluding their answers. Furthermore, they also made technical errors in calculating that 30,000/5=7,000. This occurred as a result of the students' carelessness in performing the calculations.

The student selected as a participant in the study was those who had low critical thinking skills in mathematics. The particular student could not complete the six questions given in the task according to the six indicators of critical thinking skills. The student's inability to achieve critical thinking in mathematics cases was due to the student's unfamiliarity with this type of task. The student was familiar only with the kind of questions which did not demand high critical thinking skills. As a result, it is recommended that university students become acquainted with tasks that require them to use their critical thinking skills [40], [41].

A precise strategy is required to develop the student's critical thinking skills. One way was using the teaching model, which required the students to be active learners. The material delivery focused on results and the problems or questions that provide challenges for the students. This statement in line with previous research [42], who stated that several ways to develop the critical thinking skills in mathematics in the secondary or tertiary levels of education by implementing a teaching strategy that demanded the students to be active learners, focusing learning on the process on the goals, and having assessment technique which provides students with intellectual challenges rather than memorization. Besides that, the students must also be supplied with metacognitive guidance to improve their critical thinking skills. Metacognitive guidance in the learning environment can effectively develop students' investigative and critical thinking skills [43], [44]. In addition, Lee added that providing direct feedback to the students in the form of guidance enables them to discover their answers and to learn more effectively [45].

The low critical thinking skills of first semester university students have resulted in errors in completing tasks related to the essential skills of thinking about mathematics [46]. Students' common mistakes in performing critical thinking skills in mathematics can be categorized into four errors which are the errors of interpretation, the errors of conception, the errors of procedure, and the errors of technical things. The mistake in interpretation found in the students was that the students failed to understand the questions or problems in a particular mathematics task. They were was unable to interpret the inquiry into the mathematics notions. One example can be seen in how the student transcript TVLE. They misinterpreted that 8/0 has the solution of (0,0), whereas in fact, 8/0 means there is no resolution. On another occasion, the students were failed to reach the necessary conclusion for the task. Consequently, they made mistakes in providing the correct answer to the question. The errors in clarification occurred when the student had an incomplete understanding of a particular concept. As stated by Dündar, the students failed to define or interpret two-dimensional figures due to the incomplete understanding they had of the topic [46].

The conceptual errors made by the students include the student's inability to respond to the questions about the indicators of TVLE, which does not have any solutions, and of the TVLE, which has many answers. In addition, the students were also unable to determine the solution using a graph. This was in line with previous study [47], which revealed that when a person could not understand diagrams and graphs, they could not solve it. The graph displayed the relevant mathematics information in the desired format. Trigueros and Martínez-Planellhave added that understanding two variables graph is difficult for university students when it comes to the schema structure of how the students completed a particular task [48]. Therefore, the students must understand how to draw a graph of TVLE and interpret it in the form of a distinctive representation to avoid making conceptual errors.

Students are encouraged to seek out various forms of representation that will help them gain a better understanding and find more solutions to the problems they face [46]. They also made conceptual errors regarding types of unions in mathematics because they have mistaken in mentioning the unions' members or elements. This was the evidence that the students have low understanding of mathematical concept. It was supported by Brijlall and Ndlovu [29], which showed when the students were completing particular problems in mathematics, their understanding of the concept inadequate. It may cause them to make erroneous assumptions. In line with their findings, Anne and Mangulabnan [49] stated that it would be difficult for mathematics students to learn a particular topic of mathematics without understanding the concepts related to it. Therefore, it will lead them to make errors in completing a task. The mistakes of conception occurred when in the learning of the previous stages of education. They were unable to understand the concepts in TVLE since they were only memorizing the formulas of TVLE without internalizing the concepts. The students' errors are made from the previous mathematics lesson they had before [50]. They were only doing memorization of a particular formula without understanding how to use the procedure itself. As a result, students have difficulty developing a correct and complete understanding of a concept [8].

Some procedural errors the students made can be found in how they neglected the notions of discussion in a particular mathematics problem Even if the students' operations were correct, they were incorrect in concluding the result given to that specific problem. Besides, they were also mistaken in mentioning the elements within a set of numbers covered in mathematics; they only said the sets for the integer group or whole numbers. Another error the students made was that they were mistaken in doing the sets of operations in completing task even if they have made a correct conclusion. Therefore, it is essential to employ the whole concept necessary for completing a lesson not to be mistaken in concluding. The students

must not only use symbols or notations but also be able to give correct mathematics arguments, conclude, and make generalization to complete a particular mathematics problem [51].

In terms of technical errors, the students made mistakes during the calculation process. For example, one found in the division operation of whole number (integer) into a fraction has caused them to be mistaken in determining the values of x and y. Most Saudi Arabian students have a common misconception about fractions and the mathematics calculation that involved the fractions [52]. The errors in the division of whole numbers (integer) into fractions occurred because the students were not careful in conducting the operation. Therefore, technical errors took place. Research found that frequent mistakes made by a learner in the division operation of the whole numbers (integer) were due to his carelessness in employing the procedure itself [53]. Therefore, students must possess competent basic problem-solving skills in mathematics [21], [51], [54], [55]. It is necessary to prevent the reoccurrence of similar errors from taking place.

According to the findings of the analysis, the students were still in the phase of action in the APOS theory. This occurred because they lacked mental structure of APOS, causing them to struggle with mathematics problems. The students had difficulties applying the essential operations, which were caused by a lack of mental structure related to process, object, and schema phases [56]. This has proven that the students could not manipulate the application of theorems and formulas needed for a particular mathematics problem. According to the findings of the APOS theory analysis, the students were having a limitation of understanding the system of concept related to TVLE as they enter the university level. In the secondary class, they only learned about TVLE with one single solution and its graph. They did not know about TVLE with multiple answers and its chart yet. The students' inability was the cause of their errors in internalizing the information they received from the TVLE. They lacked experience dealing with TVLE problems, did not fully comprehend TVLE materials, and had a low amount of prior knowledge related to TVLE in their previous level. For these reasons, the instructors or lecturers in a university need to have a precise strategy to uncover the students' visions related to the participants being taught. Alfrey suggested one of the strategies to deal with this phenomenon [57]. He indicated that to ease the students' frequent errors, an open discussion needs to be conducted, which focuses on overcoming the students' mistakes during the learning process.

Further stated that assessing and analyzing the misunderstanding of the homework was regarded as an essential technique to eliciting any information about the students' ways of thinking. In addition, suggested that to overcome the cognitive handicaps in completing the tasks assigned in the teaching of mathematics, students must exert sufficient mental effort and have optimal practices of thinking activities (thinking operations) that activate prior knowledge and prior experiences in solving the particular problem. The instructor's practices of thinking activities must be varied based on the specific student's level, especially when it comes to critical thinking skills, where there are differences between students in different grades.

These results indicate that instructors or lecturers, especially the mathematics department lecturers who teach the first semester students, must train students in the procedural and conceptual aspect of mathematics within the lectures. These two aspects are equally important in understanding materials in mathematics [58]. These two aspects cover TVLE of one or multiple solutions with its graphs to avoid errors. The errors identification on the students' ability in completing the critical thinking skills on the cases of TVLE in mathematics problems has assisted lecturers or instructors in recognizing the weaknesses of the students they teach. This has aided them in planning their teaching activities and working to fix the errors that occurred. The mistakes the students made were due to their low critical thinking skills in completing the mathematics tasks. According to Stapleton [59], a student who has a low conception of necessary thinking skills is likely to think incorrectly about critical thinking itself.

Consequently, the students' ability concerning their critical thinking skills needs to be improved to prevent similar errors from occurring in the future. Once the students' critical thinking has been enhanced, they will discover formulas or rules they need to complete essential thinking problems in mathematics. Research has revealed that improving students' critical thinking skills will possibly allow them to notice all the aspects necessary for helping them complete the tasks of mathematics problems which enable them to discover formulas or rules [60].

4. CONCLUSION

The study concluded that the participants selected for the study had low critical thinking skills in mathematics. Their incapability to fulfill the indicators of focus, reason, inference, situation, clarity, and overview was proven. The reason behind his incapacity was that the students made errors in completing the critical thinking cases in mathematics. Related to the APOS theory, the errors the students have made covered: i) The errors of interpretation: they were incorrect in transforming the questions into mathematics notions, and also mistaken in drawing conclusion required for the answer of the question; ii) The conceptual errors: they made errors in completing the answer about the criteria of TVLE which has one solution and TVLE with multiple solutions. They made errors in determining the solutions using graphs and also in the

mathematics concept about number; iii) The procedural errors: the students incorrectly concluded the problem. This was because they did not regard the scope of the discussion and were mistaken in doing the steps of completing a task; iv) Technical errors: the students made mistakes in the calculation which involved operations of whole number (integer) division in the form of a fraction.

Future research should develop a new teaching model to combine APOS theory and support students' critical thinking. Based on this research, generating a new teaching model to improve students' critical thinking was fundamental. The facts show us that students with critical thinking would quickly adapt when facing a new mathematical problem. On the other hand, students who do not have vital consideration will face many issues solving a problem. The upcoming research should also identify the thinking process of students during solving the problems. Identifying students thinking process will allow us to gain better understanding related on how students choose a particular way than another. When teachers acknowledge students' thinking process, they will be easier to distinguish the students' early difficulties in solving mathematical problems. Consequently, teachers can adapt the teaching delivery and materials for the next meeting.

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